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Embedded Systems and Industrial Controller EE5563 (7)

Topics

- Computers in Control Systems
- Digital Control
- The z-Transform
- Discrete Time Systems

- autopilots for aeroplanes
- satellite altitude control
- industrial and process control
- Robotics and automation
- navigational systems and radar
- energy management and control in buildings and manufacturing

Advantages of computer control are:

- inherent reliability.
- ability to control many loops simultaneously.
- flexibility of control (i.e. control algorithm can be easily modified).
- increasing cost effectiveness of implementation.
- “intelligent” or “smart” control.

Applications

» *real-time* computer control first proposed in 1950 [Brown & Campbell, Mechanical Eng., 72: 124 (1950)]

» 1954 Digitrac digital computer

» *closed-loop control*: Texaco oil refinery, 15 March 1959.

» *supervisory control*: steady state optimizations to determine set points

» *direct digital control*: ICI plant at Fleetwood, UK, using Ferranti Argus 200

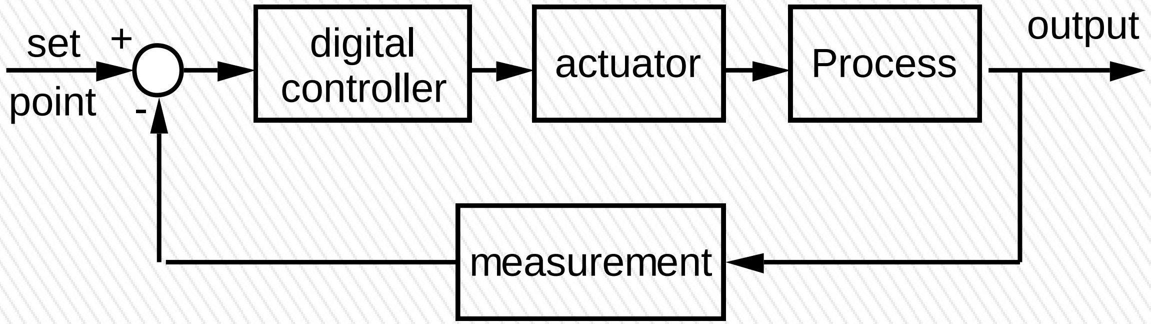
- provision for 120 control loops, 256 measurements

» 1974 microprocessor introduced

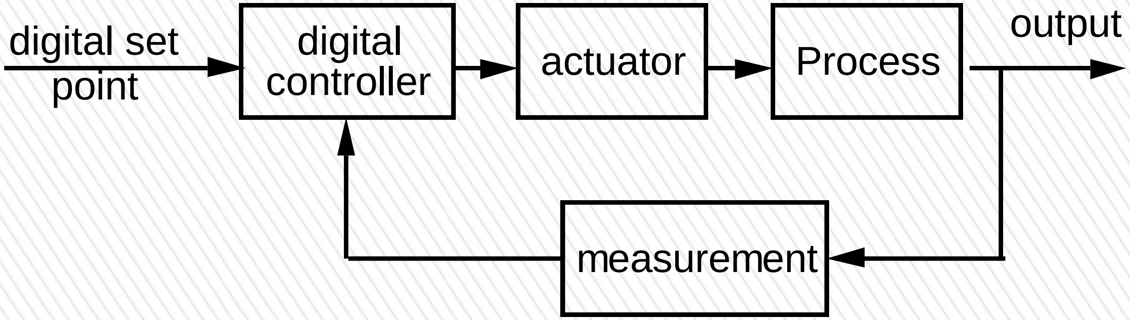
- *distributed computer control systems*
- microcontrollers (8051, 6811, PIC etc.)

Historical Development

Typical Digital Control System

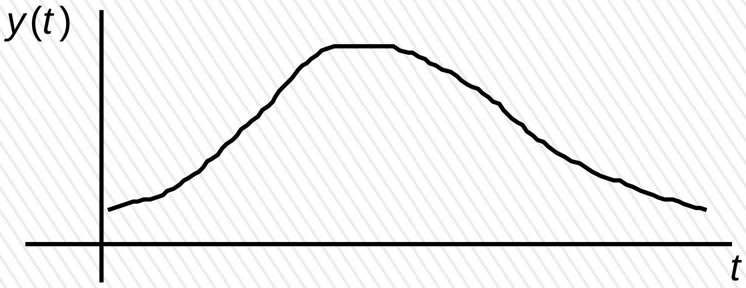


Or



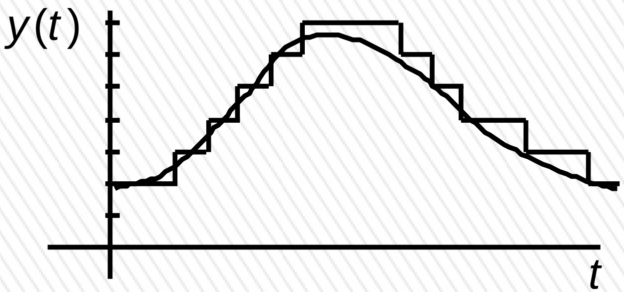
If digital control is used, it cannot be analysed as if it were a continuous control loop.

Digital Control System

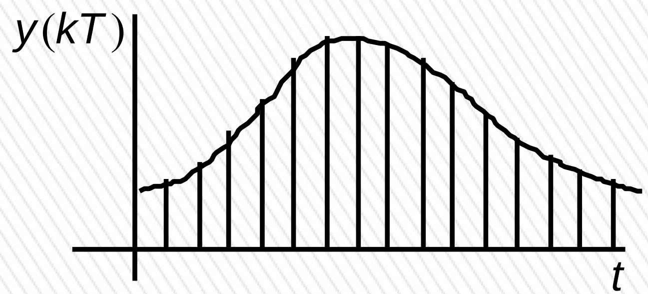


(a) continuous time analog signal

However time and/or the value of the signal may be quantized.

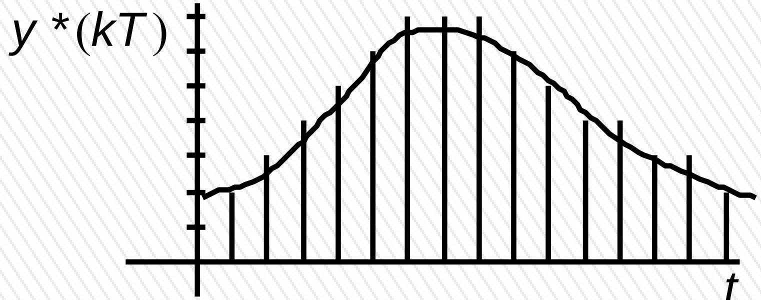


(b) continuous time quantized signal



(c) sampled data signal

Continuous vs Discrete Time Signals



(d) digital signal

A discrete-time signal is one defined **only** at discrete instants of time (c & d).

» Normally assume

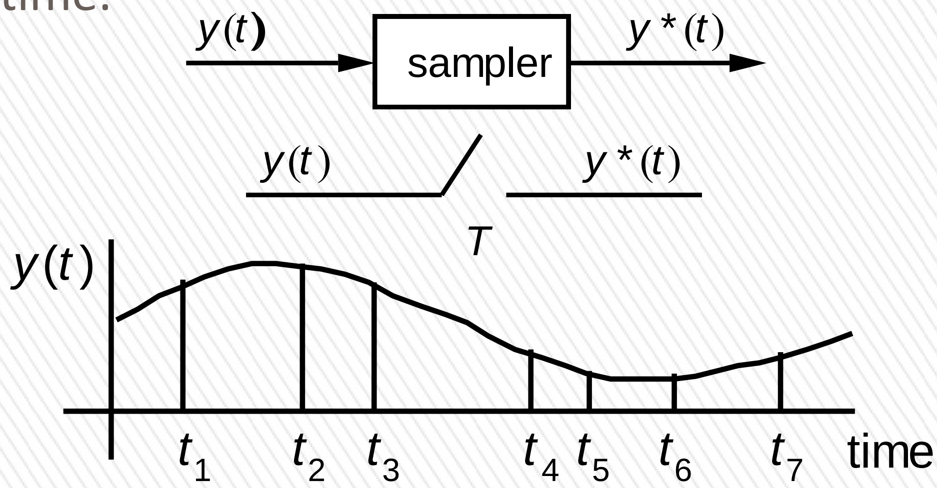
$$y^*(kT) = y(kT) \quad k = 0, 1, 2, 3, \ldots$$

If the amplitude can assume a continuous range of values it is termed a sampled-data signal.

Digital Signal

Digital Computer Interface

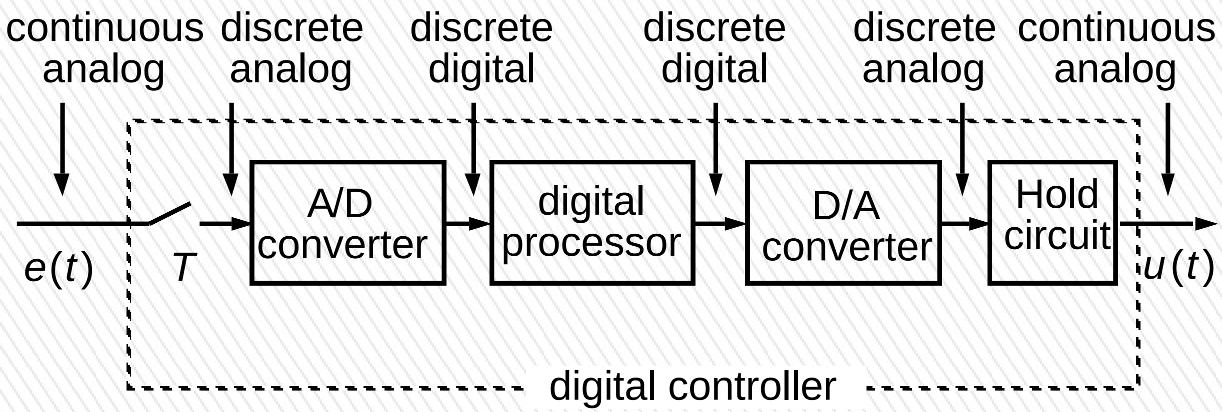
- A sampled-data signal can be generated by sampling an analog signal at discrete instants of time.



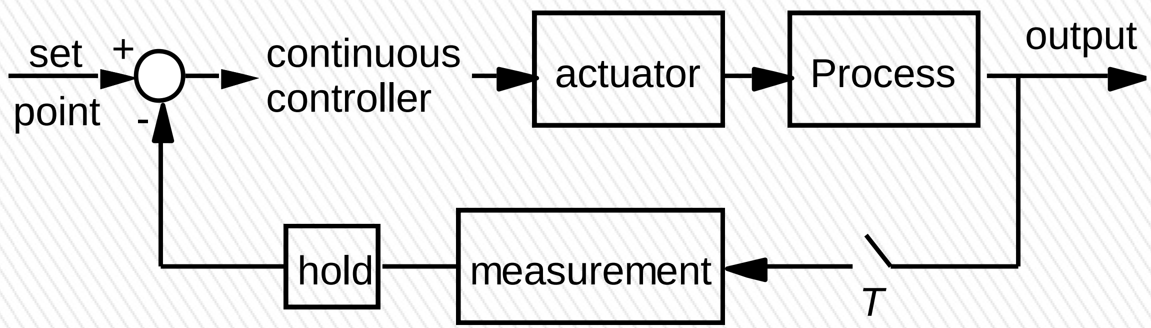
- Note that the samples need not be equally spaced. For now we shall only use periodic sampling i.e. equally spaced (T = constant).
- Loosely speaking, discrete-time control systems, sampled-data control systems and digital control systems imply the same.
- For the theory, these are normally termed discrete-time systems but for realizations in hardware and software they are normally termed digital.

Digital Controller

» Now consider the digital controller in more detail:



» Note: other parts of the loop may be discontinuous with respect to time - e.g. digital measuring device.



- Analog measurements and reference signals need to be sampled before digital processing in controllers

- Digital processing can be used for signal conditioning (DSP chips can function as Digital Controllers)

- Note that analog signals need to be **preconditioned** using analog circuitry before digitising to reduce:
  - the problem of *aliasing distortion* (*aliasing distortion: high frequency components above half the sampling frequency appearing as low frequency components. Happens when data are sampled from an analog (continuous) signal*
  - *leakage* (error due to signal truncation)
  - *noise* reduction

Digital control Components & Interfaces

The drive system of a plant normally takes an analog signal

Therefore the digital output from the controller has to be converted to analog

- Analog to digital converter (ADC, A/D)
  - simultaneous A/D
  - closed loop A/D (counter, successive approximation)

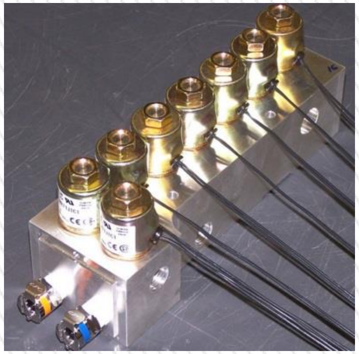
- Digital to analog converter (DAC)

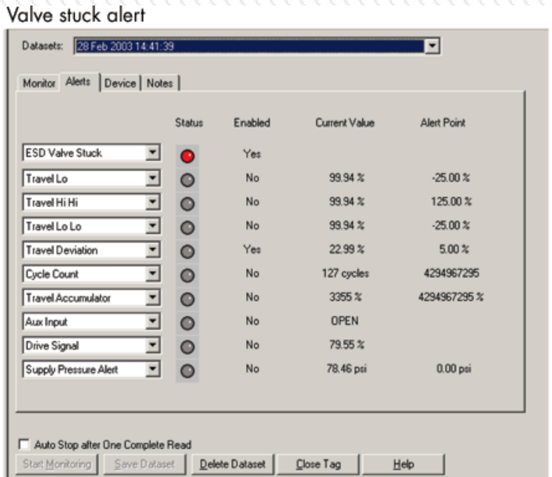
- Hold devices: are analog devices that sample the voltage of a continuously varying analog signal and holds its value at a constant level for a specified minimum period of time (e.g. analog memory devices). - used in analog-to-digital converters to eliminate variations in input signal that can corrupt the conversion process.

- Digital Actuator: for example a stepper motor which responds with incremental motion steps when driven by pulse signals. A two position (position)solenoid (binary state).

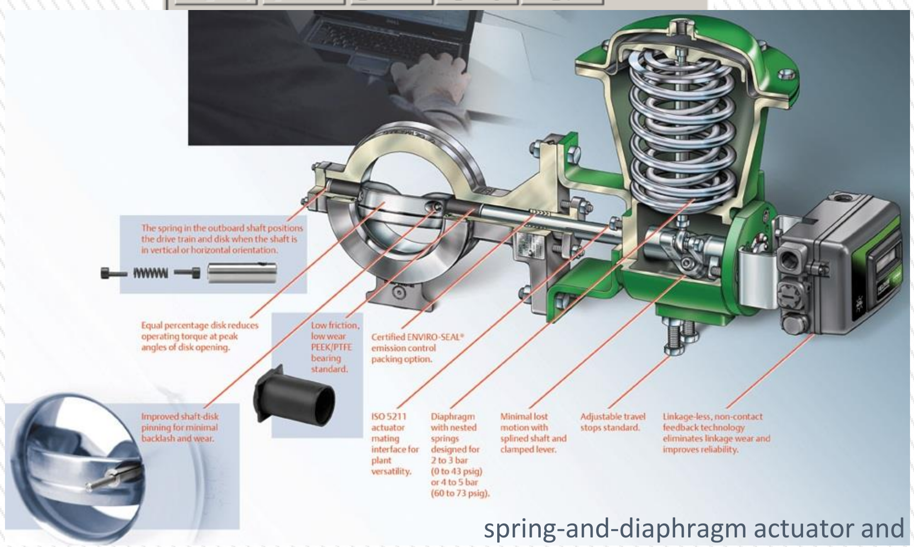
Digital control Components & Interfaces

» Digital control of flow can be achieved using digital control valve





A typical valve would have a number of Orifices each proportioned to the value Of binary word i.e. $2^0, 2^1, \ldots$



spring-and-diaphragm actuator and FIELDVUE Digital Valve Controller
Source Rotary Fischer)

Digital control Components & Interfaces

Digital Control Advantages:

1. Less susceptible to noise (presentable as discrete units)
2. Lends itself better to hardware software interfaces (fast-reliable)
3. Highly Programmable
4. Data can become compact (in case of large scale data handling)
5. Fast data transmission over long distances without excessive dynamic delays compared to analog systems
6. Low operational voltage and cost effectiveness.

Key features of Digital Control

» We need to sample signals in computer control systems.

Nyquest-Shanon Sampling Theorem:

If a signal $y(t)$ (time signal) has no frequencies higher than its bandlimit ω_c and the sampling frequency to be $\omega_s \geq 2\omega_c$ then the continuous signal $y(t)$ can be completely characterised by its sampled discrete signal $y^*(t)$.

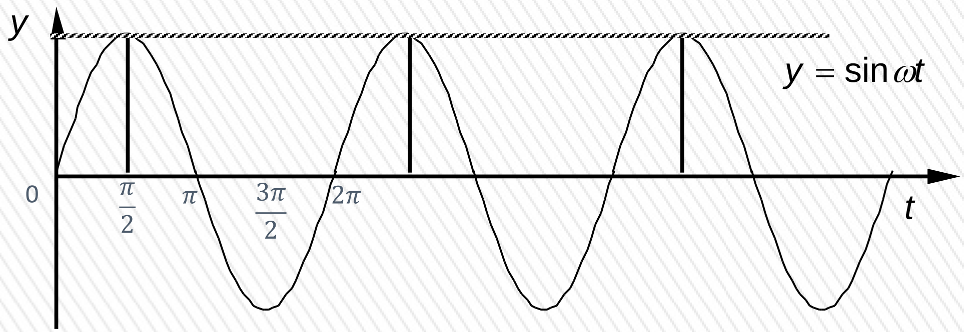
Or: The Nyquest sampling criterion requires that the sampling rate for signal to be at least twice the highest frequency of interest.

Nyquest frequency $\omega_N = 1/2\omega_s$ *of a discrete signal*

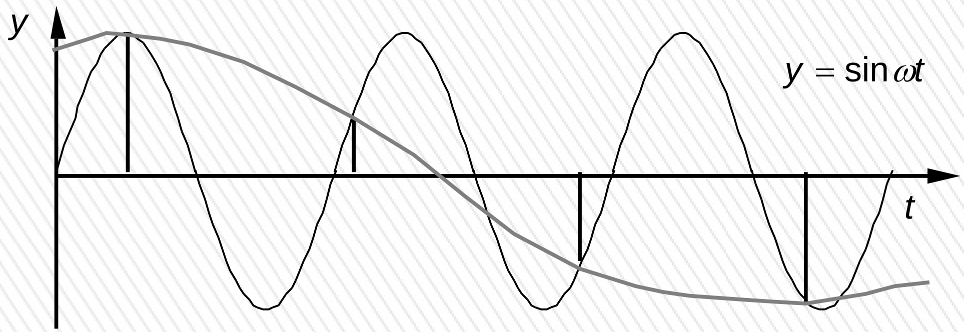
Signal Sampling, Data Presentation & Control Bandwidth

Sampling of a pure sinusoid signal with frequency of ω

$sampling\ interval\ T = \frac{2\pi}{\omega}$ in other words $\omega_s = \omega$

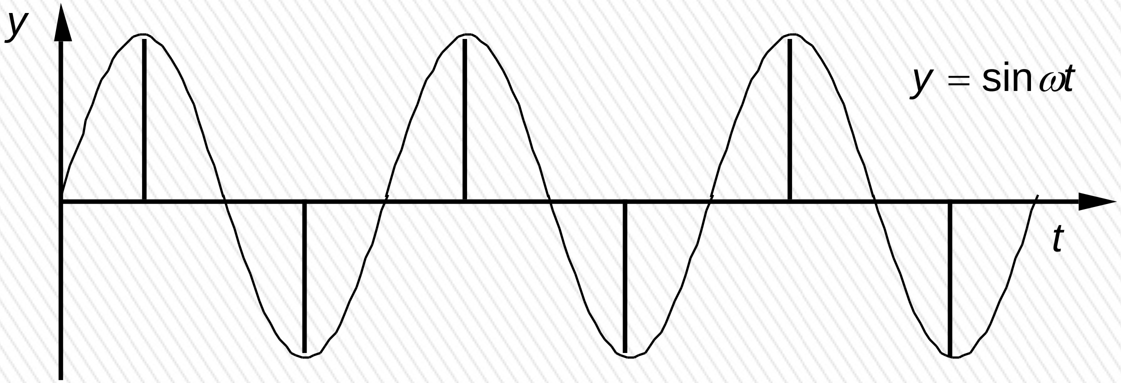


$sampling\ interval\ \frac{\pi}{\omega} < T < \frac{2\pi}{\omega}$ in other words $\omega < \omega_s < 2\omega$

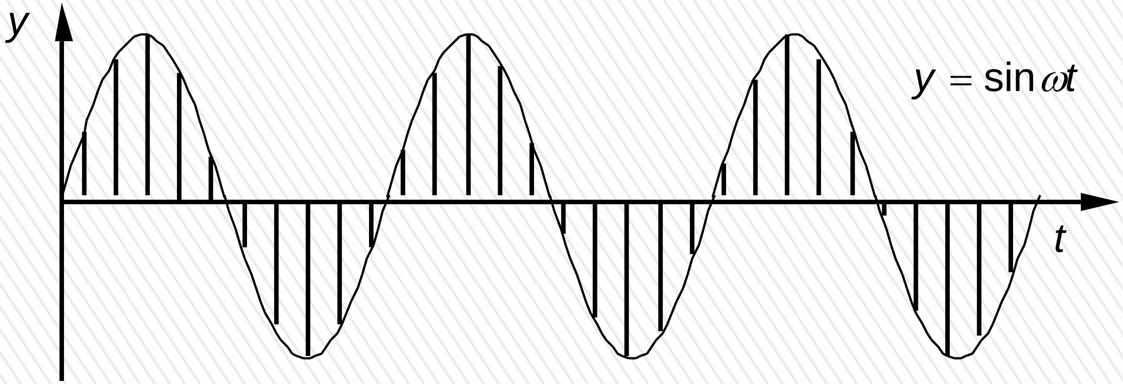


Example

$sampling\ interval\ T = \frac{\pi}{\omega}$ in other words $\omega_s = 2\omega$



$sampling\ interval\ T < \frac{\pi}{\omega}$ in other words $\omega_s > 2\omega$



Example

- The functionality of a digital controller is similar to that of an analog one, the only thing is that the I/O is in digital form.
- The control rules can be expressed by a set of *differential equations.*
- The differential equations relate the discrete outputs to the discrete inputs of the controller.
- The challenge is to formulate the correct form of the difference equation that could produce the required control signal.
- A digital controller can be defined as a number of *discrete transfer functions.* Then those transfer functions can be turned into difference equations.

Digital Control using *z-transform*

- The discrete transfer functions depends on the sampling period T *to* convert analog signals into discrete data.
- The objective of a digital controller is to develop a deference equation that represent the analog compensator, *z*-transform is used.
- When $T \to 0$ the digital control action approaches analog i.e. higher frequency of sampling increases accuracy and lessens aliasing errors.
- Obviously faster rates of sampling imposes computational and communication load on the computer system.

Digital Control using *z-transform*

An infinite sequence of data:

$$\{y_k\} = \{\dots y_{-k}, y_{-k+1}, \dots, y_0, y_1, \dots, y_k, y_{k+1}, \dots\}$$

Can be represented by a polynomial function of complex variable *z:*

$$\therefore Y(z) = \sum_{k=-\infty}^{\infty} y_k z^{-k}$$ bilateral *z*-transform

$$z = Ae^{j\varphi} = A(cos\varphi + jsin\varphi)$$

magnitude phase

Or Unilateral (single sided) *z*-transform: $Y(z) = \sum_{k=0}^{\infty} y(kT) z^{-k}$

as $z = e^{st}$

multiplication by *z* is equivalent to a pure time advance of *T* seconds (i.e. 1 sample).

Or multiplication by z^{-1} is equivalent to a pure time delay of *T* seconds (i.e. 1 sample).

z-transform

Example 1. A unit step function:

$y(t) = 1 \quad for\ t \geq 0$

$\quad\quad = 0 \quad for\ t < 0$ sampling period of *T* and the corresponding data sequence to be $\{y_k\} = \{0{,}0, \dots, 0{,}0{,}1{,}1, \dots, 1{,}1, \dots\}$

Find the *z*-transform of this sequence:

$$Y(z) = \sum_0^{\infty} z^{-k} = \frac{1}{1-z^{-1}} = \frac{z}{1-z}$$

Example 2. find the z-transform of e^{-at} , $z\{e^{-at}\}$

$y(t) = e^{-at} \Rightarrow y(kT) = e^{-akT}$

$$\therefore Y(z) = \sum_{k=0}^{\infty} e^{akT} z^{-k} = \sum_{k=0}^{\infty} (e^{-aT} z^{-1})^k = 1 + e^{-aT} z^{-1} + e^{-2aT} z^{-2} + \cdots$$

$$= \frac{1}{1 - e^{-aT} z^{-1}} = \frac{z}{z - e^{-zT}}$$

Examples

Difference Equation

» Difference equations are discrete-time models.

» The Discrete-time model can be expressed by the *nth order* linear difference equation:

$$a_0 y_k + a_1 y_{k-1} + \cdots + a_n y_{k-n} = b_0 y_k + b_1 y_{k-1} + \cdots + b_m y_{k-m}$$

» Provided that the $\{u_k\}$ is known the $\{y_k\}$ can be computed starting with the first *n* values.

» The initial *n* values can be considered as the IC.

» a_i and b_i depend on sampling period.

Properties of *z-Transform*

$Z\{x_t\} = X(z)$

Linearity : $\mathbf{Z}\ \{ay_1(t) \pm by_2(t)\} = aY_1(z) \pm bY_2(z)$

Time Shift: $\mathbf{Z}\ \{y(t-kT)\} = z^{-k}Y(z)$

Delay of *k* sampling period

Final Value Theorem: $\lim_{k\to\infty} y(kT) = \lim_{z\to 1} (1 - z^{-1})Y(z)$

$Z\{Y(s) = f(e^{Ts})X(s)\} = Y(z) = f(z)X(z)$

In a series expansion the definition of *Y(z)* can be expressed as:

$$Y(z) = y(0) + y(T)z^{-1} + y(2T)z^{-2} + \dots$$

The expanded form of *Y(z)* the coefficients of the equation represent the *y(kT).*

Example: Find *y(kT)* when $Y(z) = \dfrac{0.3z}{(z-1)(z-0.7)}$

- Simplify using partial fraction, then use standard transforms.
- Note: many standard transforms have a zero at *z=0*
- Better to expand $\frac{Y(z)}{z}$ and later multiply both sides of the equation by *z.*
- For no zeros at *z* = 0, use time-shift property

  i.e. expand *Y(z)* as usual

  then let $Y_1(z) = z(Yz)$ and find $y_1(kT)$

  and then $y(kT) = y_1((k-1)T)$

Inverse z-transform

$$Y(z) = \frac{0.3z}{(z-1)(z-0.7)} \qquad y(kT) = ?$$

$$\frac{Y(z)}{z} = \frac{0.3}{(z-1)(z-0.7)} = \frac{1}{z-1} - \frac{1}{z-0.7}$$

$$\therefore \quad Y(z) = \frac{z}{z-1} - \frac{z}{z-0.7}$$

$$\therefore \quad y(kT) = 1 - 0.7^k, \qquad k = 0, 1, 2, 3, \ldots$$

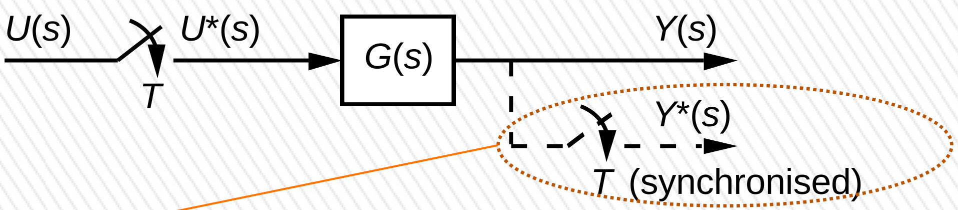
Example Continued

Pulse of *z-transform* functions of a system

» For a continuous-time system:

$$Y(s) = G(s)U(s) \quad \text{or} \quad G(s) = Y(s) \big/ U(s)$$

» If the input is sampled



» Now introduce a fictitious sampler on the output. It can be shown that:

$$Y(s) = G(s)U^*(s)$$

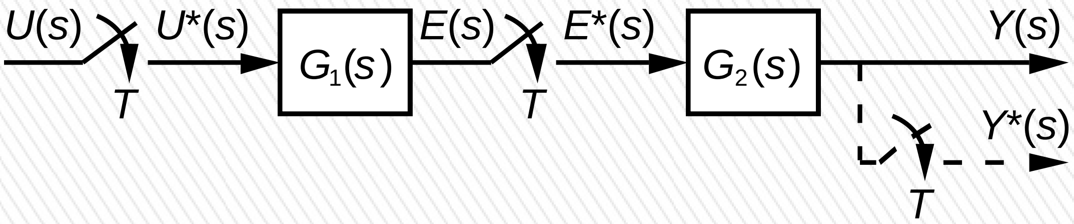
$$\therefore \quad Y^*(s) = \left(G(s)U^*(s)\right)^* = G^*(s)U^*(s)$$

$$Y(z) = G(z)U(z)$$

system impulse response sequence.

$G(z) = \sum_{k=0}^{\infty} g(kT) z^{-k}$ is defined as the pulse or *z*-transfer function of the system.

» Systems with samplers in cascade:



(assume all samplers are synchronised)

From the previous result:

$$E(z) = G_1(z)U(z)$$

And

$$Y(z) = G_2(z)E(z)$$

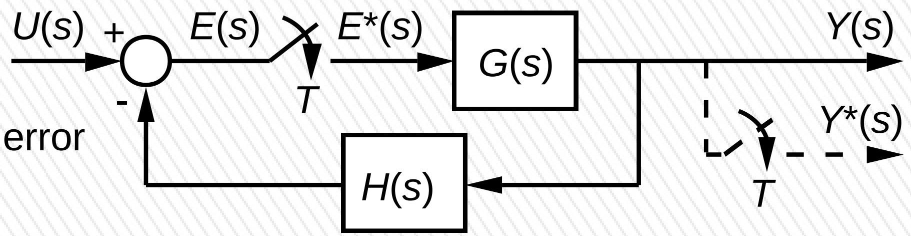
$$\therefore\ Y(z) = G_1(z)G_2(z)U(z)$$

Overall *z*-transform function: $G(z) = G_1(z)G_2(z)$

Block Diagram Analysis

Block Diagram Analysis

» Closed Loop Systems



$$Y(s) = G(s)E^*(s) \quad \text{and} \quad E(s) = U(s) - H(s)Y(s)$$

$$\therefore \quad E(s) = U(s) - H(s)G(s)E^*(s)$$

$$\therefore \quad E^*(s) = U^*(s) - GH^*(s)E^*(s) = \frac{U^*(s)}{1+GH^*(s)}$$

Also we have

$$Y^*(s) = G^*(s)E^*(s) = \frac{G^*(s)U^*(s)}{1+GH^*(s)}$$

$$\therefore \quad Y(z) = \frac{G(z)}{1+GH(z)}U(z)$$

» The equation demonstrating the mapping between complex *z*-plane and complex *s*-plane can be represented as:

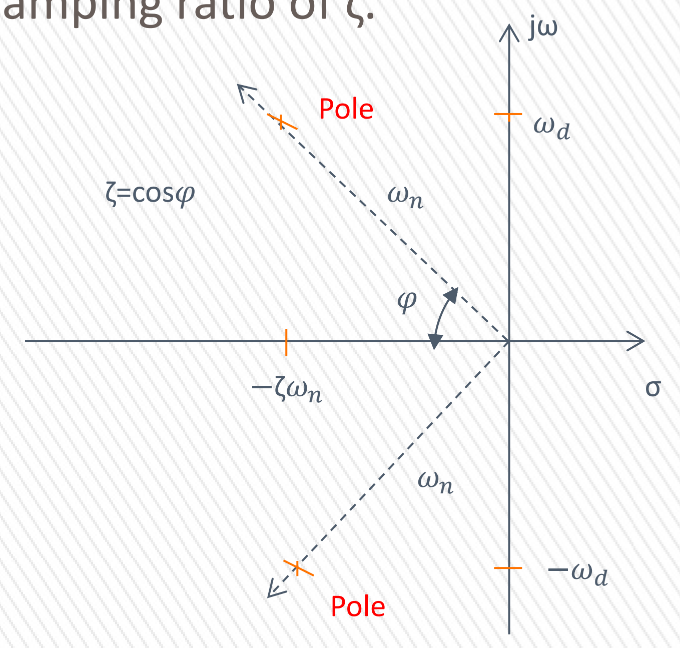
$$z = e^{Ts} \qquad \text{(a)}$$

The nature of this mapping.

Imagine a simple oscillator with undamped natural frequency ω_n and damped natural frequency ω_d and damping ratio of ζ.

The complex poles (*s-plane):*

$$s = -\zeta\omega_n \pm j\omega_d \qquad \text{(b)}$$



Complex conjugate poles on the *s*-plane

s-z Mapping

From equations (a) and (b)

$$\therefore z = e^{T(-\zeta\omega_n \pm j\omega_d)} \qquad \text{(c)}$$

The magnitude of these two poles on the *z*-plane:

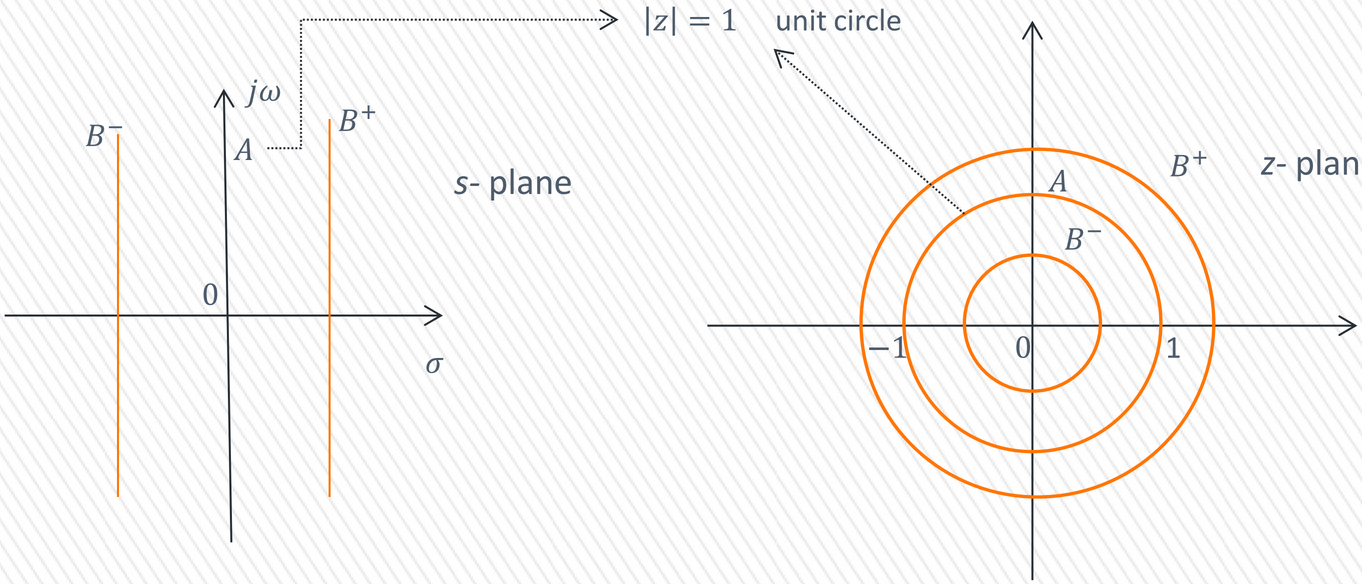
$$|z| = e^{-(T\zeta\omega_n)} \qquad \text{(d)}$$

Phase angle: $\angle z = \pm T\omega_d$

Magnitude and Phase relationships

Three conditions:

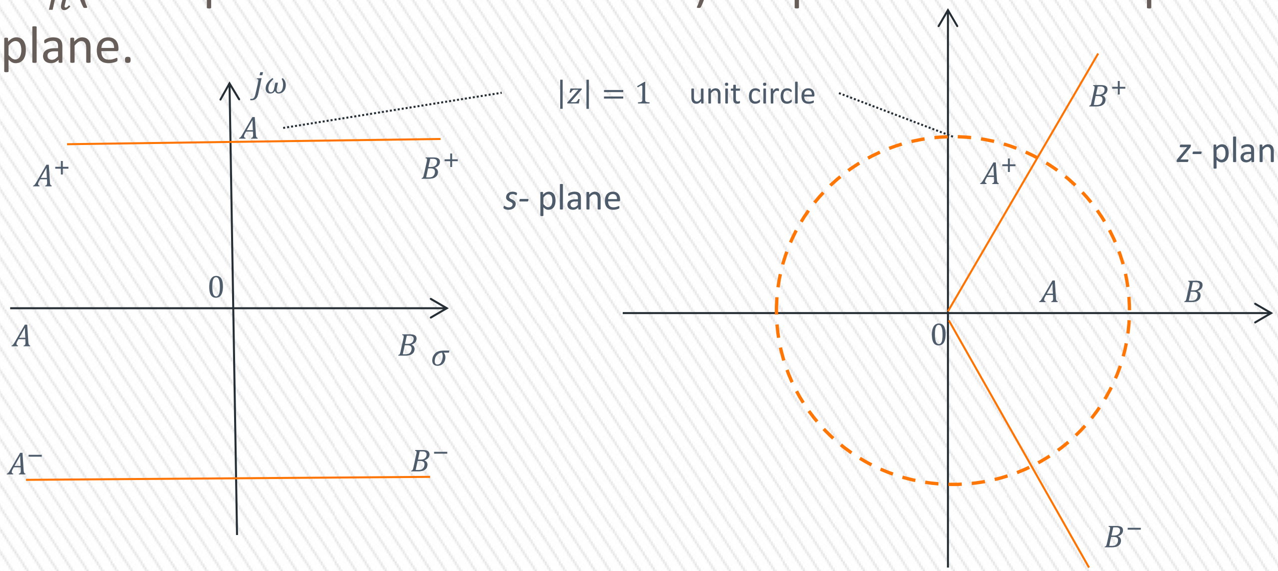
1. **<u>Constant $\zeta\omega_n$ lines:</u>** based on equation (d), when $\zeta\omega_n$ is a constant. Then $|z|$ is constant. Therefore $\zeta\omega_n$ constant lines on the *s*-plane (lines parallel to imaginary vector of the *s*-plane. Maps to circles from the origins of the *z*-plane.



Mapping Magnitude and Phase relationships

Conditions continued:

2. **Constant ω_n lines:** from the angle phase equation ($\angle z = \pm T\omega_d$), constant ω_n (lines parallel to the real axis) map onto constant phase angles on the *z*-plane.



- Note that $\angle z = \pm T\omega_d$ refers to the *many-to-one* relationship.

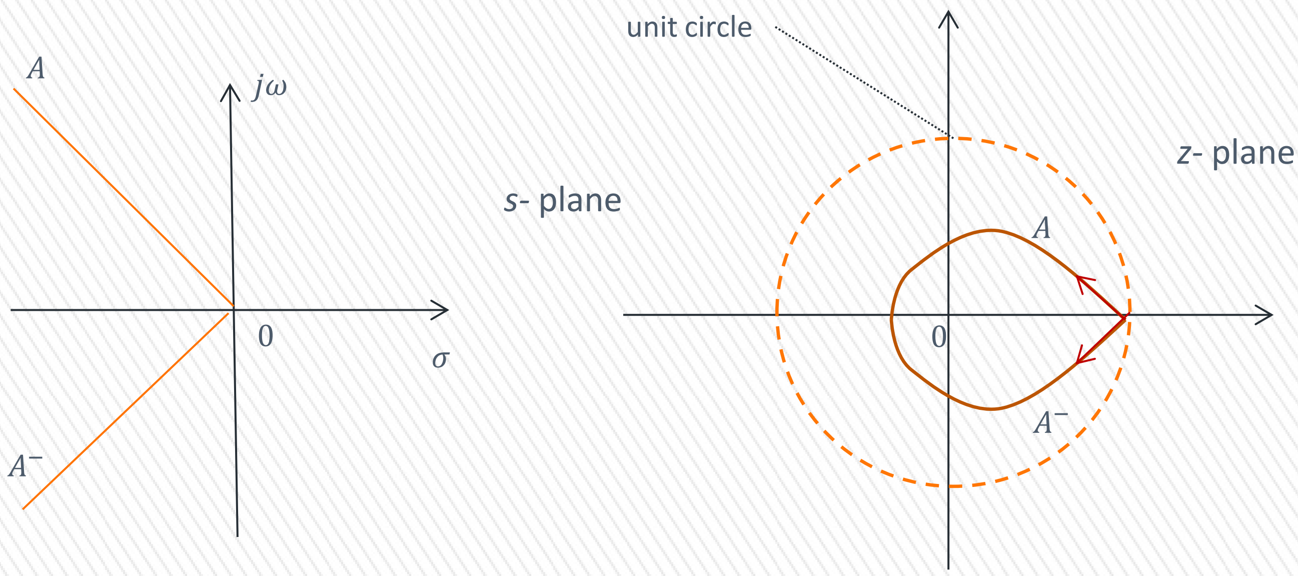
$$T\omega_d = 2\pi r + c$$

- The fact that for any change in the integer value of *r (a* constant *c)* relates to a phase change by 2π in the *z*-plane which returns the line to the original position.

Mapping Magnitude and Phase relationships

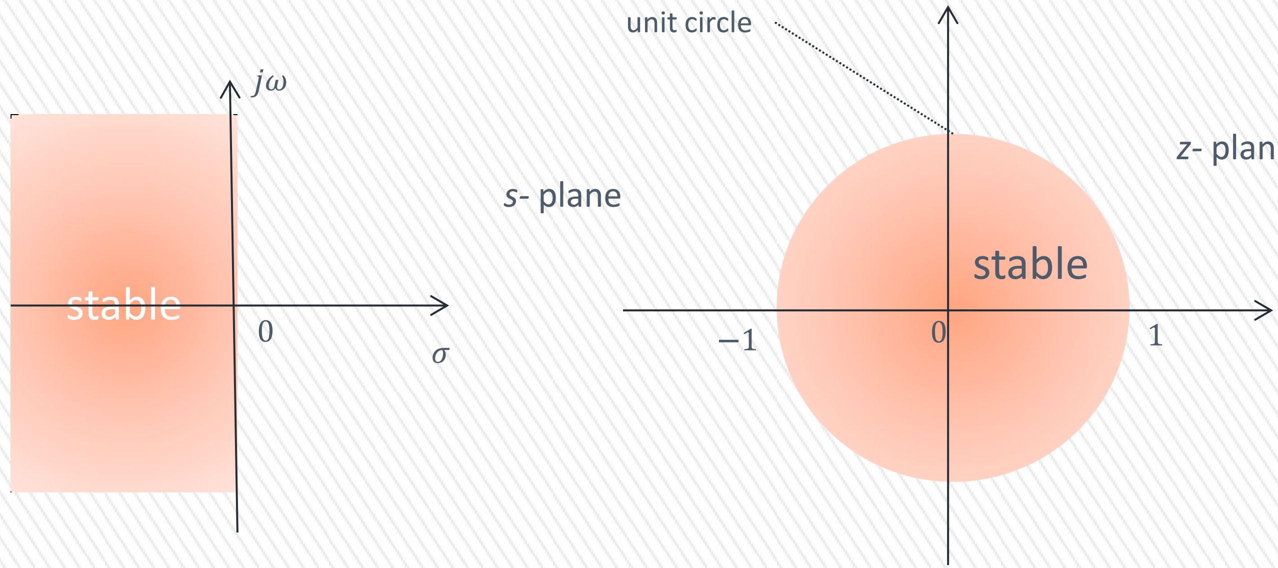
Three conditions:

3. **Constant ζ lines:** constant **ζ** lines on the *s*-plane (straight lines through the origin) map into spiral lines on the *z*-plane



Mapping Magnitude and Phase relationships

» With constant $\zeta\omega_n$ we note that the left hand side of the *s*-plane (representing stability) corresponds to the *unit circle* of the *z*-plane.



Stability of Discrete time systems

» closed-loop stability is the most important consideration of control system design.

» For continuous-time systems, there are several methods to determine stability of a closed-loop system:

- Routh-Hurwitz criterion
- Nyquist criterion
- Bode plot
- root locus plot

(i.e. locations of the CL poles/eigenvalues in the *s*-plane)

» Only root locus can be applied directly in the *z* - plane.

Stability of Discrete time systems contd.

Stability of Discrete time systems contd.

» Recall the discrete signal :

$$Y^*(s) = \sum_{k=0}^{\infty} y(kT)e^{-ksT}$$

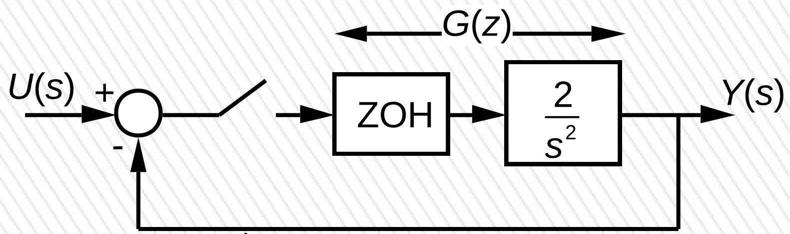
Where $s \rightarrow s + jn\omega_s$ $\quad n = 0, \pm 1, \pm 2, \ldots$

$$\therefore \; Y^*(s + jn\omega_s) = \sum_{k=0}^{\infty} y(kT)e^{-kT(s + jn\omega_s)}$$

$$= \sum_{k=0}^{\infty} y(kT)e^{-ksT}e^{-jknT\omega_s}$$

$$= Y^*(s)$$

$\therefore \; Y^*(s)$ is periodic with period $j\omega_s$



$$G(z)=\mathbf{Z}\left\{\frac{\left(1-e^{-sT}\right)}{s}\times\frac{2}{s^2}\right\}=\mathbf{Z}\left\{\frac{2}{s^3}-e^{-sT}\frac{2}{s^3}\right\}=\mathbf{Z}\left\{\frac{2}{s^3}\right\}-z^{-1}\mathbf{Z}\left\{\frac{2}{s^3}\right\}$$

$$=\left(1-z^{-1}\right)Z\left\{\frac{2}{s^3}\right\}$$

Generally $G(z)=\left(1-z^{-1}\right)Z\left\{\frac{G(s)}{s}\right\}$

$$=\frac{z-1}{z}\times\frac{T^2z(z+1)}{(z-1)^3}$$

let $T=1$

$$=\frac{z+1}{(z-1)^2}$$

CLCE: $1+G(z)=0$

$$\therefore\quad 1+\frac{z+1}{(z-1)^2}=\frac{(z-1)^2+z+1}{(z-1)^2}=0$$

$$\therefore\quad z^2-z+2=0\longrightarrow z=\frac{1\pm\sqrt{1-8}}{2}=\frac{1}{2}\pm j\frac{\sqrt{7}}{2}$$

$$\therefore\quad |z|=\sqrt{\left(\tfrac{1}{2}\right)^2+\left(\tfrac{\sqrt{7}}{2}\right)^2}=\sqrt{\tfrac{1}{4}+\tfrac{7}{4}}=\sqrt{2}>1$$

UNSTABLE

Example

Further Study on Stability tests

» Please refer to the annex of this lecture slides Lectures (no. 7b).