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Singular Stress Behavior in a Bonded Hereditarily-Elastic Aging Wedge. Part I: Problem Statement and Degenerate Case

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Stress singularity is investigated in a plane problem for a bonded isotropic hereditarily elastic (viscoelastic) aging infinite wedge. The general solution of the operator Lamé equations, which are partial differential equations in space co-ordinates and integral equations in time, respectively, is represented in terms of one-parametric holomorphic functions (the Kolosov–Muskhelishvili complex potentials depending on time) in weighted Hardy-type classes. After application of the Mellin transform with respect to the radial variable, the problem is reduced to a system of linear Volterra integral equations in time. By using the residue theory for the inverse Mellin transform, the stress asymptotics and strain estimates near the singular point are presented here for non-hereditary Dundurs parameters. The general case of the hereditary Dundurs operators is considered in Part II (see [21]).

1. Introduction, constitutive equations, and general solution

The stress singularity in bonded elastic bodies has been sufficiently well studied. Bogy [7–9], Bogy and Wang [10], and other authors investigated this problem for an infinite wedge by the Mellin transform technique. Aksentyan [1], and Chobanyan and Gevorkyan [12] analysed this problem by looking for a solution in a special power form (for homogeneous bodies such techniques was used by Williams [25, 26]).

However, if bonded parts exhibit creep and aging as, for example, reinforced concrete, plastics or composites, the stress singularity character can essentially change. The stress singularity in a plane problem for bonded hereditarily-elastic non-aging body was investigated in [4, 5, 11] using the Laplace transform with respect to time and the Mellin transform with respect to the radial variable.

The stress singularity in a corner point for a plane problem for a homogeneous hereditarily elastic aging body (for which the Laplace transform with respect to time is not effectively applicable) was investigated by using the Mellin transform with respect to the radial variable in [19]. Note that the stress asymptotics near the crack tip (the

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particular case of a corner point) in a continuously inhomogeneous aging body was analysed in [3, 27] by expansion of the solution in a Neumann-type series. In the present paper the technique of [19] is extended to investigate the stress singularity in a plane problem for a bonded isotropic hereditarily elastic aging body at the point of intersection of the interface with the external boundary. The consideration is based on the properties of functions from some Hardy-type classes [20].

Generalized Hooke's law for body parts considered has the form (see e.g. [23, 2])

$$\sigma_{ij} = \underline{\Lambda}\varepsilon_{kk}\delta_{ij} + 2\underline{\mu}\varepsilon_{ij}, \quad \varepsilon_{ij} = -(2\underline{\mu})^{-1}\underline{\nu}(1+\underline{\nu})^{-1}\sigma_{kk}\delta_{ij}$$

$$+ (2\underline{\mu})^{-1}\sigma_{ij}, \quad (i, j, k = 1 \div 3), \tag{1.1}$$

$$\underline{\nu} = (1/2)\underline{\Lambda}(\underline{\Lambda}+\underline{\mu})^{-1}, \quad \underline{\Lambda} := \Lambda^0(t) + \underline{\Lambda}^*, \quad \underline{\mu} := \mu^0(t) + \underline{\mu}^*,$$

$$(\underline{\Lambda}^*\varepsilon)(t) := \int_0^t \Lambda^*(t,\tau)\varepsilon(\tau)\,\mathrm{d}\tau, \quad (\underline{\mu}^*\varepsilon)(t) := \int_0^t \underline{\mu}^*(t,\tau)\varepsilon(\tau)\,\mathrm{d}\tau,$$

where σ_{ij} and ε_{ij} are the stress and strain tensors, $\underline{\Lambda}$ and $\underline{\mu}$ are the operator Lamé parameters different for the different body parts, $\Lambda^0(t)$, $\mu^0(t)$, $\Lambda^*(t,\tau)$, $\mu^*(t,\tau)$ are known functions, $\underline{\nu}$ is the operator Poisson ratio, and the summation is supposed in repeating subscripts. It is suggested that an action begins at the instant $t = 0$; for this reason the lower integration limits in these formulas are equal to zero. Underlined symbols designate hereditary Volterra operators of the first or the second kind; out-of-integral terms (of the operators of the second kind) are marked by zero superscript and integral ones by asterisk.

If $\Lambda^*(t,\tau) = \Lambda^*(t-\tau)$, $\mu^*(t,\tau) = \mu^*(t-\tau)$, $\Lambda^0 = \text{const.}$, $\mu^0 = \text{const.}$, then the material is hereditarily elastic but not aging. If $\Lambda^*(t,\tau) = \mu^*(t,\tau) = \mu^*(t,\tau) = 0$ and $\Lambda^0(t) \neq \text{const.}$ or $\mu^0(t) \neq \text{const.}$, then the material is elastic and aging but not hereditary. If $\Lambda^*(t,\tau) = \mu^*(t,\tau) = 0$ and $\Lambda^0(t) = \text{const.}$ and $\mu^0(t) = \text{const.}$, then the material is classical elastic.

Substituting (1.1) into the equilibrium equations

$$\sigma_{ij,\,j} = 0$$

and using the relations

$$\varepsilon_{ij} = (u_{i,j} + u_{j,i})/2$$

between strains and displacements, we come to the system of generalized Lamé equations, which are partial differential ones in space co-ordinates and integral ones in time:

$$(\underline{\Lambda}u_{j,j})_{,i} + [\underline{\mu}(u_{i,j} + u_{j,i})]_{,j} = 0. \tag{1.2}$$

For the plane strain or plane stress state $i, j = 1 \div 2$; for the plane stress case it is necessary, additionally, to replace $\underline{\Lambda}$ by $\underline{\Lambda}_* = 2\underline{\Lambda}(\underline{\Lambda} + 2\underline{\mu})^{-1}\underline{\mu}$. Here (x_1, x_2) are Cartesian co-ordinates in the plane.

The functions Λ^0, μ^0, Λ^*, μ^* are independent of the space co-ordinates x_i in each body part and we will consider the solution from function classes (see below) such that it is possible to interchange the order of the time operators $\underline{\Lambda}$ and $\underline{\mu}$ with the differentiation with respect to the space co-ordinates. Then it is possible to repeat reasoning given in [22] for the classical elasticity, considering now $\underline{\Lambda}$ and $\underline{\mu}$ as non-commutative hereditary operators. As a result, it is easy to show for the plane strain or the plane stress state that the representation of the general solution of hereditary elasticity equations (1.2) for an aging body holds in terms of the complex Kolosov–Muskhelishvili potentials $\varphi_j(z_j, t)$, $\phi_j(z_j, t)$ (see [19]). This representation in polar co-ordinates (ρ, θ) has form

$$u_\rho(\rho, \theta, t) = \frac{1}{4}\underline{\mu}^{-1}\left\{\sum_{j=1}^{2}[\mathrm{e}^{-\mathrm{i}\theta_j}\underline{\kappa}\varphi_j(z_j, \cdot) - \mathrm{e}^{-\mathrm{i}\theta_j}z_j\varphi_j'(z_j, \cdot) - \mathrm{e}^{\mathrm{i}\theta_j}\psi_j(z_j, \cdot)]\right\}(t),$$

$$u_\theta(\rho, \theta, t) = \frac{\mathrm{i}}{4}\underline{\mu}^{-1}\left\{\sum_{j=1}^{2}(-1)^j[\mathrm{e}^{-\mathrm{i}\theta_j}\underline{\kappa}\varphi_j(z_j, \cdot) + \mathrm{e}^{-\mathrm{i}\theta_j}z_j\varphi_j'(z_j, \cdot)\right.$$

$$\left. + \mathrm{e}^{\mathrm{i}\theta_j}\psi_j(z_j, \cdot)]\right\}(t),$$

$$\sigma_{\rho\rho}(\rho, \theta, t) = \frac{1}{2}\left\{\sum_{j=1}^{2}[2\Phi_j(z_j, t) - z_j\Phi_j'(z_j, t) - \mathrm{e}^{2\mathrm{i}\theta_j}\Psi_j(z_j, t)]\right\},$$

$$\sigma_{\theta\theta}(\rho, \theta, t) = \frac{1}{2}\left\{\sum_{j=1}^{2}[2\Phi_j(z_j, t) + z_j\Phi_j'(z_j, t) + \mathrm{e}^{2\mathrm{i}\theta_j}\Psi_j(z_j, t)]\right\},$$

$$\sigma_{\rho\theta}(\rho, \theta, t) = \frac{\mathrm{i}}{2}\left\{\sum_{j=1}^{2}(-1)^j[z_j\Phi_j'(z_j, t) + \mathrm{e}^{2\mathrm{i}\theta_j}\Psi_j(z_j, t)]\right\}. \tag{1.3}$$

For the case of plane strain $\sigma_{33}(\rho, \theta, t) = 2\underline{\nu}[\Phi_1(z_1, t) + \Phi_2(z_2, t)]$.

The plane elasticity parameter is $\underline{\kappa} = 3 - 4\underline{\nu}$ for the plane strain and $\underline{\kappa} = (3 - \underline{\nu})(1 + \underline{\nu})^{-1}$ for the plane stress state; $\varphi_j(z_j, t)$ and $\psi_j(z_j, t)$ are analytical functions of the complex arguments z_j and depend also on time t for a hereditary medium. The following notations are introduced:

$$\theta_1(\theta) := \theta_2(\theta) := \theta,\ z_j(\rho, \theta) := \rho\exp(\mathrm{i}\theta_j),$$

$$\Phi_j(z_j, t) := \varphi_j'(z_j, t),\ \Psi_j(z_j, t) := \psi_j'(z_j, t).$$

The prime denotes the derivative of the analytical functions $\varphi_j(z_j, t)$, $\psi_j(z_j, t)$ with respect to z_j. Here and in the following, the summation with respect to repeating indices is not supposed. It is evident that $z_1(\rho, \theta) = x_1 + \mathrm{i}x_2 = \overline{z_2(\rho, \theta)}$ and the usual properties $\varphi_1(z_1, t) = \overline{\varphi_2(z_2, t)}$, $\psi_1(z_1, t) = \overline{\psi_2(z_2, t)}$ (see [22]) will be a consequence of real-valued applied loads.

2. Definitions of some classes of holomorphic functions and Volterra operators

We present in this section the definitions of some weighted Hardy-type classes of one-parametric holomorphic functions and Volterra operators dependent on a parameter, which will be used in the next sections. A description of their properties is given in [20] (see also the Appendix).

Let us consider a segment $[0, T]$, $0 < T \leqslant \infty$, and (for $T = \infty$) a half-infinite interval $[0, \infty)$. Unless otherwise stated, all definitions and results formulated in this section for the segment $[0, T]$ hold also for $T = \infty$ after replacing $[0, T]$ by $[0, \infty)$ and the non-strict inequalities $\leqslant T$ by the strict ones $< \infty$.

Let us denote an open wedge $(0 < \rho < \infty, \theta_- < \theta < \theta_+)$ in the complex z-plane $(z = \rho e^{i\theta})$ by $W(\theta_-, \theta_+)$ and an open strip $(-\infty < \operatorname{Im}\gamma < \infty, \delta_0 < \operatorname{Re}\gamma < \delta_\infty)$ in the complex γ-plane by $S(\delta_0, \delta_\infty)$; $\bar{W}(\theta_-, \delta_+)$ is then closed wedge $(0 < \rho < \infty, \theta_- \leqslant \theta \leqslant \theta_+)$, $\bar{S}(\delta_0, \delta_\infty)$ is the closed strip $(-\infty < \operatorname{Im}\gamma < \infty, \delta_0 \leqslant \operatorname{Re}\gamma \leqslant \delta_\infty)$.

We introduce now four basic function classes: $C\hat{L}_2(\delta_0, \delta_\infty; 0, T)$ for the prescribed boundary loadings, $CH_2^0(S(\delta_0, \delta_\infty); 0, T)$ for their Mellin transforms, $CH_2(\delta_0, \delta_\infty; W(\theta_-, \theta_+); 0, T)$ for the Kolosov–Muskhelishvili potentials in a wedge, and $CH_2^\vee(\theta_-, \theta_+; S(\delta_0, \delta_\infty); 0, T)$ for their Mellin transforms. Classes $VC(0, T)$ and $VCH[S(\delta_0, \delta_\infty); 0, T]$ are also introduced for Volterra integral operators independent of and dependent on a parameter, respectively.

1^0 The class $\hat{L}_2(\delta_0, \delta_\infty)$ consists of functions $g(\rho)$ defined on the half axis such that $\|g; \delta\|_2 := [\int_0^\infty |g(\rho)\rho^\delta|^2 \rho^{-1}\, d\rho]^{1/2} < \infty$ for any $\delta \in (\delta_0, \delta_\infty)$.
The class $C\hat{L}_2(\delta_0, \delta_\infty; 0, T)$ consists of functions $g(\rho, t)$ such that $\sup_{0 \leqslant t \leqslant T} \|g(\cdot, t); \delta\|_2 < \infty$ for any $\delta \in (\delta_0, \delta_\infty)$, and $\|g(\cdot, t + \Delta t) - g(\cdot, t); \delta\|_2 \to 0$ as $\Delta t \to 0$ for any $t \in [0, T]$ and any $\delta \in (\delta_0, \delta_\infty)$.

It is easy to see that, particularly, to $C\hat{L}_2(\delta_0, \delta_\infty; 0, T)$ belong all functions $g(\rho, t)$ continuous in t at almost all ρ, whose supremus in t are locally square integrable with respect to ρ on $(0, \infty)$, and such that for any $\varepsilon > 0$ there are constants C_0, C_∞ for which the estimates hold:

$$|g(\rho, t)| < C_0 \rho^{-\delta_0 - \varepsilon} \quad (\rho \to 0), \quad |g(\rho, t)| < C_\infty \rho^{-\delta_\infty + \varepsilon} \quad (\rho \to \infty).$$

2^0 $H_2^0(S(\delta_0, \delta_\infty))$ is the class of functions $\tilde{h}(\gamma)$ holomorphic in $S(\delta_0, \delta_\infty)$ such that the norm $M_2^0(\tilde{h}; \delta) := [\int_{-\infty}^\infty |\tilde{h}(\delta + i\xi)|^2\, d\xi]^{1/2}$ is uniformly bounded with respect to the parameter δ on any $[\delta'_0, \delta'_\infty] \subset (\delta_0, \delta_\infty)$.

$CH_2^0(S(\delta_0, \delta_\infty); 0, T)$ is the class of functions $\tilde{h}(\gamma, t) \in H_2^0(S(\delta_0, \delta_\infty))$ with respect to γ for every $t \in [0, T]$ such that $\sup_{0 \leqslant t \leqslant T} M_2^0(\tilde{h}(\cdot, t); \delta)$ is uniformly bounded with respect to δ on any $[\delta'_0, \delta'_\infty] \subset (\delta_0, \delta_\infty)$ and $M_2^0(\tilde{h}(\cdot, t + \Delta t) - \tilde{h}(\cdot, t); \delta) \to 0$ as $\Delta t \to 0$ for any $t \in [0, T]$ uniformly with respect to δ on any $[\delta'_0, \delta'_\infty] \subset (\delta_0, \delta_\infty)$.

3^0 $H_2(\delta_0, \delta_\infty; W(\theta_-, \theta_+))$ is the class of functions $h(z)$ holomorphic in $W(\theta_-, \theta_+)$ such that the norm

$$M_2(h; \theta_-, \theta_+; \delta) := \sup_{\theta_- < \theta < \theta_+} \left[\int_0^\infty |h(\rho e^{i\theta})\rho^\delta|^2 \rho^{-1} d\rho \right]^{1/2} < \infty, \quad \forall \delta \in (\delta_0, \delta_\infty).$$

$CH_2\ (\delta_0, \delta_\infty; W(\theta_-, \theta_+); 0, T)$ is the class of functions $h(z, t) \in H_2(\delta_0, \delta_\infty; W(\theta_-, \theta_+))$ with respect to z for every $t \in [0, T]$ such that $\sup_{0 \leqslant t \leqslant T} M_2(h(\cdot, t); \theta_-, \theta_+; \delta) < \infty$ for any $\delta \in (\delta_0, \delta_\infty)$, and $M_2(h(\cdot, t + \Delta t) - h(\cdot, t); \theta_-, \theta_+; \delta) \to 0$ as $\Delta t \to 0$ for any $t \in [0, T]$ and any $\delta \in (\delta_0, \delta_\infty)$.

4^0 $H_2^\vee(\theta_-, \theta_+; S(\delta_0, \delta_\infty))$ is the class of functions $h^\vee(\gamma)$ holomorphic in $S(\delta_0, \delta_\infty)$ such that the norm

$$M_2^\vee(h^\vee; \theta_-, \theta_+; \delta) := \sup_{\theta_- < \theta < \theta_+} \left[\int_{-\infty}^\infty |h^\vee(\delta + i\xi) e^{\xi\theta}|^2\, d\xi \right]^{1/2}$$

is uniformly bounded with respect to δ on any $[\delta'_0, \delta'_\infty] \subset (\delta_0, \delta_\infty)$.

$\mathrm{CH}_2^{\vee}(\theta_-, \theta_+; S(\delta_0, \delta_\infty); 0, T)$ is the class of functions $h^{\vee}(\gamma, t) \in H_2^{\vee}(\theta_-, \theta_+; S(\delta_0, \delta_\infty))$ with respect to γ for all $t \in [0, T]$ such that $\sup_{0 \leqslant t \leqslant T} M_2^{\vee}(h^{\vee}(\cdot, t); \theta_-, \theta_+; \delta)$ is uniformly bounded with respect to δ on any $[\delta_0', \delta_\infty'] \subset (\delta_0, \delta_\infty)$ and $M_2^{\vee}(h^{\vee}(\cdot, t + \Delta t) - h^{\vee}(\cdot, t); \theta_-, \theta_+; \delta) \to 0$ as $\Delta t \to 0$ for any $t \in [0, T]$ uniformly with respect to δ on any $[\delta_0', \delta_\infty'] \subset (\delta_0, \delta_\infty)$.

5^0 The class $\mathrm{VC}(0, T)$ consists of Volterra operators $\underline{K}^*$ (of the first kind) such that

$$|||K^*; 0, T||| := \sup_{0 \leqslant t \leqslant T} \int_0^t |K^*(t, \tau)| \, \mathrm{d}\tau < \infty$$

and

$$\int_0^t |K^*(t + \Delta t, \tau) - K^*(t, \tau)| \, \mathrm{d}\tau \to 0, \quad \Delta t \to 0, \forall t \in [0, T].$$

Particularly, $\underline{K}^* \in \mathrm{VC}(0, T)$ may have kernels of the Abel type: $K^*(t, \tau) = K_c^*(t, \tau)(t - \tau)^{-\zeta}, 0 \leqslant \zeta < 1, K_c^*(t, \tau) \in C[0, T] \times C[0, T]$.

6^0 An operator $\underline{K}(\gamma)$ belongs to the class $\mathrm{VCH}[S(\delta_0, \delta_\infty); 0, T]$ if: for any $\gamma \in S(\delta_0, \delta_\infty)$ it is a Volterra integral operator

$$[\underline{K}(\gamma)g](t) := \int_0^t K(\gamma, t, \tau) g(\tau) \, \mathrm{d}\tau, \quad t \in [0, T];$$

the kernel $K(\gamma, t, \tau)$ is holomorphic with respect to $\gamma \in S(\delta_0, \delta_\infty)$ at every t, at a.e. τ;

$$|||K; \bar{S}'; 0, \ T||| := \sup_{0 \leqslant t \leqslant T} \int_0^t \sup_{\gamma \in \bar{S}'} |K(\gamma, t, \tau)| \, \mathrm{d}\tau < \infty;$$

and

$$\int_0^t \sup_{\gamma \in \bar{S}'} |K(\gamma, t + \Delta t, \tau) - K(\gamma, t, \tau)| \, \mathrm{d}\tau \to 0, \Delta t \to 0, \forall t \in [0, T],$$

$$\forall \bar{S}' \subset S(\delta_0, \delta_\infty).$$

The definitions of $\mathrm{VC}(0, T)$ and $\mathrm{VCH}[S(\delta_0, \delta_\infty); 0, T]$ also hold for matrix Volterra integral operators if one understands by $|K(\gamma, t, \tau)|$ a matrix norm of the kernel $K(\gamma, t, \tau)$.

3. A problem for a bonded wedge

We will consider a plane problem for a bonded infinite wedge $W^{(1)} \cup W^{(2)}$:

$$W^{(l)} := \mathrm{W}(\theta_-^{(l)}, \theta_+^{(l)}), \qquad \theta_-^{(1)} = \theta^{(1)} < \theta_+^{(1)} = \theta_-^{(2)} = 0 < \theta_+^{(2)} = \theta^{(2)}. \tag{3.1}$$

Functions and operators given in the regions $W^{(l)}$ are marked below by the corresponding supplementary subscript or superscript. We will consider a problem on a time segment $[0, T]$ for $T \leqslant \infty$. Suppose the hereditary operators $\Lambda^*, \mu^* \in \mathrm{VC}(0, T)$ (see point 5^0 of section 2) and the out-of-integral terms $\Lambda^0(t), \mu^0(t) \in C[0, T]$, $\Lambda^0(t), \mu^0(t) \neq 0$ $(t \in [0, T])$.

Let some tractions $g_i^{(l)}(\rho, t)$ be prescribed on the external boundaries $\theta = \theta^{(1)}, \theta = \theta^{(2)}$:

$$\sigma_{\theta\theta}^{(1)}(\rho, \theta^{(1)}, t) = g_\theta^{(1)}(\rho, t), \quad \sigma_{\rho\theta}^{(1)}(\rho, \theta^{(1)}, t) = g_\rho^{(1)}(\rho, t),$$

$$\sigma_{\theta\theta}^{(2)}(\rho, \theta^{(2)}, t) = g_\theta^{(2)}(\rho, t), \quad \sigma_{\rho\theta}^{(2)}(\rho, \theta^{(2)}, t) = g_\rho^{(2)}(\rho, t),$$

and jumps of tractions $g_i^{(0)}(\rho, t)$ and of displacements $f_i^{(0)}(\rho, \tau)$ on the interface $\theta = 0$, respectively:

$$\sigma_{\theta\theta}^{(1)}(\rho, 0, t) - \sigma_{\theta\theta}^{(2)}(\rho, 0, t) = g_\theta^{(0)}(\rho, t), \quad \sigma_{\rho\theta}^{(1)}(\rho, 0, t) - \sigma_{\rho\theta}^{(2)}(\rho, 0, t) = g_\rho^{(0)}(\rho, t).$$

$$u_\rho^{(1)}(\rho, 0, t) - u_\rho^{(2)}(\rho, 0, t) = f_\rho^{(0)}(\rho, t), \quad u_\theta^{(1)}(\rho, 0, t) - u_\theta^{(2)}(\rho, 0, t) = f_\rho^{(0)}(\rho, t). \tag{3.3}$$

After substituting relations (1.3) into boundary conditions (3.2) we obtain

$$\sum_{j=1}^{2} [2\Phi_j^{(1)} z_j(\rho, \theta^{(1)}), t) + z_j(\rho, \theta^{(1)})\Phi_j^{(1)\prime} (z_j(\rho, \theta^{(1)}), t)$$

$$+ \exp[-2(-1)^j \mathrm{i}\theta^{(1)}]\Psi_j^{(1)}(z_j(\rho, \theta^{(1)}), t)] = 2g_\theta^{(1)}(\rho, t), \tag{3.4}$$

$$-\sum_{j=1}^{2} (-1)^j [z_j(\rho, \theta^{(1)})\Phi_j^{(1)\prime} (z_j(\rho, \theta^{(1)}), t)$$

$$+ \exp[-2(-1)^j \mathrm{i}\theta^{(1)}]\Psi_j^{(1)}(z_j(\rho, \theta^{(1)}), t)] = 2\mathrm{i}g_\rho^{(1)}(\rho, t), \tag{3.5}$$

$$\sum_{j=1}^{2} [2\Phi_{j2}(z_j(\rho, \theta^{(2)}), t) + z_j(\rho, \theta^{(2)})\Phi_j^{(2)\prime}(z_j(\rho, \theta^{(2)}), t)$$

$$+ \exp[-2(-1)^j \mathrm{i}\theta^{(2)}]\Psi_j^{(2)}(z_j(\rho, \theta^{(2)}), t)] = 2g_\theta^{(2)}(\rho, t), \tag{3.6}$$

$$-\sum_{j=1}^{2} (-1)^j [z_j(\rho, \theta^{(2)})\Phi_j^{(2)\prime}(z_j(\rho, \theta^{(2)}), t)$$

$$+ \exp[-2(-1)^j \theta^{(2)}]\Psi_j^{(2)}(z_j(\rho, \theta^{(2)}), t)] = 2\mathrm{i}g_\rho^{(2)}(\rho, t). \tag{3.7}$$

After substituting relations (1.3) into boundary conditions (3.3) on the interface, differentiating the conditions for the displacements, and denoting $f_i^{(0)\prime}(\rho, \tau) := \partial f_i^{(0)}(\rho, \tau)/\partial\rho$, we obtain

$$\sum_{j=1}^{2} [2\Phi_1^{(1)}(z_j(\rho, 0), t) + z_j(\rho, 0)\Phi'_{j1}(z_j(\rho, 0), t) + \Psi_j^{(1)}(z_j(\rho, 0), t)]$$

$$-\sum_{j=1}^{2} [2\Phi_{j2}(z_j(\rho, 0), t) + z_j(\rho, 0)\Phi'_{j2}(z_j(\rho, 0), t)$$

$$+ \Psi_j^{(2)}(z_j(\rho, 0), t)] = 2g_\theta^{(0)}(\rho, t), \tag{3.8}$$

$$- \sum_{j=1}^{2} (-1)^j [z_j(\rho,0)\Phi'_{j1}(z_j(\rho,0),t) + \Psi_j^{(1)}(z_j(\rho,0),t)]$$

$$+ \sum_{j=1}^{2} (-1)^j [z_j(\rho,0)\Phi'_{j2}(z_j(\rho,0),t) + \Psi_j^{(2)}(z_j(\rho,0),t)] = 2ig_\rho^{(0)}(\rho,t), \tag{3.9}$$

$$(\underline{\mu}^{(1)})^{-1} \left\{ \sum_{j=1}^{2} (\underline{\kappa}^{(1)} - 1)\Phi_j^{(1)}(z_j(\rho,0),\cdot) - z_j(\rho,0)\Phi'_{j1}(z_j(\rho,0),\cdot) - \Psi_j^{(1)}(z_j(\rho,0),\cdot)] \right\}$$

$$\times (t) - (\underline{\mu}^{(2)})^{-1} \left\{ \sum_{j=1}^{2} [\underline{\kappa}^{(2)} - 1)\Phi_{j2}(z_j(\rho,0),\cdot) - z_j(\rho,0)\Phi'_{j2}(z_j(\rho,0),\cdot) \right.$$

$$\left. - \Psi_j^{(2)}(z_j(\rho,0),\cdot)] \right\} (t) = 4f_\rho^{(0)\prime}(\rho,t), \tag{3.10}$$

$$(\underline{\mu}^{(1)})^{-1} \left\{ \sum_{j=1}^{2} (-1)^j [(\underline{\kappa}^{(1)} + 1)\Phi_j^{(1)}(z_j(\rho,0),\cdot) + z_j(\rho,0)\Phi'_{j1}(z_j(\rho,0),\cdot) \right.$$

$$\left. + \Psi_j^{(1)}(z_j(\rho,0),t)] \right\} (t) - (\underline{\mu}^{(2)})^{-1} \left\{ \sum_{j=1}^{2} (-1)^j [(\underline{\kappa}^{(2)} + 1)\Phi_{j2}(z_j(\rho,0),\cdot) \right.$$

$$\left. + z_j(\rho,0)\Phi'_{j2}(z_j(\rho,0),\cdot) + \Psi_j^{(2)}(z_j(\rho,0),\cdot)] \right\} (t) = -4\mathrm{i}f'_{\theta 0}(\rho,t). \tag{3.11}$$

Here only equations (3.10) and (3.11) include the integral operators and their operator coefficients are expressed in terms of four hereditary operators $\underline{\mu}^{(l)}$, $\underline{\kappa}^{(l)}$ ($l = 1$–2). Using equations (3.8) and (3.9), we reduce equations (3.10) and (3.11) to equivalent ones, whose operator coefficients are expressed in terms of only two combinations of these operators.

Let us introduce the hereditary operator analogs $\underline{\alpha}$ and $\underline{\beta}$ of the Dundurs parameters represented for classical elasticity problems (without heredity) in [14, 15] (see also [8, 9]):

$$\tilde{\underline{\mu}} := [(\underline{\mu}^{(1)})^{-1}(\underline{\kappa}^{(1)} + 1) + (\underline{\mu}^{(2)})^{-1}(\kappa^{(2)} + 1)]^{-1},$$

$$\underline{\alpha} := \tilde{\underline{\mu}}[(\underline{\mu}^{(1)})^{-1}(\underline{\kappa}^{(1)} + 1) - (\underline{\mu}^{(2)})^{-1}(\underline{\kappa}^{(2)} + 1)],$$

$$\underline{\beta} := \tilde{\underline{\mu}}[(\underline{\mu}^{(1)})^{-1}(\underline{\kappa}^{(1)} - 1) - (\underline{\mu}^{(2)})^{-1}(\underline{\kappa}^{(2)} - 1)]. \tag{3.12}$$

Let the operator $(\underline{\mu}_1^{-1} + \underline{\mu}_2^{-1})$ act on equations (3.8) and (3.9), add then the equations obtained, respectively, to doubled equations (3.10) and (3.11). After acting the operator $\tilde{\underline{\mu}}$ on the both equations resulting, we have

$$\sum_{j=1}^{2} \{(\underline{\beta} + 1)\Phi_j^{(1)}(z_j(\rho,0),\cdot) - \tfrac{1}{2}(\underline{\alpha} - \underline{\beta})[z_j(\rho,0)\Phi_j^{(1)\prime}(z_j(\rho,0),\cdot)$$

$$+ \Psi_j^{(1)}(z_j(\rho,0),\cdot)]\}(t) + \sum_{j=1}^{2} \{(\underline{\beta} - 1)\Phi_{j2}(z_j(\rho,0),\cdot) - \tfrac{1}{2}(\underline{\alpha} - \underline{\beta})$$

$$\times [z_j(\rho,0)\Phi_j^{(2)\prime}(z_j(\rho,0),\cdot) + \Psi_j^{(2)}(z_j(\rho,0),\cdot)]\}(t) = 2\tilde{g}_\theta^{(0)}(\rho,t), \tag{3.13}$$

$$\sum_{j=1}^{2}(-1)^j\{(\underset{\sim}{\alpha}+1)\Phi_j^{(1)}(z_j(\rho,0),\cdot)+\tfrac{1}{2}(\underset{\sim}{\alpha}-\underset{\sim}{\beta})[z_j(\rho,0)\Psi_j^{(1)\prime}(z_j(\rho,0),\cdot)$$

$$+\Psi_j^{(1)}(z_j(\rho,0)\cdot,)]\}(t)$$

$$+\sum_{j=1}^{2}(-1)^j\{(\underset{\sim}{\alpha}-1)\Phi_{j2}(z_j(\rho,0),\cdot)+\tfrac{1}{2}(\underset{\sim}{\alpha}-\underset{\sim}{\beta})[z_j(\rho,0)\Phi_j^{(2)\prime}(z_j(\rho,0),\cdot)$$

$$+\Psi_j^{(2)}(z_j(\rho,0),\cdot)]\}(t)=2\mathrm{i}\tilde{g}_\rho^{(0)}(\rho,t), \tag{3.14}$$

$$\tilde{g}_\theta^{(0)}(\rho,t):=\underset{\sim}{\tilde{\mu}}\{[(\underset{\sim}{\mu}^{(1)})^{-1}+(\underset{\sim}{\mu}^{(2)})^{-1}]g_\theta^{(0)}(\rho,t)+4f_\rho^{(0)\prime}(\rho,t)\},$$

$$\tilde{g}_\rho^{(0)}(\rho,t):=\underset{\sim}{\tilde{\mu}}\{[(\underset{\sim}{\mu}^{(1)})^{-1}+(\underset{\sim}{\mu}^{(2)})^{-1}]g_\rho^{(0)}(\rho,t)-4f_\theta^{(0)\prime}(\rho,t)\},$$

Suppose henceforth that $\underset{\sim}{\tilde{\mu}}=\tilde{\mu}^0(t)+\underset{\sim}{\tilde{\mu}}^*$, $(\underset{\sim}{\mu}^{(l)})^{-1}=[1/\mu^{(l)0}(t)]+\underset{\sim}{\tilde{\tilde{\mu}}}^{(l)*}$, where $\tilde{\mu}^0(t)$, $1/\mu^{(l)0}(t)\in C[0,T]$ and $\underset{\sim}{\tilde{\mu}}^*$, $\underset{\sim}{\tilde{\tilde{\mu}}}^{(l)*}\in \mathrm{VC}(0,T)$.

To take into consideration the boundary loads having power singularities with respect to radial variable, suppose that the prescribed functions $g_i^{(l)}, f_i^{(0)\prime}\in \mathrm{C}\hat{\mathrm{L}}_2(\delta_g,1;0,T)$ for some $\delta_g<1$. Then (see point 8^0 of the Appendix) also $\tilde{g}_i^{(0)}\in \mathrm{C}\hat{\mathrm{L}}_2(\delta_g,1;0,T)$.

Thus, we have obtained the system of eight linear boundary value intergo-differential equations (3.4)–(3.9), (3.13), (3.14) for the eight unknown complex potentials $\Phi_j^{(l)}(z_j,t)$, $\Psi_j^{(l)}(z_j,t)$ $(j,l=1\div 2)$, $z_j\in W_j^{(l)}=W(\theta_{j-}^{(l)},\theta_{j+}^{(l)})$, $\theta_{j\mp}^{(l)}:=-(-1)^j\theta_{\mp}^{(l)}$ (see (3.1)), $t\in[0,T]$. In system (3.4)–(3.9), (3.13), (3.14) only the last two equations are integral ones in reality and their left-hand sides are expressed in terms of only two Dundurs operators $\underset{\sim}{\alpha}$ and $\underset{\sim}{\beta}$. The original operators $\underset{\sim}{\mu}^{(l)}$, $\underset{\sim}{\kappa}^{(l)}$ will affect the solution only through the right-hand sides $\tilde{g}_i^{(0)}$ of equations (3.13), (3.14).

Let us consider the following linear combinations of the complex potentials: $\tilde{\Psi}_{j\mp}^{(l)}(z_j,\tau):=z_j\Phi_j'(z_j,\tau)+\exp(2\mathrm{i}\theta_{j\mp}^{(l)})\Psi_j(z_j,\tau)$. In fact, only these combinations and the original potentials $\Phi_j(z_j,\tau)$ are involved in the boundary equations (3.4)–(3.9), (3.13), (3.14). We shall write that some functions $\Phi_j^{(l)},\Psi_j^{(l)}\in \mathrm{C}\tilde{\mathrm{H}}_2(\delta_0,\delta_\infty;W_j^{(l)};0,T)$ if $\Phi_j^{(l)}\in \mathrm{CH}_2(\delta_0,\delta_\infty;W_j^{(l)};0,T)$ and $\tilde{\Psi}_{j\mp}^{(l)}\in \mathrm{CH}_2(\delta_0,\delta_\infty;\tilde{W}_{j\mp}^{(l)};0,T)$, where $\tilde{W}_{j-}^{(l)}:=W(\theta_{j-}^{(l)},\tilde{\theta}_j^{(l)})$, $\tilde{W}_{j+}^{(l)}:=W(\tilde{\theta}_j^{(l)},\theta_{j+}^{(l)})$, for any $\tilde{\theta}_j^{(l)}\in(\theta_{j-}^{(l)},\theta_{j+}^{(l)})$.

We look for a solution of the problem (3.4)–(3.9), (3.13), (3.14) such that the Kolosov–Muskhelishvili potentials

$$\Phi_j^{(l)}(z_j,\tau),\ \Psi_j^{(l)}(z_j,\tau)\in \mathrm{C}\tilde{\mathrm{H}}_2(\tilde{\delta}_0,1;W_j^{(l)};0,T) \tag{3.15}$$

for some appropriate $\tilde{\delta}_0<1$.

Note that this also provides the membership of $z_j\Phi_j^{(l)\prime}(z_j,\tau),\Psi_j^{(l)}(z_j,\tau)$ to $\mathrm{CH}_2(\tilde{\delta}_0,1;W_j^{(l)\prime};0,T)$ for any $\bar{W}_j^{(l)\prime}\subset W_j^{(l)}$. Since $\tilde{\delta}_0<1$, the memberships $\Phi_j^{(l)}(z_j(\rho,\theta),\tau)\in L_2(W_{j0}^{(l)})$, $z_j\Phi_j^{(l)\prime}(z_j(\rho,\theta),\tau)+\exp(\mathrm{i}\theta_j)\Psi_j^{(l)}(z_j(\rho,\theta),\tau)\in L_2(W_{j0}^{(l)})$ for any finite part $W_{j0}^{(l)}$ of the wedge $W_j^{(l)}$ and for any $\tau\in[0,T]$ follow from Lemmas 1.17, 1.18 and Remark 1.19 in [20]. Consequently, the functions $u_{i,k}(\rho,\theta,\tau)$, $\sigma_{ik}(\rho,\theta,\tau)\in L_2(W_0^{(l)})$ for any $\tau\in[0,T]$ and the stored elastic energy is finite in each finite part of the wedges for any $\tau\in[0,T]$ and $\tilde{\delta}_0<1$. Moreover, $u_{i,k}$ and σ_{ik} satisfy estimates of type (A.4) for any $\delta_\infty'<1$ and $\delta_0'>\tilde{\delta}_0$.

If (3.15) holds, then (see point 2^0 of the Appendix) the following Mellin transforms with respect to the complex variables z_j exist for any $\gamma \in S := S(\tilde{\delta}_0, 1)$:

$$\Phi_2^{(l)\vee}(\gamma, \tau) := \int_0^\infty \Phi_j^{(l)}(z_j, \tau) z_j^{\gamma-1}\, dz_j,$$

$$\Psi_1^{(l)\vee}(\gamma, \tau) := \int_0^\infty \Psi_j^{(l)}(z_j, \tau) z_j^{\gamma-1}\, dz_j,\ z_j \in W_j^{(l)},$$

$$\tilde{\Psi}_{j\mp}^{(l)\vee}(\gamma, \tau) = -\gamma \Phi_j^{(l)\vee}(\gamma, \tau) + \exp(2i\theta_{j\mp}^{(l)}) \Psi_{j\mp}^{(l)\vee}(\gamma, \tau);$$

$$\Phi_j^{(l)\vee}, \Psi_j^{(l)\vee} \in \mathrm{C}\tilde{\mathrm{H}}_2^{\vee}(\theta_{j-}^{(l)}, \theta_{j+}^{(l)}; S; 0, T).$$

Here and in the following, we write that functions $\Phi_j^{(l)\vee}, \Psi_j^{(l)\vee} \in \mathrm{C}\tilde{\mathrm{H}}_2^{\vee}(\theta_{j-}^{(l)}, \theta_{j+}^{(l)}; S; 0; T)$ if $\Phi_j^{(l)\vee} \in \mathrm{CH}_2^{\vee}(\theta_{j-}^{(l)}, \theta_{j+}^{(l)}; S; 0, T)$ and $\tilde{\Psi}_{j-}^{(l)\vee} \in \mathrm{CH}_2^{\vee}(\theta_{j-}^{(l)}, \tilde{\theta}_j^{(l)}; S; 0, T)$, $\tilde{\Psi}_{j+}^{(1)\vee} \in \mathrm{CH}_2^{\vee}(\tilde{\theta}_j^{(l)}, \theta_{j+}^{(l)}; S; 0, T)$ for any $\tilde{\theta}_j^{(l)} \in (\theta_{j-}^{(l)}, \theta_{j+}^{(l)})$. Let us also introduce the notations

$$\mathfrak{F}(\gamma, t) := \{\Phi_1^{(1)\vee}, \Phi_2^{(1)\vee}, \Psi_1^{(1)\vee}, \Psi_2^{(1)\vee}, \Phi_1^{(2)\vee}, \Phi_2^{(2)\vee}, \Psi_1^{(2)\vee}, \Psi_2^{(2)\vee}\}, \tag{3.16}$$

$$\Theta_{\mp} := \{\theta_{1\mp}^{(1)}, \theta_{2\mp}^{(1)}, \theta_{1\mp}^{(1)}, \theta_{2\mp}^{(1)}, \theta_{1\mp}^{(2)}, \theta_{2\mp}^{(2)}, \theta_{1\mp}^{(2)}, \theta_{2\mp}^{(2)}\}.$$

Then the membership $\mathfrak{F} \in \mathrm{C}\tilde{\mathrm{H}}_2^{\vee}(\Theta_-, \Theta_+; S; 0, T)$ means that $\Phi_j^{(l)\vee}, \Psi_j^{(l)\vee} \in \mathrm{C}\tilde{\mathrm{H}}_2^{\vee}(\theta_{j-}^{(l)}, \theta_{j+}^{(l)}; S; 0, T)$ $(l, j = 1 \div 2)$.

From (3.15) it also follows (see point 3^0 of the Appendix) that the Mellin transforms of the potentials with respect to the complex variables are expressed in terms of the Mellin transforms of their boundary values with respect to the radial variable:

$$\Phi_j^{(l)\vee}(\gamma, \tau) = \exp(i\gamma\theta_{j\mp}^{(l)}) \langle \Phi(\rho \exp(i\theta_{j\mp}^{(l)}), \tau) \rangle(\gamma),$$

$$\tilde{\Psi}_{j\mp}^{(l)\vee}(\gamma, \tau) = -\gamma \Phi_j^{(l)\vee}(\gamma, \tau) + \exp(2i\theta_{j\mp}^{(l)}) \Psi_j^{(l)\vee}(\gamma, \tau) =$$
$$\exp(i\gamma\theta_{j\mp}^{(l)}) \langle \tilde{\Psi}_{j\mp}^{(l)\vee}(\rho \exp(i\theta_{j\mp}^{(l)}), \tau) \rangle(\gamma),$$

where $\langle g \rangle(\gamma, t)$ denotes the Mellin transform of a function $g(\rho, t)$ with respect to the radial variable ρ (see (A.1)).

Taking this into account, we apply the Mellin transform with respect to radial variable to equations (3.4)–(3.9), (3.13), (3.14) and obtain the system of 8 Volterra integral equations (from which only the last two are integral ones in fact) with a parameter γ for the 8 unknown transforms of the complex potentials $\Phi_j^{(l)\vee}(\gamma, t)$, $\Psi_j^{(l)\vee}(\gamma, t)$. After denoting 8-dimensional vectors and (8×8) matrices by bold or gothic letters, the system can be written in the form:

$$[\underline{\mathbf{B}}(\gamma)\mathfrak{F}(\gamma, \cdot)](t) = \mathbf{G}(\gamma, t), \tag{3.17}$$

$$[\underline{\mathbf{B}}(\gamma)\mathfrak{F}(\gamma, \cdot)](t) := \left\{ \mathbf{B}^0(\gamma, t)\mathfrak{F}(\gamma, t) + \int_0^t \mathbf{B}^*(\gamma, t, \tau)\mathfrak{F}(\gamma, \tau)\, d\tau \right\},$$

The unknown vector $\mathfrak{F}(\gamma, t)$ is given by (3.16) while the known vector $\mathbf{G}(\gamma, t) := 2\ \{\langle g_\theta^{(1)} \rangle, i\langle g_\rho^{(1)} \rangle, \langle g_\theta^{(2)} \rangle, i\langle g_\rho^{(2)} \rangle, \langle g_\theta^{(0)} \rangle, i\langle g_\rho^{(0)} \rangle, \langle \tilde{g}_\theta^{(0)} \rangle, i\langle \tilde{g}_\rho^{(0)} \rangle\}$. Since $g_j^{(l)}, \tilde{g}_j^{(0)} \in \mathrm{C}\hat{\mathrm{L}}_2(\delta_g, 1; 0, T)$, we see from point 1^0 of the Appendix that $\mathbf{G} \in \mathrm{CH}_2^0(S(\delta_g, 1); 0, T)$.

The matrix operator $\underline{\mathbf{B}}(\gamma)$ has the form

$$
\begin{matrix}
(2-\gamma)e^{-i\gamma\theta^{(1)}} & (2-\gamma)e^{-i\gamma\theta^{(1)}} & e^{i(2-\gamma)\theta^{(1)}} & e^{i(2-\gamma)\theta^{(1)}} & 0 & 0 & 0 & 0 \\
-\gamma e^{-i\gamma\theta^{(1)}} & \gamma e^{i\gamma\theta^{(1)}} & e^{i(2-\gamma)\theta^{(1)}} & -e^{-(2-\gamma)\theta^{(1)}} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & (2-\gamma)e^{-i\gamma\theta^{(2)}} & (2-\gamma)e^{i\gamma\theta^{(2)}} & e^{i(2-\gamma)\theta^{(2)}} & e^{-i(2-\gamma)\theta^{(2)}} \\
0 & 0 & 0 & 0 & -\gamma e^{-i\gamma\theta^{(2)}} & \gamma e^{i\gamma\theta^{(2)}} & e^{-i(2-\gamma)\theta^{(2)}} & -e^{-i(2-\gamma)\theta^{(2)}} \\
(2-\gamma) & (2-\gamma) & 1 & 1 & -(2-\gamma) & -(2-\gamma) & -1 & -1 \\
-\gamma & \gamma & 1 & -1 & \gamma & -\gamma & -1 & 1 \\
\underline{\beta}+1+\frac{\gamma}{2}(\underline{\alpha}-\underline{\beta}) & \underline{\beta}+1+\frac{\gamma}{2}(\underline{\alpha}-\underline{\beta}) & -\frac{1}{2}(\underline{\alpha}-\underline{\beta}) & -\frac{1}{2}(\underline{\alpha}-\underline{\beta}) & \underline{\beta}-1+\frac{\gamma}{2}(\underline{\alpha}-\underline{\beta}) & \underline{\beta}-1+\frac{\gamma}{2}(\underline{\alpha}-\underline{\beta}) & -\frac{1}{2}(\underline{\alpha}-\underline{\beta}) & -\frac{1}{2}(\underline{\alpha}-\underline{\beta}) \\
-\underline{\alpha}-1+\frac{\gamma}{2}(\underline{\alpha}-\underline{\beta}) & \underline{\alpha}+1+\frac{\gamma}{2}(\underline{\alpha}-\underline{\beta}) & -\frac{1}{2}(\underline{\alpha}-\underline{\beta}) & \frac{1}{2}(\underline{\alpha}-\underline{\beta}) & -\underline{\alpha}+1+\frac{\gamma}{2}(\underline{\alpha}-\underline{\beta}) & \underline{\alpha}-1+\frac{\gamma}{2}(\underline{\alpha}-\underline{\beta}) & -\frac{1}{2}(\underline{\alpha}-\underline{\beta}) & \frac{1}{2}(\underline{\alpha}-\underline{\beta})
\end{matrix}
\tag{3.18}
$$

To obtain the solution of the original problem for a wedge, it is necessary to solve system (3.17) and to invert the Mellin transformed potentials $\Phi_j^{(l)\vee}$ $\Psi_j^{(j)\vee}$ in order to obtain stresses and displacements after substituting $\Phi_j^{(l)}$, $\Psi_j^{(l)}$ into relations (1.3). We will consider now the case when the operator $\underline{\mathbf{B}}(\gamma)$ degenerates into an algebraic matrix.

4. Degenerate case

For a degenerate case of hereditary operators we obtain here the power-logarithmic stress asymtotic stress asymptotics for the solution from the function class $C\tilde{H}_2$. The methods and results of this section will be used in the following sections. Particularly, the same asymptotics hold for some non-degenerate cases too. However, we show also in this section that even for the degenerate case it is impossible, in general, to obtain the analogous power-logarithmic asymptotics for strains but appropriate estimates are given instead.

Suppose $\underline{\alpha} = \alpha^0(t)$ and $\underline{\beta} = \beta^0(t)$, i.e. the operators $\underline{\alpha}$, $\underline{\beta}$ are not hereditary although may be aging. This happens particularly when the Poisson ratios $\underline{\nu}^{(l)}$ (and consequently the parameters $\underline{\kappa}^{(l)}$) are not hereditary (i.e. $\underline{\nu}^{(l)} = \nu^{(l)0}(t)$) and the operators $\underline{\mu}^{(1)}$ and $\underline{\mu}^{(2)}$ are proportional to one operator (i.e. $\underline{\mu}^{(l)} = m^{(l)}(t)\underline{\mu}$, where $\underline{\mu} \in \mathrm{VC}(0, T)$ is a hereditary operator and functions $m^{(l)}(t) \in C[0, T]$). In this case the operator $\underline{\mathbf{B}}(\gamma)$ degenerates into the algebraic matrix $\mathbf{B}^0(\gamma, t)$.

Then for any fixed time instant $t \in [0, T]$, we arrive at the system of linear algebraic equations

$$\mathbf{B}^0(\gamma, t)\mathfrak{F}(\gamma, t) = \mathbf{G}(\gamma, t) \tag{4.1}$$

which is also obtained by solving the corresponds classical elasticity problem for a wedge by using the complex Kolosov–Muskhelishvili potentials.

The solution of (4.1) is represented in the form

$$\mathfrak{F}(\gamma, t) = (\mathbf{B}^0)^{-1}(\gamma, t)\mathbf{G}(\gamma, t), \tag{4.2}$$

$$(\mathbf{B}^0)^{-1}(\gamma, t) := \mathbf{A}(\gamma, t)/\Delta(\gamma, t), \tag{4.3}$$

where $\Delta(\gamma, t)$ is the determinant of the matrix $\mathbf{B}^0(\gamma, t)$, $\mathbf{A}(\gamma, t)$ is the transposed matrix of its algebraic complements, and $(\mathbf{B}^0)^{-1}(\gamma, t)$ is the inverse matrix to $\mathbf{B}^0(\gamma, t)$. The function $\Delta(\gamma, t)$ coincides with the corresponding function $\mathscr{D}(\theta^{(2)}, -\theta^{(1)}, \alpha^0(t), \beta^0(t), 1-\gamma)$ given in [9], where the classical elasticity problem is solved for bonded wedges.

Let t be a fixed time instant. The matrixes $\mathbf{B}^0(\gamma, t)$ and $\mathbf{A}(\gamma, t)$ are entire functions of γ (see (3.18)). The function $\Delta(\gamma, t)$ is also an entire one, is not equal to zero identically (see e.g. [9]), and consequently may have only zeros of finite multiplicities N_k^0 at isolated points $\gamma_k(t)$ (see e.g. [6, chapter V, section 1]). By (4.3), consequently $(\mathbf{B}^0)^{-1}(\gamma, t)$ is a meromorphic function in the γ-plane with poles of finite multiplicities in $\gamma_k(t)$. Moreover, according to [16, Theorem 7.1], $(\mathbf{B}^0)^{-1}(\gamma, t)$ has the form

$$(B^0)^{-1}_{jw}(\gamma, t) = \sum_{n=1}^{N_k} \sum_{q=1}^{P_{kn}} [\gamma - \gamma_k(t)]^{-q} \sum_{p=0}^{P_{kn}-q} \phi_{jk}^{(np)}(t)\chi_{wk}^{(n, P_{kn}-q-p)}(t) + \Gamma_{jw}(\gamma, t) \tag{4.4}$$

in a neighbourhood of $\gamma_k(t)$. Here $j, w = 1 \div 8$; $N_k(t)$ is the dimension of the eigenspace of the matrix $B^0_{jw}(\gamma_k, t)$; $\phi_{jk}^{(np)}(t)$ and $\chi_{jk}^{(np)}(t)$ ($n = 1 \div N_k(t)$, $p = 0 \div P_{kn} - 1$) are some canonical systems of eigenvectors and associated vectors of the matrix $B^0_{jw}(\gamma, t)$ corresponding to $\gamma_k(t)$ and of the conjugate matrix $\overline{B^0_{wj}(\gamma, t)}$ corresponding to $\bar{\gamma}_k(t)$ at a fixed instant t; $\Gamma_{jw}(\gamma, t)$ is a matrix function holomorphic in $\gamma_k(t)$. In addition, due to a result in [16, point 3 of section 1], $\sum_{n=1}^{N_k(t)} P_{kn}(t) = N_k^0$, i.e. the algebraic multiplicity of the eigenvalue $\gamma_k(t)$ of the matrix $B_{jw}^0(\gamma_k, t)$ is equal to the multiplicity of the zero $\gamma_k(t)$ of the function $\Delta(\gamma, t)$.

Due to (3.16), we can write (4.2) and (4.3) also in the form

$$\Phi_j^{(1)\vee}(\gamma, t) = \sum_{w=1}^{8} (B^0)^{-1}_{jw}(\gamma, t) G_w(\gamma, t),$$
$$\Phi_j^{(2)\vee}(\gamma, t) = \sum_{w=1}^{8} (B^0)^{-1}_{4+j,w}(\gamma, t) G_w(\gamma, t),$$
$$\Psi_j^{(1)\vee}(\gamma, t) = \sum_{w=1}^{8} (B^0)^{-1}_{2+j,w}(\gamma, t) G_w(\gamma, t)$$
$$\Psi_j^{(2)\vee}(\gamma, t) = \sum_{w=1}^{8} (B^0)^{-1}_{6+j,w}(\gamma, t) G_w(\gamma, t),$$
$$\tilde{\Psi}_{j\mp}^{(l)\vee}(\gamma, t) = \sum_{w=1}^{8} \tilde{D}_{jw\mp}^{(l)}(\gamma, t) G_w(\gamma, t), \quad j = 1 \div 2,$$
$$\tilde{D}_{jw\mp}^{(l)}(\gamma, t) := -\gamma (B^0)^{-1}_{4l-4+j,w}(\gamma, t) + \exp(2\mathrm{i}\theta_{j\mp}^{(l)}) (B^0)^{-1}_{4l-2+j,w}(\gamma, t). \tag{4.5}$$

Hereafter, let $d(r, \tau)$ be a set of circular neighbourhoods with a radius $r > 0$ of all the roots $\gamma_k(\tau)$ and let for a strip S' in the γ-plane, $S'_r(\tau) := S' \backslash d(r, \tau)$ be a perforated strip.

The analysis of the elements of the matrix $\mathbf{B}^0(\gamma, \tau)$ (see (3.18)) shows that for any τ and any perforated strip S'_r there exist finite positive numbers $M' := M'(S'_r, \tau)$ and $M^\vee := M^\vee(S'_r, \tau)$ such that

$$|1/\Delta(\gamma, \tau)| < M' \exp[\mp 2(\theta^{(2)} - \theta^{(1)}) \operatorname{Im} \gamma] \quad (\operatorname{Im} \gamma \to \pm \infty), \tag{4.6}$$
$$|(B^0)^{-1}_{jw}(\gamma, \tau)| < M^\vee |\mathrm{e}^{\mathrm{i}\gamma\theta}|, \quad \forall (\gamma, \theta) \in \bar{S}'_r \times [\theta_{j-}^{(1)}, \theta_{j+}^{(1)}], \tag{4.7}$$
$$|(B^0)^{-1}_{4+j,w}(\gamma, \tau)| < M^\vee |\mathrm{e}^{\mathrm{i}\gamma\theta}|, \quad \forall (\gamma, \theta) \in \bar{S}'_r \times [\theta_{j-}^{(2)}, \theta_{j+}^{(2)}], \tag{4.8}$$
$$\tilde{D}_{jw-}^{(l)}(\gamma, \tau) < M^\vee |\mathrm{e}^{\mathrm{i}\gamma\theta}|, \quad \forall (\gamma, \theta) \in \bar{S}'_r \times [\theta_{j-}^{(l)}, \tilde{\theta}_j^{(l)}], \tag{4.9}$$
$$\tilde{D}_{jw+}^{(l)}(\gamma, \tau) < M^\vee |\mathrm{e}^{\mathrm{i}\gamma\theta}|, \quad \forall (\gamma, \theta) \in \bar{S}'_r \times [\tilde{\theta}_j^{(l)}, \theta_{j+}^{(l)}], \tag{4.10}$$

for $j = 1 \div 2$, $w = 1 \div 8$, and any $\tilde{\theta}_j^{(l)} \in (\theta_{j-}^{(l)}, \theta_{j+}^{(l)})$. Moreover, if a strip S' does not include zeros $\gamma_k(\tau)$ of $\Delta(\gamma, \tau)$ for $\tau \in [0, t]$, then $S'_r = S'$ and the parameteres M', $M^\vee$ can be considered as independent of time τ on the segment $[0, t]$.

Let us introduce

$$\delta_{g+}(t) := \max \left[\sup_k (\operatorname{Re} \gamma_k(t)), \delta_g \right] \quad (\operatorname{Re} \gamma_k(t) < 1),$$
$$\delta_{s+}(t) := \sup_{k,\tau} [\operatorname{Re} \gamma_k(\tau)] \quad (0 \leqslant \tau \leqslant t, \quad \operatorname{Re} \gamma_k(\tau) < 1),$$
$$\delta_{gs+}(t) := \max[\delta_{s+}(t), \delta_g] = \sup_\tau \delta_{g+}(\tau) \quad (0 \leqslant \tau \leqslant t). \tag{4.11}$$

If the instantaneous Dundurs parameters $\alpha^0(t)$, $\beta^0(t)$ are such that $\delta_{g+}(t) < 1$, then there are no zeros of $\Delta(\gamma, t)$ in the strip $S(\delta_{g+}(t), 1)$. If, in addition, $\alpha^0(\tau)$, $\beta^0(\tau)$ are such that $\delta_{gs+}(t) < 1$, then there are no zeros of $\Delta(\gamma, \tau)$ for any $\tau \in [0, t]$ in the strip $S(\delta_{gs+}(t), 1)$.

Let $\delta_{gs+}(t) < 1$. Taking into account representations (4.5), estimates (4.7)–(4.10) for $S'_r = S(\delta_{gs+}(t), 1)$, and the membership $\mathbf{G} \in \mathrm{CH}_2^0(S(\delta_g, 1); 0, T)$, we obtain from point 7^0 of the Appendix that $\Phi_j^{(l)\vee}, \Psi_j^{(l)\vee} \in \mathrm{C}\widetilde{\mathrm{H}}_2^{\vee}(\theta_{j-}^{(l)}, \theta_{j+}^{(l)}; S(\delta_{gs+}(t), 1); 0, t)$. Consequently (see point 2^0 of the Appendix), the inverse Mellin transform exists for the solution of (4.2) in $S(\delta_{gs+}(t), 1)$:

$$\Phi_j^{(l)}(z_j, \tau) = \frac{1}{2\pi \mathrm{i}} \int_{\delta - \mathrm{i}\infty}^{\delta + \mathrm{i}\infty} \Phi_j^{(l)\vee}(\gamma, \tau) z_j^{-\gamma}\, \mathrm{d}\gamma,$$

$$\Psi_j^{(l)}(z_j, \tau) = \frac{1}{2\pi \mathrm{i}} \int_{\delta - \mathrm{i}\infty}^{\delta + \mathrm{i}\infty} \Psi_j^{(l)\vee}(\gamma, \tau) z_j^{-\gamma}\, \mathrm{d}\gamma, \tag{4.12}$$

and $\Phi_j^{(l)}, \Psi_j^{(l)} \in \mathrm{C}\widetilde{\mathrm{H}}_2(\delta_{gs+}(t), 1; W_j^{(l)}; 0, t)$. Here $\delta \in (\delta_{gs+}(t), 1)$; $\Phi_j^{(l)\vee}, \Psi_j^{(l)\vee}$ are given by (4.5). Setting $t = T$, we get memberships (3.15) (where $\tilde{\delta}_0 = \delta_{gs+}(T)$) as was a priori supposed. Thus, the solution looked for is obtained. By point 6^0 of the Appendix, we obtain for any $t \in [0, T]$ the estimate, which is rather coarse but uniform with respect to $\tau \in [0, t]$:

$$|\Phi_j^{(l)}(z_j(\rho, \theta), \tau)|, \quad |\Psi_j^{(l)}(z_j(\rho, \theta), \tau)|, \quad |\tilde{\Psi}_{j\mp}^{(l)}(z_j(\rho, \theta), \tau)| < \tilde{\tilde{M}}(t) \rho^{-\delta_{gs+}(t) - \varepsilon},$$
$$\forall \varepsilon \in (0, 1 - \delta_{gs+}(t)), \forall \theta \in [\theta_-^{(l)\prime}, \theta_+^{(l)\prime}] \subset (\theta_-^{(l)}, \theta_+^{(l)}), \tag{4.13}$$

where the bound $\tilde{\tilde{M}}(t)$ is independent of τ and θ.

Let us investigate now the asymptotics of the solution as $\rho \to 0$. Let

$$\delta_-(t) := \min \{\min_k [\operatorname{Re} \gamma_k(t)], 1\} \quad (\operatorname{Re} \gamma_k(t) > \delta_g).$$

As usual (see e.g. [18]), we shift the investigation path in (4.12) to the left into the strip $S(\delta_g, \delta_-(t))$, calculating residues of the integrands at the zeros $\gamma_k(t)$ of the function $\Delta(\gamma, t)$. This is possible due to estimates (4.7)–(4.10) and to point 5^0 of the Appendix applied to $\mathbf{G}(\gamma, t)$. Thus, we obtain the asymptotic representation for the Kolosov–Muskhelishvili potentials as $\rho \to 0$:

$$\Phi_j^{(l)}(z_j, t) = \sum_{\delta_g < \operatorname{Re}\gamma_k < 1} \operatorname*{res}_{\gamma = \gamma_k} \left[\sum_{w=1}^{8} (B^0)^{-1}_{4l-4+j,w}(\gamma, t) G_w(\gamma, t) z_j^{-\gamma} \right] + \Phi_{*j}^{(l)}(z_j, t)$$
$$= \sum_{\delta_g < \operatorname{Re}\gamma_k < 1} z_j^{-\gamma_k} \sum_{n=1}^{N_k} \sum_{p=0}^{P_{kn}-1} K_{knp}(t) \sum_{q=0}^{p} \frac{1}{q!} (-\ln z_j)^q \phi_{4l-4+j,k}^{(n,p-q)}(t) + \Phi_{*j}^{(l)}(z_j, t), \tag{4.14}$$

$$\Psi_j^{(l)}(z_j, t) = \sum_{\delta_g < \operatorname{Re}\gamma_k < 1} z_j^{-\gamma_k} \sum_{n=1}^{N_k} \sum_{p=0}^{P_{kn}-1} K_{knp}(t) \sum_{q=0}^{p} \frac{1}{q!}$$
$$\times (-\ln z_j)^q \phi_{4l-2+j,k}^{(n,p-q)}(t) + \Psi_{*j}^{(l)}(z_j, t), \tag{4.15}$$

$$K_{knp}(t) := \sum_{v=0}^{P_{kn}-p-1} \sum_{w=1}^{8} \chi_{wk}^{(n, P_{kn}-v-p-1)}(t) G_{wk}^{(v)}(t), \quad G_{wk}^{(v)}(t) := \frac{1}{v!} \frac{\partial^v}{\partial \gamma^v} G_w(\gamma, t)|_{\gamma = \gamma_k}.$$

Representation (4.4) was used for the residue calculations. The terms $\Phi_{*j}^{(l)}, \Psi_{*j}^{(l)}$ have the form (4.12) for $\delta \in (\delta_g, \delta_-(t))$. Using estimates (4.7)–(4.10) for the strip $S'_r = S(\delta_g, \delta_-(t))$, the membership $\mathbf{G} \in \mathrm{CH}_2^0(S(\delta_g, 1); 0, T)$, and also point 2^0 from the Appendix, we see that $\Phi_{*j}^{(l)}(\cdot, t) \in H_2(\delta_g, \delta_-(t); W_j^{(l)})$ and $\tilde{\Psi}_{*j\mp}^{(l)}(\cdot, t) \in H_2(\delta_g, \delta_-(t); \tilde{W}_{j\mp}^{(l)})$ for any $\tilde{\theta}^{(l)} \in (\theta_-^{(l)}, \theta_+^{(l)})$ and for all $t \in [0, T]$.

After substituting (4.14) into (1.3), we obtain the stress asymptotics

$$\sigma_{ij}^{(l)}(\rho, \theta, t) = \sum_{\delta_g < \mathrm{Re}\, \gamma_k < 1} \rho^{-\gamma_k(t)} \sum_{n=1}^{N_k} \sum_{p=0}^{P_{kn}-1} K_{knp}(t)$$

$$\times \sum_{q=0}^{p} \ln^q\left(\frac{1}{\rho}\right) F_{ijkn,p-q}^{(l)}(\theta, t) + \sigma_{*ij}^{(l)}(\rho, \theta, t) \quad (i, j, l = 1 \div 2), \tag{4.16}$$

where

$$|\sigma_{*ij}^{(l)}(\rho, \theta, t)| < M_*(t)\rho^{-\delta_g - \varepsilon}, \quad \forall\, \varepsilon \in (0, \delta_-(t) - \delta_g),$$

$$\forall\, \theta \in [\theta_-^{(l)\prime}, \theta_+^{(l)\prime}] \subset (\theta_-^{(l)}, \theta_+^{(l)}).$$

The parameters $N_k(t)$ and $P_{kn}(t)$ in (4.14) and (4.16) are described above. The stress intensity factors $K_{knp}(t)$ depend on the right-hand sides of the boundary conditions and they are expressed by (4.15) for the infinite wedge. It is easy to see that for each γ_k, the number of stress intensity factors $K_{knp}(t)$ is equal to $\sum_{n=1}^{N_k(t)} P_k(t) = N_k^0(t)$, i.e. to the multiplicity of the zero of the determinant $\Delta(\gamma, t)$. The functions $F_{ijkns}^{(l)}(\theta, t)$ are infinitely smooth with respect to θ and can be written explicitly. For example,

$$F_{\theta\theta kns}^{(l)}(\theta, t) := \frac{1}{2} \sum_{j=1}^{2} \mathrm{e}^{-\mathrm{i}\gamma_k \theta_j} \sum_{w=0}^{s} \frac{1}{w!} [-\mathrm{i}\theta_j]^w$$

$$\times [(2 - \gamma_k)\phi_{4l-4+j,k}^{(n,s-w)}(t) - (1 - \delta_{sw})\phi_{4l-4+j,k}^{(n,s-w-1)}(t) + \exp(2\mathrm{i}\theta_j)\phi_{4l-2+j,k}^{(n,s-w)}(t)]$$

for $\sigma_{\theta\theta}^{(l)}$, where δ_{sw} is the Kronecker delta. The estimate for $\sigma_{*ij}^{(l)}$ in (4.16) follows from the memberships of $\Phi_{*j}^{(l)}$, $\tilde{\Psi}_{*j\mp}^{(l)}$ given above and from point 6^0 of the Appendix; the bound $M_*(t)$ is independent of θ on any internal segment $[\theta_-^{(l)\prime}, \theta_+^{(l)\prime}] \subset (\theta_-^{(l)}, \theta_+^{(l)})$.

Asymptotics (4.16) holds for any $t \in [0, T]$, the stress intensity factors $K_{knp}(t)$ and the parameter $M_*(t)$ are finite for every t but may be unbounded on $[0, T]$.

Thus, for the non-hereditary Dundurs parameters $\underset{\sim}{\alpha} = \alpha^0(t)$ and $\underset{\sim}{\beta} = \beta^0(t)$, the same power-logaritmic asymptotics of the solution from $\mathrm{C}\tilde{\mathrm{H}}_2$ is obtained near the wedge corner at any time instant t as for the corresponding classical elasticity problem, in which the elasticity moduli coincide with the instantaneous moduli in the hereditary problem considered, at the same instant. Such elastic problems (in other function classes) were considered in [1, 9, 12].

Note that for the non-hereditary but aging Dundurs parameters $\alpha^0(t)$, $\beta^0(t)$, form (4.16) of the stress asymptotics may be unstable in the vicinity of some t, where the multiplicity of $\gamma_k(t)$ change. It seems possible to rewrite the asymptotics in a stable form using methods displayed in [13, 24].

Now let $\underset{\sim}{\alpha}$ and $\underset{\sim}{\beta}$ not only be non-hereditary but also non-aging, i.e., $\underset{\sim}{\alpha} = \alpha^0$ and $\underset{\sim}{\beta} = \beta^0$ are independent of time. Then the zeros γ_k of the function $\Delta(\gamma)$ as well as the associated vector chains $\phi_{wk}^{(np)}$, $\chi_{wk}^{(np)}$, the functions $F_{ijkns}^{(l)}(\theta)$, and the parameters N_k, P_{kn}, N_k^0, M_* are independent of time too. The factors $K_{knp}(t)$ dependent on time by means of $G_{wk}^{(v)}(t)$, i.e. of loads only. Since $G_w \in CH_2^0(S(\delta_g, 1); 0, T)$, we obtain (see point 4^0 of the Appendix) that $\partial^v G_w(\gamma, t)/\partial\gamma^v \in \mathrm{CH}_2^0(S(\delta_g, 1); 0, T)$ too, and consequently $K_{knp}(t)$ given by (4.15) are continuous and due to point 6^0 of the Appendix are also bounded on $[0, T]$. After substituting asymptotics (4.16) into hereditary Hooke's law (1.1), we obtain for strains analogous power-logarithmic asymptotics with the same singularity powers γ_k as for the stress asymptotics

$$\varepsilon_{ij}^{(l)}(\rho,\theta,t) = \sum_{\delta_g<\mathrm{Re}\,\gamma_k<1} \rho^{-\gamma_k}(t) \sum_{n=1}^{N_k} \sum_{p=0}^{P_{kn}-1} \sum_{q=0}^{p} \ln^q\left(\frac{1}{\rho}\right)$$

$$\times\left[-K_{knp}^{(l1)}(t)\delta_{ij}\sum_{v=1}^{2} F_{vvkn,p-q}^{(l)}(\theta) + K_{knp}^{(l2)}(t)F_{ijkn,p-q}^{(l)}(\theta)\right]$$

$$+\varepsilon_{*ij}^{(l)}(\rho,\theta,t) \quad (i,j,l=1\text{–}2), \tag{4.17}$$

where

$$|\varepsilon_{*ij}^{(l)}(\rho,\theta,t)| < \tilde{M}_*(t)\rho^{-\delta_g-\varepsilon}, \forall\,\varepsilon\in(0,\delta_-(t)-\delta_g), \forall\,\theta\in[\theta_-^{(l)\prime},\theta_+^{(l)\prime}]\subset(\theta_-^{(l)},\theta_+^{(l)}).$$

$$K_{knp}^{(l1)}(t) := [(2\underline{\mu}^{(l)})^{-1}\underline{\nu}^{(l)}K_{knp}](t), \quad K_{knp}^{(l2)}(t) := [(2\underline{\mu}^{(l)})^{-1}K_{knp}](t)$$

for the plane strain case. For the plane stress case $\underline{\nu}^{(l)}$ must be replaced by $\underline{\nu}_*^{(l)} := (1-\underline{\nu}^{(l)})^{-1}-1$. Taking into account that $[(\underline{\mu}^{(l)})^{-1}]^*$, $\underline{\nu}^{(l)*}$, $\underline{\nu}_*^{(l)*}\in \mathrm{VC}(0,T)$, we obtain (see point 8^0 of the appendix) that the corresponding strain intensity factors $K_{knp}^{(lj)}(t)$ are continuous and bounded on $[0,T]$ like the stress intensity factors $K_{knp}(t)$.

For the case of parameters $\alpha^0(t)$ and $\beta^0(t)$ dependent on time, the stress singularity powers $\gamma_k(t)$ are functions of time, and power-logarithmic asymptotics of type (4.17) for the strains $\varepsilon_{ij}^{(l)}$ with the same singularity powers as for the stresses will occur only for degenerate (non-hereditary) operators $\underline{\mu}^{(l)} = \mu^{(l)0}(t)$, $\underline{\nu}^{(l)} = \nu^{(l)0}(t)$. If the operator $\underline{\mu}^{(l)}$ or $\underline{\nu}^{(l)}$ is non-degenerate (has non-zero integral term $\underline{\mu}^{(l)*}$ or $\underline{\nu}^{(l)*}$, respectively), then after substituting into (1.1) we obtain that the strain asymptotics is generally not power-longarithmic (with a finite number of logarithm terms) due to the action of integral operators, however an estimate can be obtained for this case. Since $(\underline{\mu}^{(l)})^{-1}$, $\underline{\kappa}^{(l)}\in\mathrm{VC}(0,T)$, it follows from point 8^0 of the Appendix and from the memberships $\Phi_j^{(l)}, \Psi_j^{(l)}\in \mathrm{C}\tilde{\mathrm{H}}_2(\delta_{gs+}(t),1;W_j^{(l)};0,t)$ that the functions $(\underline{\mu}^{(l)})^{-1}\underline{\kappa}^{(l)}\Phi_j^{(1)}$, $(\underline{\mu}^{(l)})^{-1}\Psi_1^{(1)}$ belong to the same class and estimates (4.13) are true also for these functions. Substituting the estimates into Kolosov–Muskhelishvili representation (1.3) and into Hook's law (1.1), we get

$$|\sigma_{ij}^{(l)}(\rho,\theta,\tau)|, |\varepsilon_{ij}^{(l)}(\rho,\theta,\tau)| < \tilde{\tilde{M}}_0(t)\rho^{-\delta_{gs+}(t)-\varepsilon}$$

$$\forall\,\varepsilon\in(0,1-\delta_{gs+}(t)), \forall\,\theta\in[\theta_-^{(l)\prime},\theta_+^{(l)\prime}]\subset(\theta_-^{(l)},\theta_+^{(l)}), \forall\,\tau\in[0,t], \tag{4.18}$$

where the bound $\tilde{\tilde{M}}_0(t)$ is independent of ρ,θ,τ. The estimate holds for all $\tau\in[0,t]$. For $\sigma_{ij}^{(l)}(\rho,\theta,\tau)$ this estimate is coarser than asymptotics (4.16) but it is uniform with respect to $\tau\in[0,t]$. In addition, $\sigma_{ij}^{(l)}(\rho,\theta,\tau)$, $\varepsilon_{ij}^{(l)}(\rho,\theta,\tau)\in C[0,t]$ uniformly with respect to (ρ,θ) in any $\bar{W}^{(l)\prime}\subset W^{(l)}$.

Thus, if $g_i^{(l)}$, $\tilde{g}_i^{(0)}$ are sufficiently smooth near the corner point (i.e., $\delta_g(t)<0$) and $\Delta(\gamma,t)$ has no zeros in the strip $0\leqslant \mathrm{Re}\,\gamma<1$, then stresses and strains are bounded near the corner point, otherwise stress and strain singularities of the form (4.16)–(4.18) may occur.

Appendix

Several properties of one-parametric holomorphic functions from some Hardy-type classes, Mellin transform, and Volterra operators

We present here short information about properties of function classes defined in Section 2 and about their interplay with the Mellin transform and Volterra operators. Details are given in [20].

1^0 If $g(\rho, t) \in \mathrm{C}\hat{\mathrm{L}}_2(\delta_0, \delta_\infty; 0, T)$, then (see Theorem 2.5 in [20]) its Mellin transform with respect to the real argument ρ

$$\langle g \rangle(\gamma, t) = \int_0^\infty g(\rho, t)\rho^{\gamma-1}\,\mathrm{d}\rho \tag{A.1}$$

belongs to $\mathrm{CH}_2^0(S(\delta_0, \delta_\infty); 0, T)$.

2^0 It follows from Theorem 2.12 in [20] that if a function $h(z, t) \in \mathrm{CH}_2(\delta_0, \delta_\infty; W(\theta_-, \theta_+); 0, T)$, then its Mellin transform with respect to the complex argument z

$$h^\vee(\gamma, t) := \int_0^\infty h(z, t) z^{\gamma-1}\,\mathrm{d}z, \quad z \in W(\theta_-, \theta_+) \tag{A.2}$$

is independent of the integration path in $W(\theta_-, \theta_+)$, belongs to $\mathrm{CH}_2^\vee(\theta_-, \theta_+; S(\delta_0, \delta_\infty); 0, T)$, and

$$h(z, t) := \frac{1}{2\pi \mathrm{i}} \int_{\delta - i\infty}^{\delta + i\infty} h^\vee(\gamma, t) z^{-\gamma}\,\mathrm{d}\gamma \quad (\delta_0 < \delta < \delta_\infty) \tag{A.3}$$

for $z \in W(\theta_-, \theta_+)$.

Conversely, if a function $h^\vee(\gamma, t) \in \mathrm{CH}_2^\vee(\theta_-, \theta_+; S(\delta_0, \delta_\infty); 0, T)$, then the function $h(z, t)$ defined by (A.3) belongs to $\mathrm{CH}_2(\delta_0, \delta_\infty; W(\theta_-, \theta_+); 0, T)$, is independent of $\delta \in (\delta_0, \delta_\infty)$, and (A.2) holds for $\gamma \in S(\delta_0, \delta_\infty)$.

The corresponding statements for functions $h(z) \in H_2(\delta_0, \delta_\infty; W(\theta^-, \theta_+))$, $h^\vee(\gamma) \in H_2^\vee(\theta_-, \theta_+; S(\delta_0, \delta_\infty))$ of one variable also hold (see [20, Theorem 1.7]).

3^0 It follows from Lemma 2.13 in [20] that if $h(z, t) \in \mathrm{CH}_2(\delta_0, \delta_\infty; W(\theta_-, \theta_+); 0, T)$, then for every $t \in [0, T]$ and for almost every $\rho \in (0, \infty)$ the functions $h^\mp(\rho, t) := \lim_{\theta \to \theta_\mp} h(\rho e^{i\theta}, t) \in \mathrm{C}\hat{\mathrm{L}}_2(\delta_0, \delta_\infty; 0, T)$ exist and

$$h^\vee(\gamma, t) = \exp(\mathrm{i}\gamma\theta_\mp)\langle h^\mp \rangle(\gamma, t), \quad \gamma \in S(\delta_0, \delta_\infty).$$

4^0 It follows from Lemma 2.4 in [20] that if $\tilde{h} \in \mathrm{CH}_2^0(S; 0, T)$, then $\partial\tilde{h}(\gamma, t/\partial\gamma \in \mathrm{CH}_2^0(S; 0, T)$ too.

5^0 It is proved in Lemma 2.4 in [20] that if $\tilde{h} \in \mathrm{CH}_2^0(S; 0, T)$, then for any internal strip $\bar{S}' \subset S(\delta_0, \delta_\infty)$ there exists a number $\tilde{\tilde{M}}^0 < \infty$ such that $\sup_{0 \leqslant t \leqslant T} |\tilde{h}(\gamma, t)| \leqslant \tilde{\tilde{M}}^0$, $\gamma \in \bar{S}'$, and $\tilde{h}(\gamma, t) \in C[0, T]$ with respect to t uniformly with respect to $\gamma \in \bar{S}'$.

6^0 It is proved in Lemma 1.10 in [20] that if $h \in H_2(\delta_0, \delta_\infty; W)$, then for any internal wedge $\bar{W}' \subset W$ and for any $[\delta_0', \delta_\infty'] \in (\delta_0, \delta_\infty)$ there exists a number $\tilde{\tilde{M}} < \infty$ such that $|h(z)| \leqslant \tilde{\tilde{M}}|z|^{-\delta}$, $\{z, \delta\} \in \bar{W}' \times [\delta_0', \delta_\infty']$.

It is proved in Lemma 2.8 in [20] that if $h \in \mathrm{CH}_2(\delta_0, \delta_\infty; W; 0, T)$, then for any internal wedge $\bar{W}' \subset W$ and for any $[\delta_0', \delta_\infty'] \in (\delta_0, \delta_\infty)$ there exists a number

$\tilde{\tilde{M}} < \infty$ such that

$$\sup_{0 \leqslant t \leqslant T} |h(z,t)| \leqslant \tilde{\tilde{M}} |z|^{-\delta}, \quad \{z, \delta\} \in \bar{W}' \times [\delta_0', \delta_\infty'] \tag{A.4}$$

and $h(z,t)z^{\delta} \in C[0,T]$ with respect to t, uniformly with respect to $\{z, \delta\} \in \bar{W}' \times [\delta_0', \delta_\infty']$.

7^0 Let a function $\tilde{h} \in \mathrm{CH}_2^0(S; 0, T)$. Let for all $t \in [0, T]$ a function $\tilde{h}_1(\gamma, t)$ be holomorphic in S with respect to γ and be such that for some interval (θ_-, θ_+) and for every $\bar{S}' \subset S$ there exists a number $\tilde{\tilde{M}}(\tilde{h}_1; \theta_-, \theta_+; S') < \infty$ such that $\sup_{0 \leqslant t \leqslant T} |\tilde{h}_1(\gamma, t)| \leqslant \tilde{\tilde{M}}^{\vee}(\tilde{h}_1; \theta_-, \theta_+; S')|e^{i\gamma\theta}|, (\gamma, \theta) \in \bar{S}' \times (\theta_-, \theta_+)$, and $\tilde{h}_1(\gamma, t)e^{i\gamma\theta} \in C[0, T]$ with respect to t, uniformly with respect to $\{\gamma, \theta\} \in \bar{S}' \times (\theta_-, \theta_+)$; then $\tilde{h}\tilde{h}_1 \in \mathrm{CH}_2^{\vee}(\theta_-, \theta_+; S; 0, T)$. (See Lemma 2.11 in [20]).

8^0 Let $\underline{K} = K^0(t) + K^*$, $K^0(t) \in C[0, T]$, $\underline{K}^* \in \mathrm{VC}(0, T)$. Then $\underline{K}$ acts in $C[0, T]$ as well as in all the classes of functions dependent on t and defined in Section 2. If, additionally, $T < \infty$ and $\det[K^0(t)] \neq 0$ $(t \in [0, T])$, then $\underline{K}^{-1} = [K^0(t)]^{-1} - \underline{R}^*$, where $\underline{R}^* \in \mathrm{VC}(0, T)$ (see e.g. [17]).

If $g(\rho, t) \in \mathrm{C\hat{L}}_2(\delta_0, \delta_\infty; 0, T)$ and if an operator $\underline{K}^* \in \mathrm{VC}(0, T)$, then (see Theorem 3.3 in [20]) $\langle \underline{K}^* g \rangle (\gamma, t) = (\underline{K}^* \langle g \rangle)(\gamma, t) \in \mathrm{CH}_2^0$ $(S(\delta_0, \delta_\infty); 0, T)$.

9^0 Let $\underline{K}^*(\gamma) \in \mathrm{VCH}(S; 0, T)$, then $\underline{K}^*(\gamma)$ and $[\underline{I} + \underline{K}^*(\gamma)]^{-1}$ act in $\mathrm{CH}_2^0(S; 0, T)$ as well as in $\mathrm{CH}_2^{\vee}(\theta_-, \theta_+; S; 0, T)$ (see Theorems 3.12, 3.13 in [20]).

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