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Embedded Systems and Industrial Controller EE5563 (6)

Topics

- Control Process
- Control Systems Design & Analysis

Open-Loop Control:

Is normally a simple switch on and switch off process, for example a light in a room is switched on or off.

Closed-Loop Control:

A continuous exercise of monitoring a condition (comparing the output of the system with a specified condition) and conducting an action to reduce the error. For example an air-conditioning system that keeps the temperature/humidity at a specified level.

Control modes:

A way that controllers react to an error signal.

Continuous and Discrete Control

Control Processes

- Process Control is all about controlling a defined operation or condition, either by time factors or events.

- For example an automated production line in a factory that follows a specific sequence of operations to complete the assembly of a part.

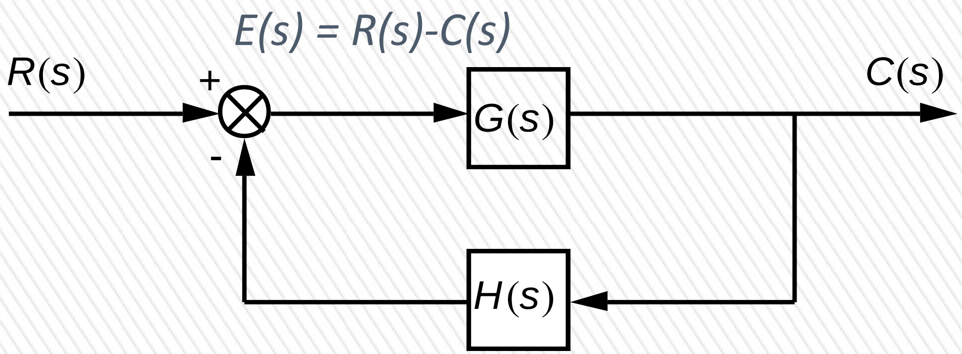
- With time based (clock-based) systems specific actuation (activity) is stipulated. With an event-based system a specific event should occur e.g. a sensor detecting a product on a production line that switches the conveyor on.

- Therefore a control system requires to monitor a situation and *measure* the output variable *compare* with a “desired” variable.

“Feedback”

The difference is a measure of the *error* in the system.

General Block Diagram Definitions for Control Systems



R(s) : system input ⇛ some measure of *desired* output

C(s) : system output ⇛ *actual* output of dynamical process

G(s) : forward-path transfer function

H(s) : feedback transfer function

if *H(s) = 1* have a **unity feedback** system (important practical)

G(s).H(s) : open-loop transfer function (OLTF)

total TF around the loop if it is broken at any point.

E(s) : Error

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1 + G(s)H(s)}$$

$$= \frac{G(s)}{1+G(s)}$$ for unity feedback function

Therefore;

$$E(s) = R(s) - C(s) = R(s) - \frac{R(s)G(s)}{1 + G(s)} = \frac{1}{1 + G(s)}R(s)$$

Error is shown to be dependent on the input *R(s)*

Feedback

» A closed-loop system uses the measurements of the system output and compares it to the expected (desired) value of the system to generate the error signal.

» The error signal occurs when a control variable changes (value/state).

» When the change occurs there is likely to be a ***transient*** effects, which die away in time.

» The term ***Steady-State*** is used for *the difference between the desired set value input and output* after the transient period dies away.

Steady State error

Performance of a Feedback System

The primary reasons for using feedback are:

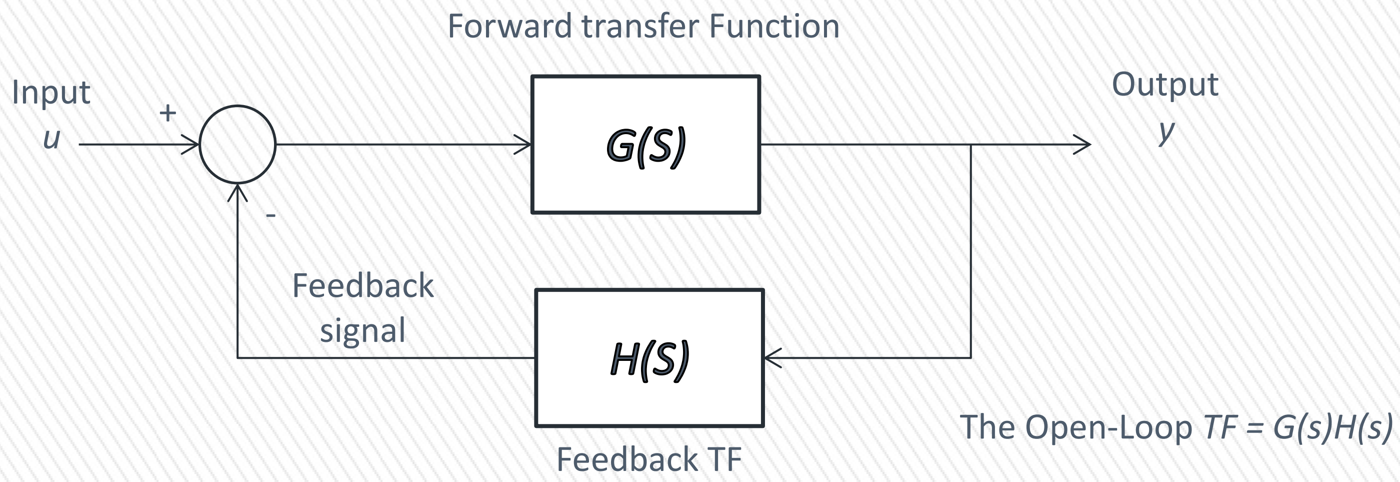
- to reduce sensitivity of the closed-loop performance to parameter variations and imperfections of the plant model.
- to reduce sensitivity of the closed-loop performance to disturbance inputs and noise.

Considerations when designing feedback control systems:

- reducing steady state errors
- improving transient response

Root Locus Method

Consider the general feedback system:



The root locus analysis is a graphical method for examining how the roots of a system change with variation of a certain system parameter (i.e. gain within a feedback system).

» Mainly used for stability criterion Walter R. Evans (1948) which can determine stability of the system.

» The root locus plots the poles of the closed loop transfer function as a function of a gain parameter.

- **loop gain** is the sum of the gain expressed as a ratio
- Feedback loops are widely used in electronics in amplifiers and oscillators, and more generally in both electronic and nonelectronic industrial control systems to control industrial plant and equipment.

Loop Gain

Root locus cont.

- The Overall closed-loop TF:

$$\frac{Y(s)}{U(s)} = \frac{G(s)}{1 + G(s)H(s)}$$

The closed-loop poles are the roots of the characteristic equation:

$$1 + G(s)H(s) = 0$$

- Stability of a closed-loop system depends on the poles and the zeros of the loop TF.

- The root locus show how these poles move in the *s*-plane when parameter of *G(s)H(s)* changes.

  - Normally this parameter is the root locus gain *K*

Plotting Root Locus

- Root locus can be plotted by varying any of the parameters of the system and solving the corresponding closed-loop characteristic equation.
- Most computer programs use this method.
- For preliminary analysis a rough sketch suffices, only critical for important parameters and the roots to be accurate.

Some rules:

$1 + G(s)H(s) = 0$ or *GH = -1*

Since the *GH* can be expressed as a ratio of two *monic polynomials* $(c_n x^n + c_{n-1}x^{n-1}, \ldots + c_0)$; (i.e. $c_n = 1$), then:

$K\frac{N(s)}{D(s)} = -1$ or $KN(s) + D(s) = 0$

N(s) : numerator polynomial of the loop TF

D(s): denominator polynomial of the loop TF

K: loop gain

The polynomials can be factorised, so:

$$K\frac{(s-z_1)(s-z_2)\ldots(s-z_m)}{(s-p_1)(s-p_2)\ldots(s-p_n)}=-1$$

Or

$$(s-p_1)(s-p_2)\ldots(s-p_n)+K(s-z_1)(s-z_2)\ldots(s-z_m)=0$$

Where:

z_i= a zero of the loop of the TF

p_i= a pole of the loop of the TF

m= the order of the numerator polynomial=number of zeros of *GH*

n= the order of the denominator polynomial = number of poles of *GH*

Plotting cont.

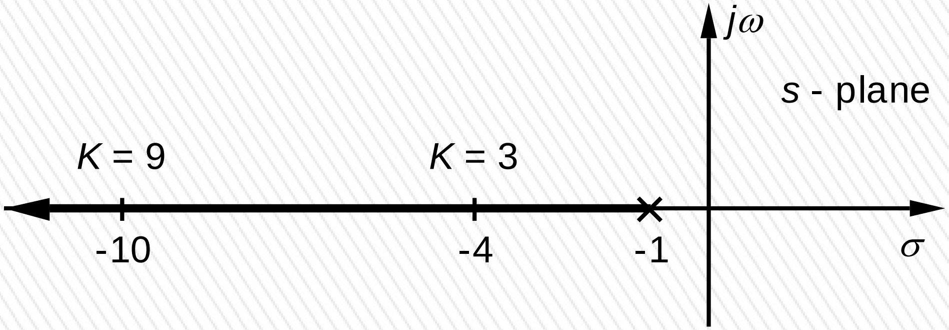
Consider $G(s)H(s) = \frac{K}{s+1}$

The open-loop pole *s = -1*

The closed-loop pole is determined by $1 + \frac{K}{s+1} = 0 \Rightarrow s + 1 + K = 0$

Or $s = -1 - K$

The root locus diagram for various *K=0, K=3 and K=9*



By increasing the *K* the pole moves to the left – the time constant of the closed-loop system decreases continuously

Example 1

Consider $G(s)H(s) = \frac{K}{(s+1)(s+5)}$

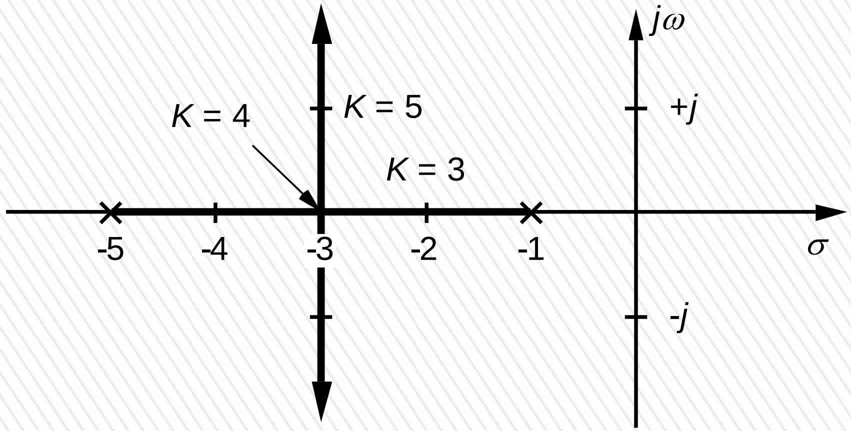
The open-loop pole *s = -1, -5*

The closed-loop pole is determined by $1 + \frac{K}{(s+1)(s+5)} = 0 \Rightarrow$

$$s^2 + 6s + 5 + K$$

The root locus diagram for:

$K = 3 :\ s = -2, -4,\qquad K = 4\ \ s = -3, 3 \qquad K = 5 \quad s = -3 \pm j$



Example 2

Refer to W. de Silva, Modelling and Control of Engineering Systems, (2009), pp. 338-340

For reminder on complex numbers

The general open-loop TF can be expressed as:

$$G(s)H(s) = K\frac{\prod_{i=1}^{m}(s+z_i)}{\prod_{i}^{n}(s+p_i)}$$

Where z_i are the open-loop system zeros , and p_i are the open-loop systems poles. K is the root locus gain.

The closed-loop character equation can be expressed as:

$$1 + K\frac{\prod_{i=1}^{m}(s+z_i)}{\prod_{i=1}^{n}(s+p_i)} = 0 \text{ or } K\frac{(s+z_1)(s+z_2)\ldots(s+z_m)}{(s+p_1)(s+p_2)\ldots(s+p_n)} = -1$$

So any point on the root locus must satisfy the *magnitude and phase condition:*

$$K\frac{|s-z_1||s-z_2|\ldots|s-z_m|}{|s-p_1||s-p_2|\ldots|s-p_n|} = 1$$

The phase angle: $\sum_{i=1}^{n}\angle(s-p_i) - \sum_{i=1}^{n}\angle(s-z_i) = \pi + 2\pi r$

$r = 0, \pm1, \pm2, \ldots$

The general root locus rules

» Computer packages (e.g. MATLAB) for plotting the root - locus are now readily available, but it is important to know the basic rules for sketching the loci.

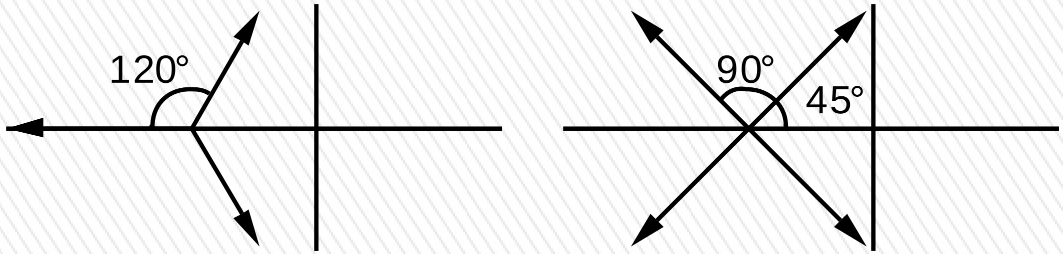
» Manual plotting of the root - locus diagram is considerably eased by a series of rules which give a good indication of the shape of the loci.

1. The loci start (i.e. $K = 0$) at the *n* poles of the open-loop TF $G(s)H(s)$
2. The number of loci is equal to the order of the Character Equation. (The plot is symmetrical about the real axis of the *s*-plane.)
3. The root loci end (i.e. $K \to \infty$) at the *m* zeros of $G(s)H(s)$, and if *m* < *n* (usual) then the remaining *n* - *m* loci tend to infinity.
4. Portions of the real axis are sections of a root locus if the number of poles and zeros lying on the axis to the right is odd.
5. Those loci terminating at infinity tend towards asymptotes at angles relative to the positive real axis given by:

$$\frac{\pi}{(n-m)}, \frac{3\pi}{(n-m)}, \frac{5\pi}{(n-m)}, \ldots, \frac{\{2(n-m)-1\}\pi}{(n-m)}$$

Root Lucas rules cont.

Examples



3 excess pole 4 excess poles

6. The intersection of the asymptotes on the real axis occurs at the 'centre of gravity' of the pole - zero configuration of $G(s)H(s)$, i.e. at

$$s = \frac{\sum \text{poles of } G(s)H(s) - \sum \text{zeros of } G(s)H(s)}{(n-m)}$$

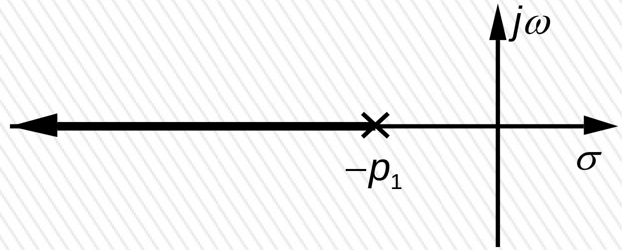
7. The intersection of the root - loci with the imaginary $(j\omega)$axis can be calculated by Routh - Hurwitz or geometrical analysis (only on some plots).

8. The breakaway points (points at which multiple roots of the characteristic polynomial occur) of the root locus are the solutions of $\frac{dK}{ds} = 0$ (not all the solutions are necessarily breakaway points.)

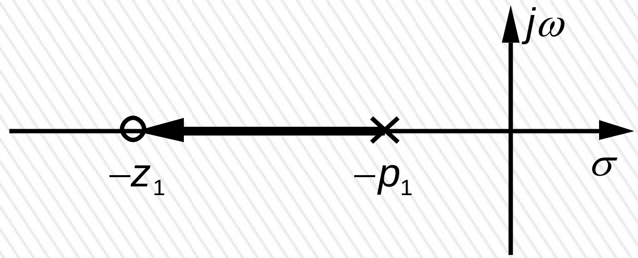
Root Lucas rules cont.

Examples

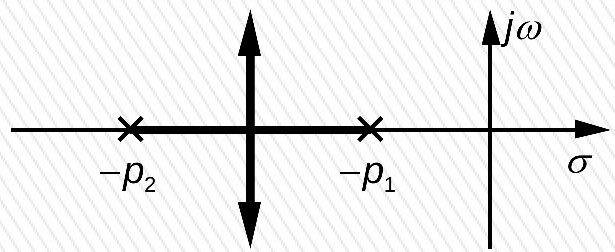
$$G(s)H(s) = \frac{K}{s+p_1}$$



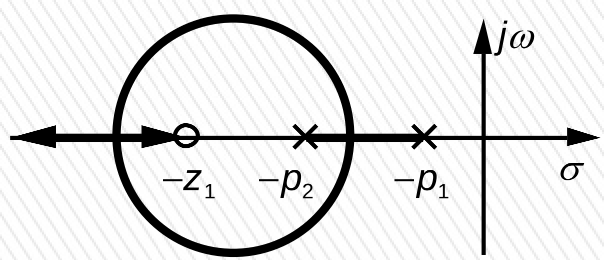
$$G(s)H(s) = \frac{K(s+z_1)}{s+p_1}$$



$$G(s)H(s) = \frac{K}{(s+p_1)(s+p_2)}$$

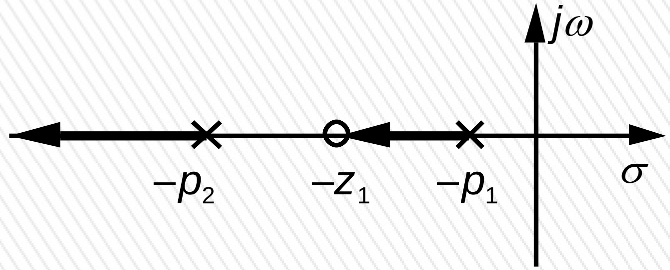


$$G(s)H(s) = \frac{K(s+z_1)}{(s+p_1)(s+p_2)}$$

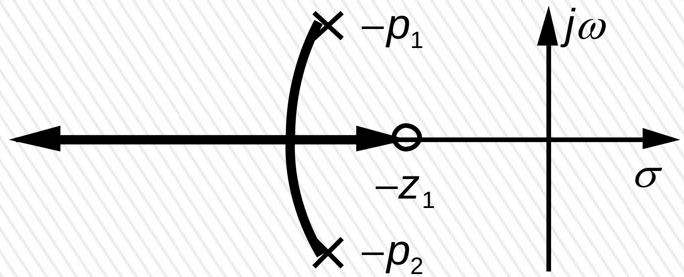


Examples cont.

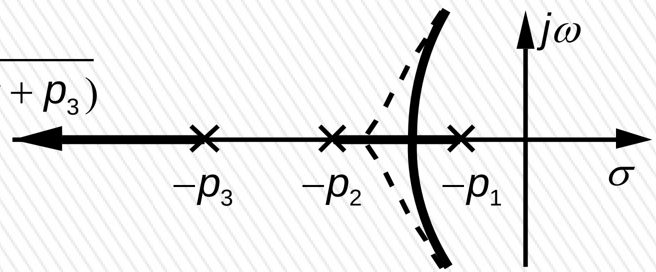
$$G(s)H(s) = \frac{K(s+z_1)}{(s+p_1)(s+p_2)}$$



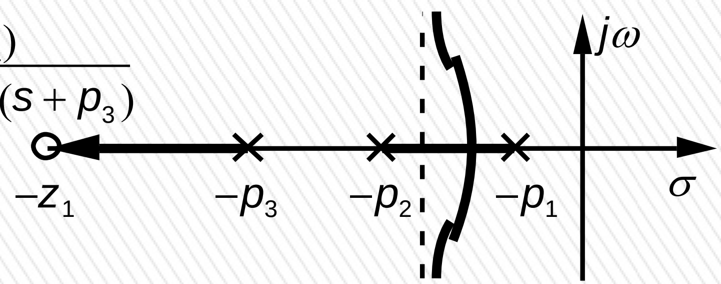
$$G(s)H(s) = \frac{K(s+z_1)}{(s+p_1)(s+p_2)}$$



$$G(s)H(s) = \frac{K}{(s+p_1)(s+p_2)(s+p_3)}$$



$$G(s)H(s) = \frac{K(s+z_1)}{(s+p_1)(s+p_2)(s+p_3)}$$

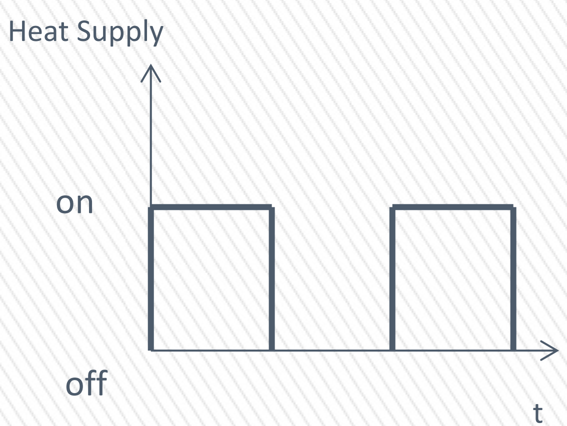


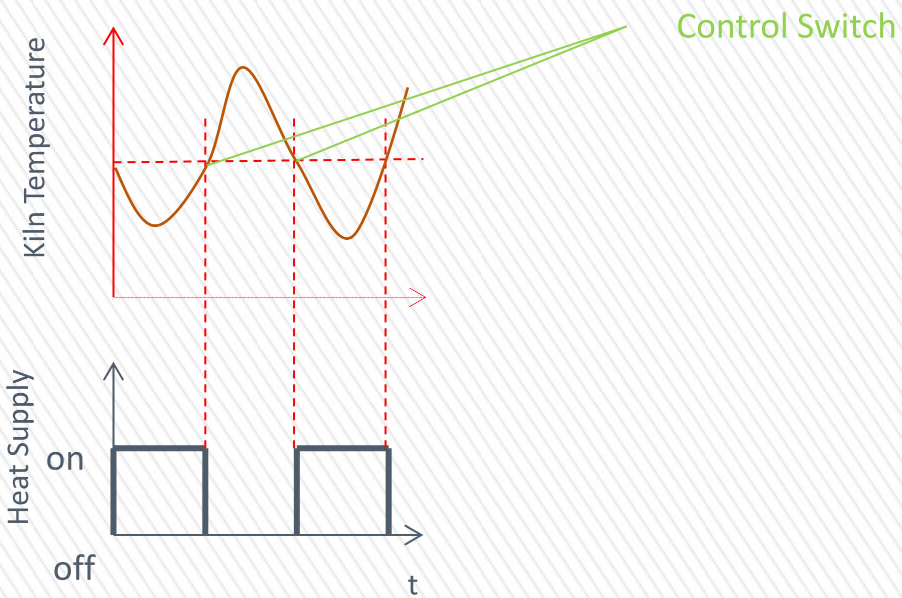
Control Modes

The are a number of ways in which a control system can react to an error signal and correct the error by supplying an output.

1. The two-step mode:

The controller is just a switch (e.g. thermostat) activated by an error signal e.g. temperature becoming higher than desired.





- This example is a single temperature setting. So it almost continuously switches the thermostat on off. A better solution would be to have two temperature settings and the heater is switched on at a lower value and switched off when the kiln reaches a specified temperature.

- With two step mode, the controller output is either on or off signal, regardless of the magnitude of the error.

Control Modes (controller reacting to an error)

In case of proportional control mode, the response of the controller is determined by the magnitude of error.

The correction signal is proportional to the size of correction required.

Therefore: Controller output = $K_p e$

Where K_p is a constant and *e* is the error.

The Laplace Transforms:

Controller Output *(s)* = $K_p E(s)$

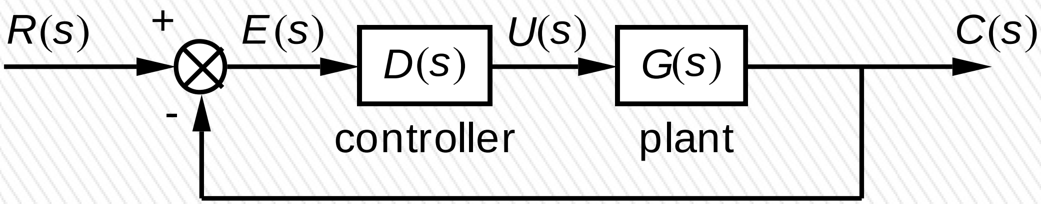
TF of the controller

Control Mode Proportional

Consider *unity* feedback controller with a controller *D(s)* in forward path.

» using the output value of the system to help us prepare the next output value. In this way, we can create systems that correct errors.

» here with Proportional control we have a gain element with TF of $D(s) = K_p$ with forward path element of *G(s)*

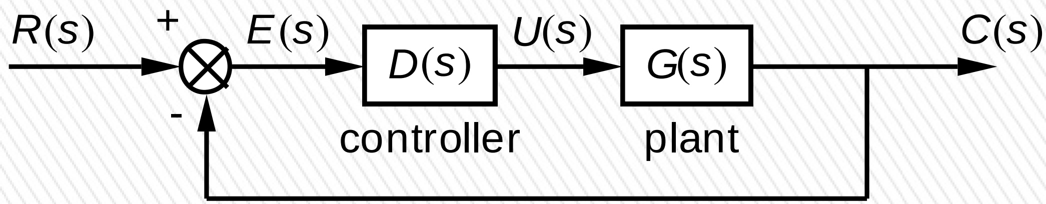


The error is therefore:

$$E(s) = \frac{K_p G(s)}{1 + K_p G(s)} R(s)$$

This is a basic feedback structure. Here, we are using the output value of the system to help us prepare the next output value. In this way, we can create systems that correct errors. Here we see a feedback loop with a value of one. We call this a **unity feedback**.

Control Mode Proportional cont.



The controller is to be designed to achieve desired CL characteristics

- » steady state response (specified steady state error)
- » transient response (settling time, overshoot)
- » *U(s)* is termed the control signal (i.e. the output of the controller and input to the plant)

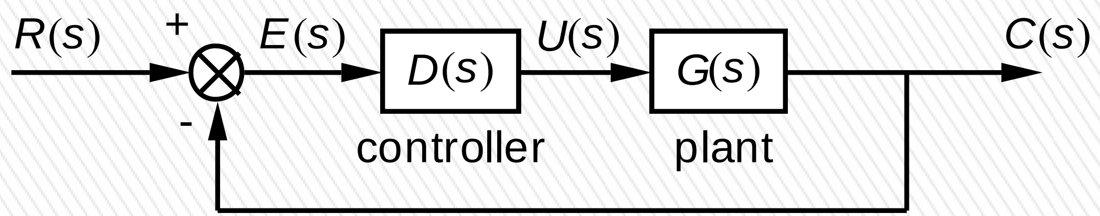
- » *U(s)* is proportional to the error *E(s)*

For a step input the steady state error e_{SS}*:*

$$e_{SS} = \lim_{s\to\infty} sE(s) = \lim_{s\to 0}\left[s\frac{1}{1+1/K_pG(s)}\frac{1}{s}\right]$$

Control Mode Proportional cont.

Consider a unity feedback system with a controller *D(s)* in the forward path:



The controller is to be designed to achieve desired CL characteristics

» steady state response (specified steady state error)

» transient response (settling time, overshoot)

U(s) is the control signal (the product of the output of the controller and input to the plant)

When $D(s) = K_p$ *(a constant of proportionality) i.e. U(s) is proportional to error E(s)* → ***Proportional Control***

Note: the effect on the closed loop poles is given by the root locus

Controller Design (P, PI and PID)

In case of Derivative mode of control the controller output is proportional to the rate of change in error signal in time:

$$Controller\ Outpt = K_D \frac{de}{dt}$$

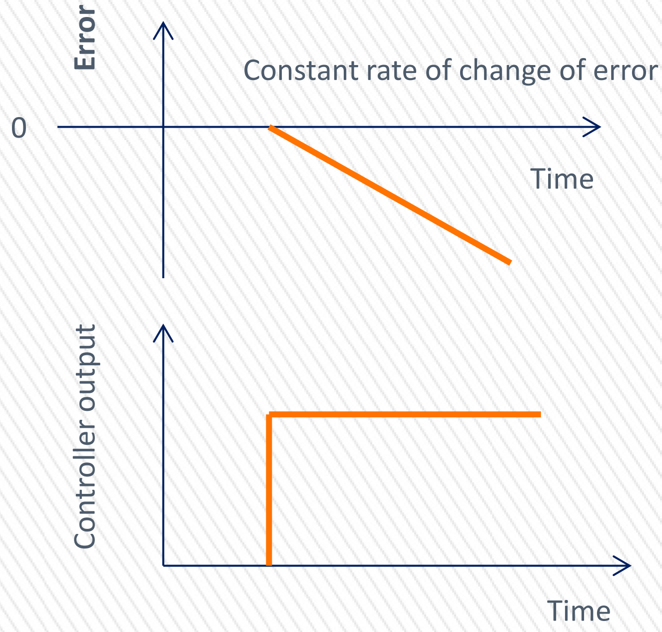
K_D is the constant of proportionality.

Therefore, the TF is obtained by the Laplace transform:

$$Controller\ Outpt(s) = K_D s E(s)$$

Where the TF is $K_D s$

Controller Design (Derivative Control)



On the derivative control:

» Once the error signal changes the controller output signal can be large (it is proportional to rate and not the value of the error)

» The controller output is constant because rate of change is constant.

» Derivative controllers do not respond to steady-state error signals, because with steady-state error the rate of change in time is zero (i.e. constant) – due to this derivative controllers are always combined with proportional control. (the proportional control responses to all error signals including the steady-state)

Derivative Control

Proportional + Derivative Control

The Controller output for PD controller can be given by:

$$Controller\ Output = K_p e + K_D \frac{de}{dt}$$

Proportionality constant

Derivative constant

Rate of error change

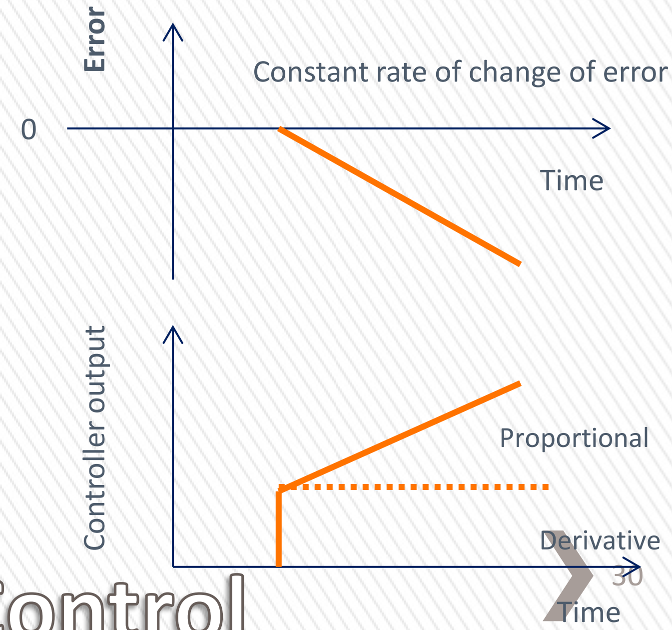
The system has a TF that can be given by:

$$Controller\ output(s) = K_p E(s) + K_D sE(s)$$

The TF is $K_p + K_D s$ that can be written as:

$$TF = K_D\left(s + \frac{1}{T_D}\right)$$

T_D is the derivative time constant $T_D = {}^{K_D}/_{K_P}$



Integral Control

In the integral mode of control the rate of change of the output control is proportional to the input error signal:

$$\frac{dI}{dt} = K_I e$$

Where K_I is the constant of proportionality $(1/s)$ by integrating both sides of the above equation:

$$\int_{I_0}^{I_{out}} dI = \int_0^t K_I e dt$$

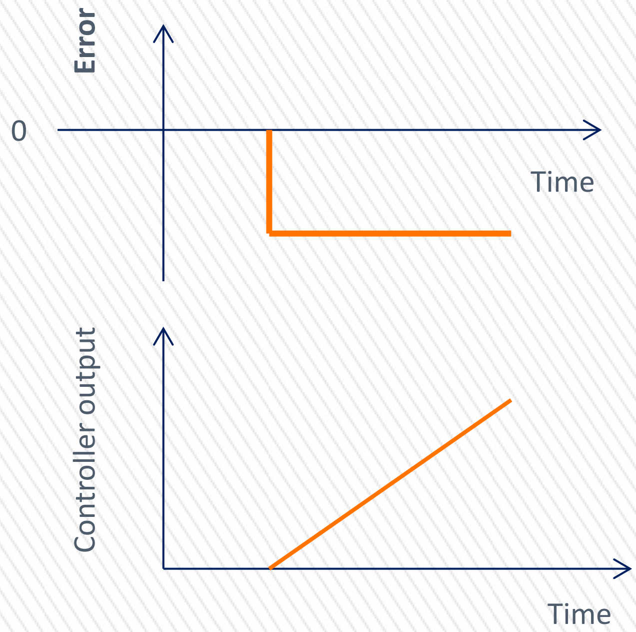
$$\therefore \; I_{out} - I_0 = \int_0^t K_I e dt$$

Controller output at *t=0*

The TF is obtained by taking the Laplace transform:

$$(I_{out} - I_0)(s) = \frac{1}{s} K_I E(s)$$

The $Transfer\ function = \frac{1}{s} K_I$



Integral Control Cont.

PID Control

A combination of the proportional, integral and derivative offers the three-mode controller or the PID controller:

$$controller\ output = K_p e + K_I \int edt + K_D \frac{de}{dt}$$

Taking the Laplace transform gives:

$$controller\ output\ (s) = K_p E(s) + \frac{1}{s} K_I E(s) + sK_D(s)$$

Therefore, the TF:

$$Transfer\ function = K_p e + \frac{1}{s} K_I + sK_D$$

Further reading: W. Bolton, Mechatronics: electronic control systems in mechanical and electrical engineering, the 4th edition 2008.