COMPUTATIONAL MODELLING OF A DIRECTIONAL SOLIDIFICATION PROCESS

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Aims of the Research Project

- To generate a computational mesh that represents an idealised geometry for the investment casting of a turbine blade. The assembly consists of the investment shell and core, the alloy component and the chill.
- To develop a moving boundary condition which models the withdrawal of the assembly from the furnace, thus controlling the direction and speed of solidification.
- To perform a fully coupled thermo-mechanical analysis of the withdrawal of the assembly from the furnace.

Approach

- An idealised symmetric geometry has been assumed to reduce the computational effort required to perform the analysis.
- The thermo-elastic analysis has been modelled using a 3D continuum approach.
- Finite volume techniques have been used to discretize the governing equations.

Investment Casting

- Investment casting is used to produce components for use in aircraft, electronic, optical and medical equipment.
- The process enables manufacturers to produce complex components with a high degree of precision.
- The components have excellent surface quality.

Directional Solidification

- Allows the manufacturer to control the speed of solidification and govern growth of solid grain in the material.
- Directional solidification improves the quality and structural integrity of the component thus increasing the life cycle and reliability of the product.

Geometry

The geometry consists of four sections:

- ceramic core
- ceramic shell
- metal alloy
- copper chill

The top and bottom of the core are attached to the shell, and the chill is in place at all times.

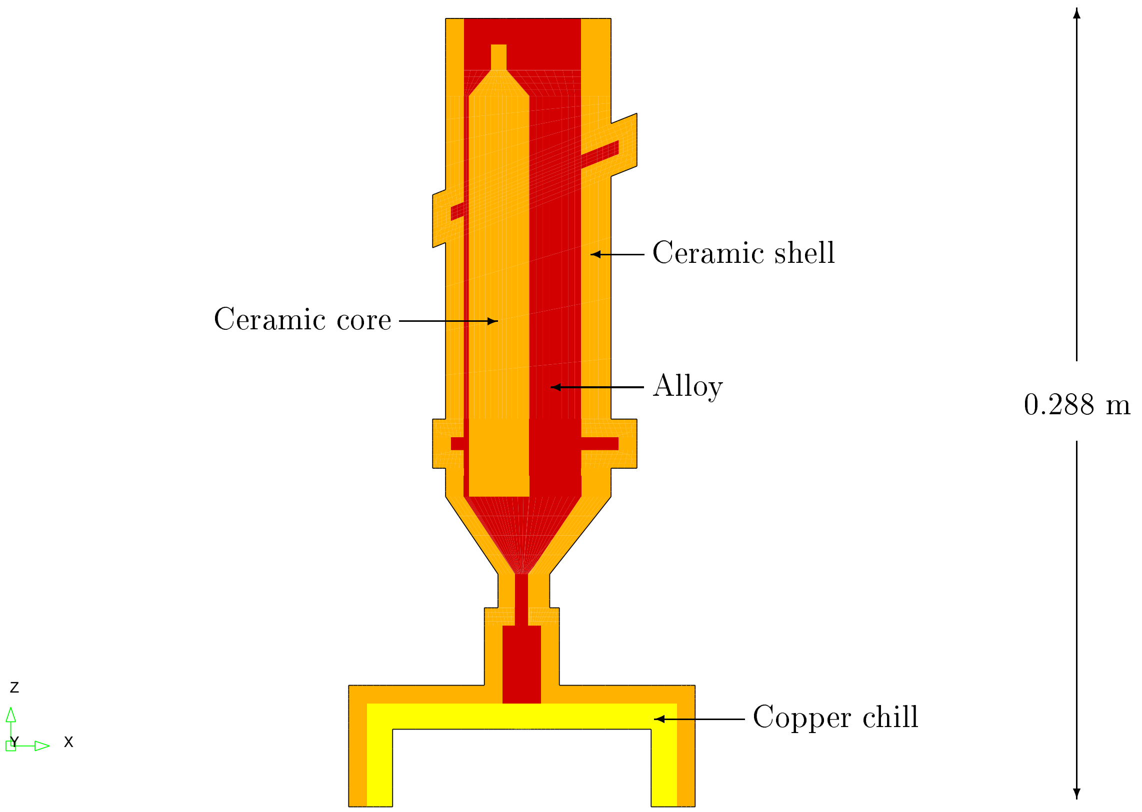


Figure 1: A vertical cross-section of the geometry

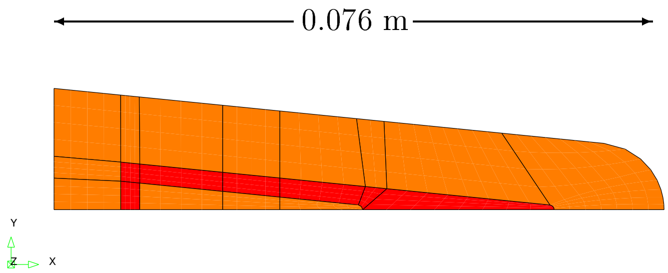
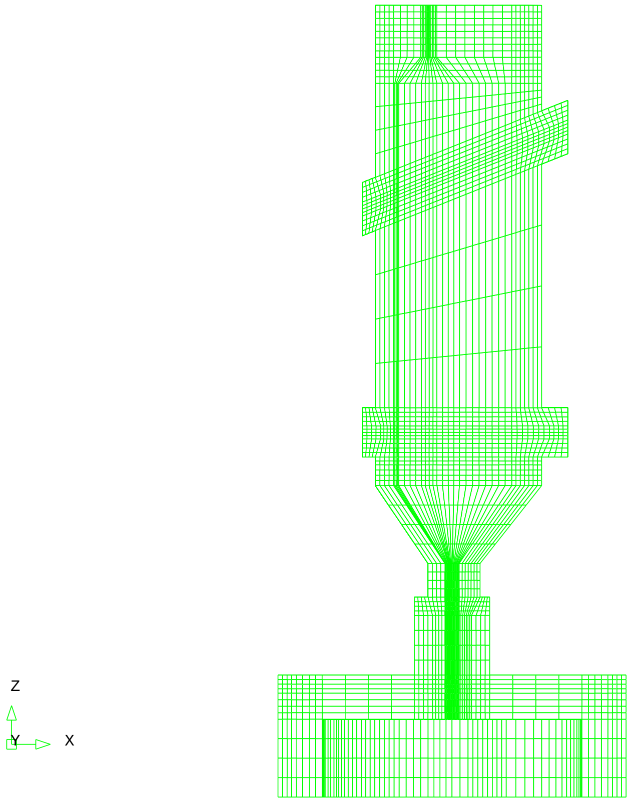


Figure 2: A horizontal cross-section through main body of geometry

Mesh

The symmetric mesh is made up of 8-noded hexahedral elements.



$$\begin{aligned} \text{No. of elements} &= 45156 \\ \text{No. of nodes} &= 53647 \end{aligned}$$

Initial Conditions

The temperatures of the shell, alloy and core are set initially to 1773K. The chill is given an initial temperature of 293K.

Material Properties

Temperature dependent material properties are used for the thermo-elastic analysis.

Governing Equations

To model the thermo-elastic behaviour of the process the following equations must be solved in a coupled fashion.

The scalar heat equation with phase change is

$$\frac{\partial(\rho c_P T)}{\partial t} = \mathrm{div}(k \mathrm{grad}\ T) - L_h \frac{\partial(\rho f)}{\partial t}$$

where

ρ - density	c_P - specific heat capacity
k - thermal conductivity	f - liquid fraction
L_h - latent heat of solidification	

The equilibrium equations describing the solid mechanics behaviour are, in matrix form,

$$[L]^T\{\Delta\sigma\} = 0$$

where $[L]$ is the differential operator and $\{\Delta\sigma\}$ is the Cauchy stress increment.

The stress is related to elastic strain such that

$$\{\Delta\sigma\} = [D]\{\Delta\epsilon_e\} = [D]\left(\{\Delta\epsilon\} - \{\Delta\epsilon_{th}\}\right)$$

where $[D]$ is the elasticity matrix, and $\{\Delta\epsilon_e\}$, $\{\Delta\epsilon\}$ and $\{\Delta\epsilon_{th}\}$ are the elastic, total and thermal incremental strain, respectively.

The thermal strain is related to the incremental temperature change, ΔT, as follows

$$\{\Delta\epsilon_{th}\} = \{\alpha\Delta T, \alpha\Delta T, \alpha\Delta T, 0, 0, 0\}^T$$

where α is the coefficient of thermal expansion.

Thermal and Mechanical Boundary Conditions

To model the withdrawal from the furnace the geometry is fixed in space and a moving radiative boundary condition is applied to the external surface of the shell.

Initially the ambient temperature T_A satisfies the following temperature profile, where the top of the chill is located at $z = 0\,\mathrm{m}$.

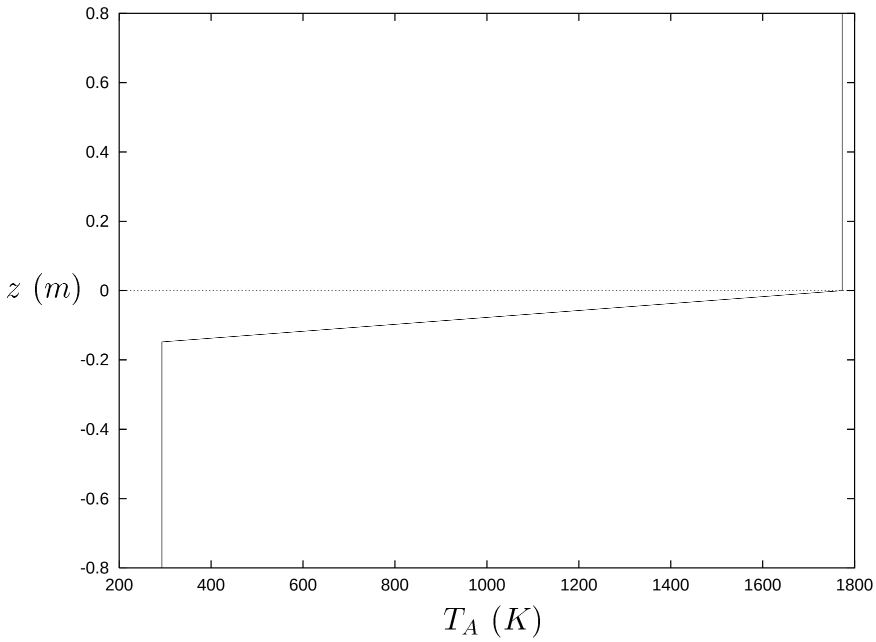


Figure 3: Ambient temperature profile

To simulate the withdrawal the ambient temperature profile moves upwards at a constant rate.

Newton cooling boundary conditions are applied to the base of the shell and chill, where the heat transfer coefficients are 10W/mK and 6000W/mK repectively. An adiabatic condition is applied to the top of the geometry and the symmetry plane.

Any gap which occurs between the shell and alloy as the alloy cools and contracts is treated as a vacuum.

Fixed displacement boundary conditions are applied to inhibit penetration of the shell or core by the alloy.

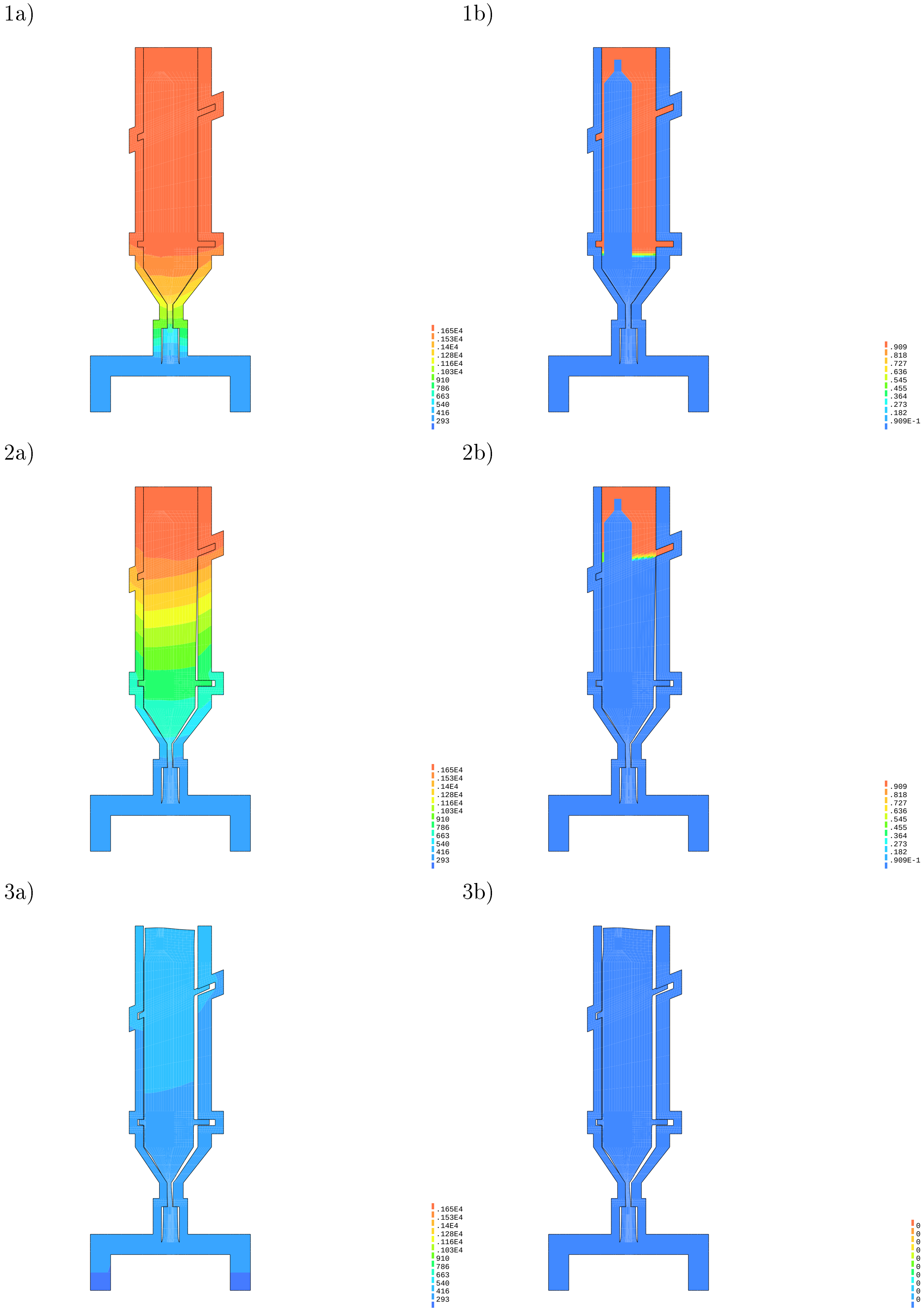


Figure 4: The contours of a) temperature and b) liquid fraction at 1) $t = 30$, 2) $t = 60$ and 3) $t = 120$ mins.

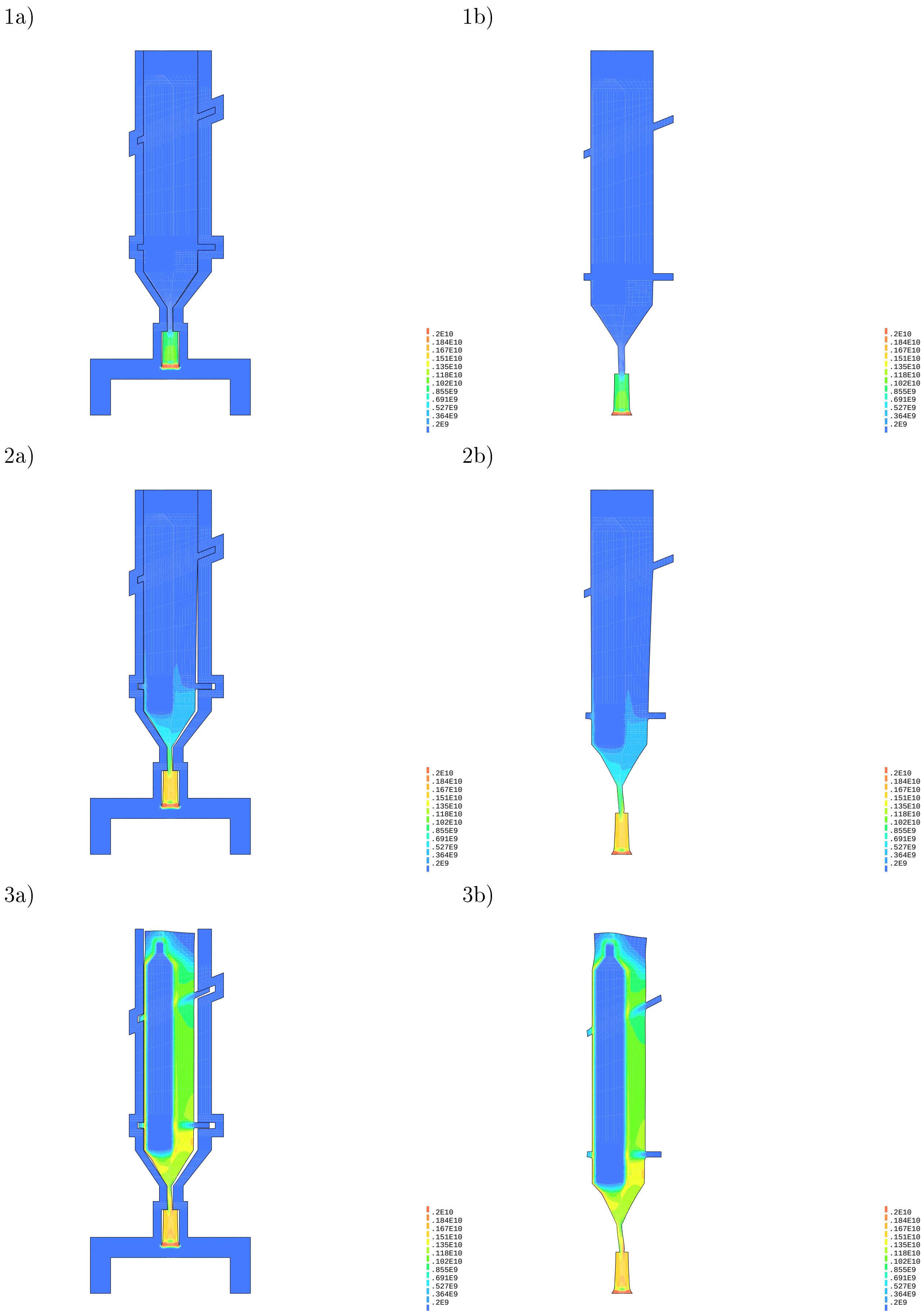


Figure 5: The effective stress contours on a) whole geometry and b) the alloy with the deformation magnified ($\times 10$) at 1) $t = 30$, 2) $t = 60$ and 3) $t = 120$ mins.