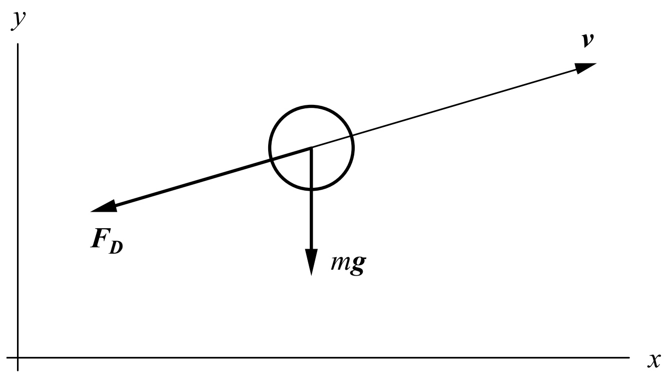
3. Soccer Kick – Effects of Aerodynamic Drag

OBJECTIVES

- Use sophisticated software (*Mathematica*) to calculate the trajectory of a soccer ball under the influence of aerodynamic drag.
- Examine how aerodynamic drag affects the shape of the flight trajectory, the range of a kick, and the optimum launch angle.

INTRODUCTION

When a projectile is launched through a vacuum, the flight is governed by gravity and the projectile traces a parabolic path. However, when the projectile is launched through air the motion of the projectile becomes more complicated. Because air is a fluid with non-zero density and viscosity, it produces aerodynamic effects that may substantially influence the trajectory of the projectile. Consider a soccer ball that has been launched through still air. Here, we analyse the trajectory of the ball in a rectangular coordinate system where the positive x-axis is in the forward horizontal direction and the positive y-axis is vertically upwards. We assume that the ball has no spin.



During the flight through the air, the ball is subject to the force of gravity and to an aerodynamic drag force that arises from the interaction of the ball with the surrounding air. The drag force F_D is in the opposite direction to the instantaneous velocity of the ball and acts to decrease the speed of the ball. The magnitude of the drag force depends on the speed of the ball through the air, the cross-sectional area of the ball, and the shape and surface characteristics of the ball. The shape and surface characteristics are accounted for by a scale factor called the drag coefficient. At velocities that are typical of a soccer kick, the drag force acting on the ball is given by

$$F_D = \frac{1}{2}\rho S C_D v^2 \qquad (1)$$

where ρ is the air density (1.225 kg/m^3 at sea level and 15ºC), S is the cross-sectional area of the ball (0.038 m^2), C_D is the drag coefficient (0.2), and v is the velocity of the ball relative to the air (up to about 40 m/s).

The flight trajectory equations of the soccer ball are

$$\frac{d^2x}{dt^2} = -kv\,C_D\frac{dx}{dt} \qquad (2)$$

and $$\frac{d^2y}{dt^2} = -kv\,C_D\frac{dy}{dt} - g\,, \qquad (3)$$

where v is the instantaneous velocity of the ball, and g is the acceleration due to gravity (9.81 m/s^2). The constant k is given by $k = \rho S/2m$, where m is the mass of the ball (0.43 kg).

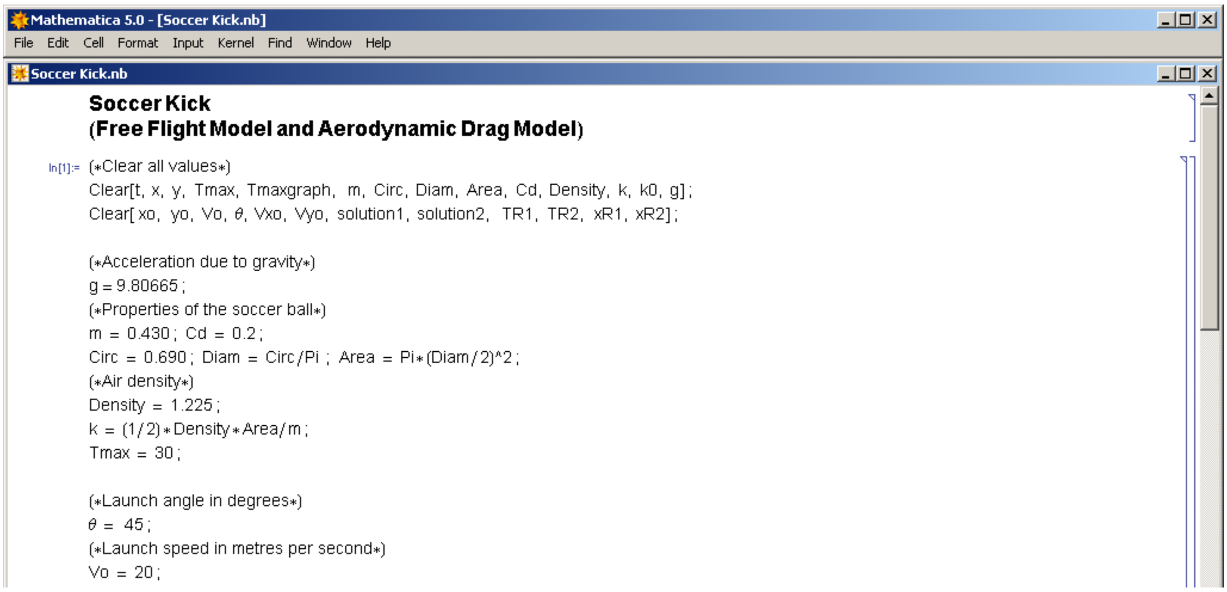
The flight trajectory equations are second-order differential equations in time, t, and the initial conditions are the values of the projection velocity, projection height, and projection angle. There are no closed-form solutions for $x(t)$ and $y(t)$, and so solutions must be obtained using numerical methods. Here we will obtain numerical solutions for $x(t)$ and $y(t)$ using a technical computing software package (*Mathematica*). The trajectory of the ball will be computed and the range and flight time of the ball will be determined.

PROCEDURE

Go to one of the University computers and log on to the computer in the usual way. Open **My Computer**, then double-click on the L Drive (**depapps on 'academic.windsor' (L:)**). Open the **CC** folder, then open the **Mathematica** folder. Start up *Mathematica* by double-clicking the **Mathematica 5.0** icon.

Open a web browser (*Netscape* or *Internet Explorer*) and go to the "Laboratory Items" page on the *Biomechanics of Sport and Exercise* u-Link site. **Soccer Kick.nb** is a *Mathematica* notebook with a program that calculates the trajectory of a soccer ball under the influence of aerodynamic drag. Click on the icon and download the file from the u-Link site to your computer hard disk. (I suggest you place the files in the **My Documents** folder.)

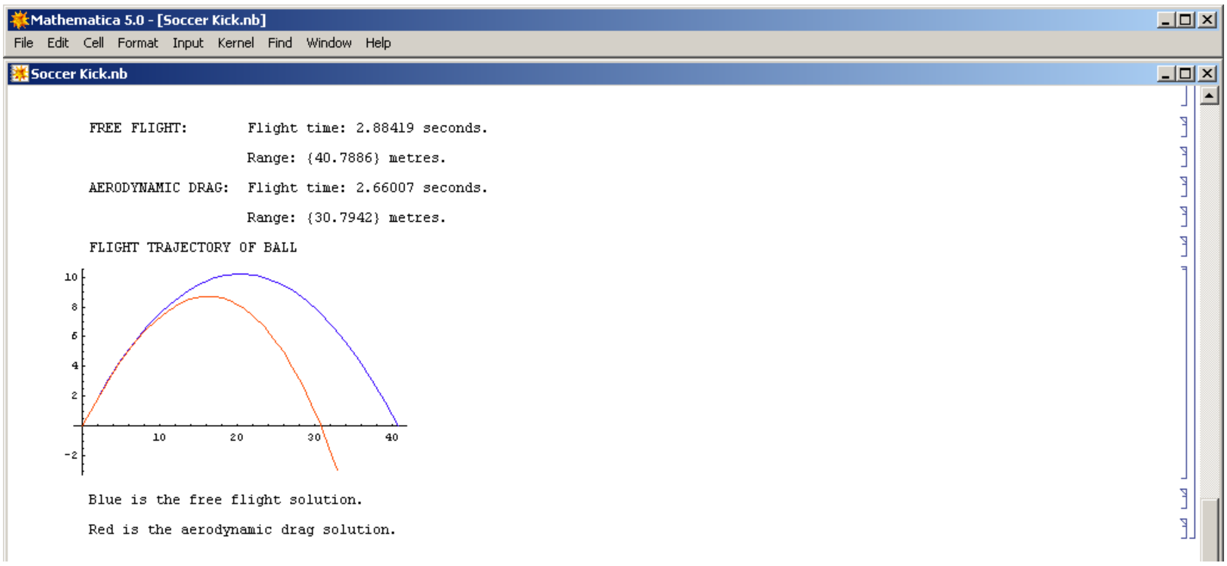
Return to the **Mathematica 5.0** window and select **Open** from the **File** menu. The **Open** dialog box will appear. Find and open the **Soccer Kick** notebook file that you downloaded to your computer.



The **Soccer Kick** program calculates the range and flight time of a soccer ball that is launched at a specified velocity and angle. The flight trajectory is calculated with a 'free flight model' in which air resistance is ignored, and an 'aerodynamic model' which includes the effects of air resistance. Have a quick look at the program and try to make some sense of it.

The last few cells at the bottom contain the output of the computer program. Here the output is a calculation of the flight time and range for the free flight model and for the aerodynamic model.

There is also a plot of the flight trajectory of the ball. The blue curve is for the free flight model, and the red curve is for the aerodynamic model.



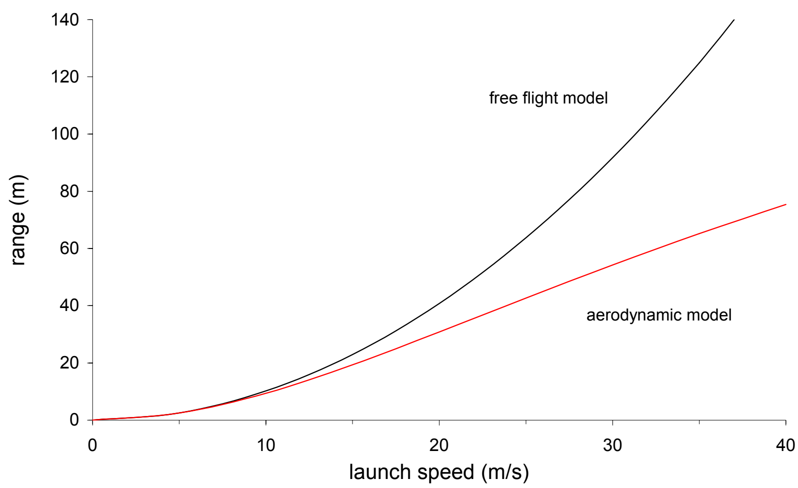
Test Trajectories

The calculations shown above are with a launch velocity of 20 m/s and a launch angle of 45º. You may change the launch velocity and launch angle by typing in new values for *Vo* and *θ*. To run the **Soccer Kick** program with the new values, select **Evaluation** from the **Kernel** menu, then select **Evaluate Cells**. Note the changes in the calculated flight time, range, and flight trajectories.

Experiment with various combinations of launch velocity and launch angle. For example, keep the launch velocity constant at 20 m/s and note the differences between launch angles of 10º, 45º, and 90º. Now keep the launch angle constant at 30º and note the differences between launch velocities of 2 m/s, 10 m/s, and 30 m/s.

Effect of Aerodynamic Drag on the Range of a Soccer Ball

The effect of air resistance on the flight of a ball becomes more pronounced at higher launch velocities. To investigate this effect, keep the launch angle constant at 45º and record the range achieved (for both models) at a selection of launch velocities between 0 and 40 m/s. Plot your data in *Excel*. (The graph you should obtain is shown below.)



For each launch velocity, calculate the reduction in range due to air resistance, and the reduction in range as a percentage of the free flight range. Plot your data.

Effect of Aerodynamic Drag on the Optimum Launch Angle

The well-known result that the optimum launch angle for a projectile is 45º only applies if certain conditions are satisfied. The conditions are: 1) the launch and landing are at the same height, 2) the projectile is launched at the same velocity at all launch angles, and 3) air resistance is negligible. Here we will investigate the effect of air resistance on the optimum launch angle. We will assume that the player can kick the ball at the same velocity at all launch angles. (The results from the Shot Put Lab suggest that this may not be so, but I do know of anyone who has tested it.)

To calculate the optimum launch angle, keep the launch velocity constant at 30 m/s and record the range achieved (for both models) at a selection of launch angles between 30 and 60º. Plot your data. How has air resistance affected the optimum launch angle?

Mathematica Tutorial

If you want to learn about the capabilities of the *Mathematica* software, have a look at the on-line tutorial. From the **Help menu**, select **Tutorial**. The **Mathematica Tutorial** window will open and you will be asked to begin the tutorial. It is claimed that the tutorial takes only ten minutes, but you would probably spend considerable more time on it if you are keen to learn how to use the software. There are copies of the user manual ("The Mathematica Book") in the Brunel University Library.

References

Linthorne, N. P., & Everett, D. J. (2006). Release angle for attaining maximum distance in the soccer throw-in. *Sports Biomechanics*, *5*, 243–260.