STRESS SINGULARITY IN A COMPOUND ARBITRARILY ANISOTROPIC BODY, AND APPLICATIONS TO COMPOSITES

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The article considers stress singularity in the neighborhood of the line of intersection of the free surface of a compound elastic arbitrarily anisotropic body and the junction surface of its different parts. A solution is constructed for the model problem for a two-dimensional compound wedge, the result being an analytic function whose zeros are equal to the degree of stress singularity of the initial three-dimensional body. The degree of singularity is determined for some composites consisting of layers with different fiber directions, and for composites bonded to aluminum.

On the basis of the results of [2-4], it was shown in [1] that the degree of stress singularity in the neighborhood of an edge is determined by the solution of some model problem in a two-dimensional composite wedge. When in the vicinity of the edge the body has a plane of symmetry of the elastic properties that is perpendicular to the edge, the model problem breaks down into one of plane strain and one of torsion (or antiplane strain). The first was considered in [5], the second in [6]. On the basis of [5,6], the author in [1] computed the degrees of singularity for composites with a plane of symmetry of the elastic properties perpendicular to the edge. In the present paper, the case of arbitrary anisotropy is analyzed.

1. Let us first consider the problem of generalized plane strain of a composite rectilinear-anisotropic wedge with anisotropy of general form, i.e., the problem for an infinite dihedral wedge in which the stresses do not vary along the edge, the lateral loads are self-equilibrated, and there is no tension, torsion, or bending. Assume that stresses are specified on the lateral faces, while there is rigid adhesion on the junction surface. In the cylindrical r, φ, x_3 coordinate system with origin at the apex of the wedge and x_3 axis directed along the edge, the boundary conditions have the form

$$\begin{gathered}
\sigma_\varphi'(r,\varphi_1)=t_1(r),\quad \tau_{r\varphi}'(r,\varphi_1)=t_2(r),\quad \tau_{3\varphi}'(r,\varphi_1)=t_3(r)\\
\sigma_\varphi''(r,\varphi_2)=t_4(r),\quad \tau_{r\varphi}''(r,\varphi_2)=t_5(r),\quad \tau_{3\varphi}''(r,\varphi_2)=t_6(r)\\
\sigma_\varphi'(r,0)=\sigma_\varphi''(r,0),\ \tau_{r\varphi}'(r,0)=\tau_{r\varphi}''(r,0),\ \tau_{3\varphi}'(r,0)=\tau_{3\varphi}''(r,0)\\
u_r'(r,0)=u_r''(r,0),\ u_\varphi'(r,0)=u_\varphi''(r,0),\ u_3'(r,0)=u_3''(r,0)
\end{gathered}\tag{1.1}$$

where one prime denotes quantities in one part of the wedge, while two primes denote quantities in the other part.

Here φ_1 and φ_2 are the angles of the free surfaces of the first and second part with the junction surface, where $\varphi=0$ ($\varphi_2\leqslant 0\leqslant\varphi_1$, $\varphi_1-\varphi_2\leqslant 2\pi$), t_1 are specified functions. The stressed state of each part of the compound wedge is defined by the solutions of the equation [7]

$$\begin{gathered}
(L_4L_2-L_3^2)f=0,\quad L_4=b_{22}\frac{\partial^4}{\partial x_1^4}-2b_{26}\frac{\partial^4}{\partial x_1^3\,\partial x_2}+\\
+(2b_{12}+b_{66})\frac{\partial^4}{\partial x_1^2\,\partial x_2^2}-2b_{16}\frac{\partial^4}{\partial x_1\,\partial x_2^3}+b_{11}\frac{\partial^4}{\partial x_2^4}\\
L_3=-b_{24}\frac{\partial^3}{\partial x_1^3}+(b_{25}+b_{46})\frac{\partial^3}{\partial x_1^2\,\partial x_2}-(b_{14}+b_{56})\frac{\partial^3}{\partial x_1\,\partial x_2^2}+b_{15}\frac{\partial^3}{\partial x_2^3}\\
L_2=b_{44}\frac{\partial^2}{\partial x_1^2}-2b_{45}\frac{\partial^2}{\partial x_1\,\partial x_2}+b_{55}\frac{\partial^2}{\partial x_2^2},\qquad b_{ij}=a_{ij}-a_{i3}a_{j3}a_{33}^{-1}
\end{gathered}\tag{1.2}$$

where a_{ij} are the elastic constants of the material in the generalized Hooke's law (see [7]). The general solution of (1.2) can be written using the roots of the characteristic equation

$$l_4(\mu)l_2(\mu)-l_3^2(\mu)=0 \tag{1.3}$$

where the polynomials l_i are obtained from L_i by replacing $\partial^j/\partial x_1^j$ by 1 and $\partial^j/\partial x_2^j$ by μ^j.

Assume that μ_j are the roots of (1.3):

$$a_j=\cos\varphi+\mu_j\sin\varphi,\quad b_j=\mu_j\cos\varphi-\sin\varphi$$
$$z_j=x_1+\mu_j x_2=ra_j(\varphi),\ D_j=d/dz_j$$
$$\lambda_j=-l_3(\mu_j)l_2(\mu_j)^{-1},\quad l_2(\mu_j)\neq 0;\quad \lambda_j=1,\quad l_2(\mu_j)=0$$
$$\delta_j=1,\ l_2(\mu_j)\neq 0;\ \delta_j=0,\ l_2(\mu_j)=0$$
$$p_j=\delta_j(b_{11}\mu_j^2+b_{12}-b_{16}\mu_j)+\lambda_j(b_{15}\mu_j-b_{14})$$
$$q_j=\delta_j(b_{12}\mu_j+b_{22}\mu_j^{-1}-b_{26})+\lambda_j(b_{25}-b_{24}\mu_j^{-1})$$
$$r_j=\delta_j(b_{14}\mu_j+b_{24}\mu_j^{-1}-b_{46})+\lambda_j(b_{45}-b_{44}\mu_j^{-1})$$
$$c_j=p_j\cos\varphi+q_j\sin\varphi;\quad d_j=q_j\cos\varphi-p_j\sin\varphi$$
$$h_1=b_{12}-\mu_1\mu_2 b_{11},\ h_2=h_1-b_{11}(\mu_1-\mu_2)^2$$

A material for which the roots μ_j of (1.3) do not include any equal ones will be assigned to type A. For this material (see [7])

$$\sigma_r=\sum_{j=1}^{6}\delta_j b_j^2 D_j^2 f_j,\quad \sigma_\varphi=\sum_{j=1}^{6}\delta_j a_j^2 D_j^2 f_j$$
$$\tau_{r\varphi}=-\sum_{j=1}^{6}\delta_j a_j b_j D_j^2 f_j,\quad \tau_{3r}=\sum_{j=1}^{6}\lambda_j b_j D_j^2 f_j,\quad \tau_{3\varphi}=-\sum_{j=1}^{6}\lambda_j a_j D_j^2 f_j \tag{1.4}$$
$$u_r=\sum_{j=1}^{6}c_j D_j f_j,\quad u_\varphi=\sum_{j=1}^{6}d_j D_j f_j,\quad u_3=\sum_{j=1}^{6}r_j D_j f_j.$$

Let us also consider a material for which $l_3(\mu)\equiv 0$, while the equation $l_4(\mu)=0$ has pairwise equal roots; we assign this material to type B. In particular, type B includes isotropic materials and transversely isotropic materials with isotropy axis perpendicular to the plane of the wedge. Assume that μ_1 and μ_2 are two different roots of the equation $l_4(\mu)=0$, while μ_5,μ_6 are the roots of the equation $l_2(\mu)=0; m=3-j$. Then for the given material we obtain the following from [5,7]:

$$\sigma_r=\sum_{j=1}^{2}[b_j^2(D_j^2 f_j+z_m D_j^2 g_j)+2b_j b_m D_j g_j]$$
$$\sigma_\varphi=\sum_{j=1}^{2}[a_j^2(D_j^2 f_j+z_m D_j^2 g_j)+2a_j a_m D_j g_j]$$
$$\tau_{r\varphi}=-\sum_{j=1}^{2}[a_j b_j(D_j^2 f_j+z_m D_j^2 g_j)+(a_j b_m+a_m b_j)D_j g_j]$$
$$u_r=\sum_{j=1}^{2}[h_1 a_j(D_j f_j+z_m D_j g_j)+h_2 a_m g_j] \tag{1.5}$$
$$u_\varphi=\sum_{j=1}^{2}[h_1 b_j(D_j f_j+z_m D_j g_j)+h_2 b_m g_j]$$
$$\tau_{3r}=\sum_{j=5}^{6}b_j D_j^2 f_j,\quad \tau_{3\varphi}=-\sum_{j=5}^{6}a_j D_j^2 f_j,\quad u_3=\sum_{j=5}^{6}r_j D_j f_j$$

Here and henceforth, if there is no summation sign, summation is not performed over repeating indexes. The quantities $f_j(z_j)$, $g_j(z_j)$ in (1.4) and (1.5) are analytic functions of their arguments, which are the solution of (1.2) and are to be determined from the boundary conditions.

We can write expressions for the stresses and displacements in terms of certain analytic functions for other relationships among the roots μ_j of (1.3), $l_2(\mu)=0$ and $l_3(\mu)=0$, but they are evidently of no practical significance, and therefore we will confine ourselves to the above two cases.

Consider the Mellin transformation of complex function $v(z)=v(x_1+\mu x_2)$, where μ is some complex constant, while x_1 and x_2 are Cartesian coordinates on the plane.

Assume that $\langle v\rangle$ is the Mellin transform of v(z); then

$$\langle v\rangle=\int_0^\infty v(x_1+\mu x_2)r^{s-1}\,dr=a^{-s}(\varphi)V(s)$$

$$a(\varphi)=\cos\varphi+\mu\sin\varphi,\qquad V(s)=\int_0^\infty v(z)z^{s-1}\,dz$$

If v(z) is analytic in the sector $\psi_1<\varphi<\psi_2$, $0<r<\infty$ and $v(z)=O(r^{\xi})\ (r\to0)$, $v(z)=O(r^{\eta})\ (r\to\infty)$, $\eta<\xi$, then V(s) exists in the strip $-\xi<\mathrm{Re}\,s<-\eta$ and is independent of φ.

We define the inverse transformation in the customary fashion:

$$v(z)=\frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty}\langle v\rangle r^{-s}\,ds$$

We will seek a solution satisfying the following conditions:

$$\sigma_{ij}=O(r^{-1+\alpha}),\ r\to0;\ \sigma_{ij}=O(r^{-1-\beta}),\ r\to\infty,\ \alpha>0,\ \beta\geqslant0 \tag{1.6}$$

We will assume that $u_{i,r}$ also satisfies (1.6) (this eliminates rotation of the wedge as a rigid whole). In this case the Melling transformation can be applied to σ_{ij}, $u_{i,r}$.

Let us consider two possible cases: A-A: both parts of the wedge consist of materials of type A; A-B: one part consists of type A material, the other of type B. The third possible case, B-B, reduces to plane strain and torsion and was considered in [1,5,6].

As in [5], we substitute the stresses and displacements from (1.4) and (1.5) into (1.1), we differentiate the last three of the resultant equations with respect to r, and we apply the Mellin transformation with respect to r to all 12 expressions. As a result, we obtain a system of equations in U_i for both cases (A-A and A-B):

$$\sum_{j=1}^{12}B_{ij}U_j=T_i\qquad (i=1,2,\ldots,12) \tag{1.7}$$

$$T_j^*=\|\langle t_1\rangle,\ \langle-t_2\rangle,\ \langle-t_3\rangle,\ \langle t_4\rangle,\ \langle-t_5\rangle,\ \langle-t_6\rangle,\ 0,0,0,0,0,0\|$$

In case A-A

$$U_j^*=\|F_1',F_2',F_3',F_4',F_5',F_6',F_1'',F_2'',F_3'',F_4'',F_5'',F_6''\|$$

In case A-B

$$U_j^*=\|F_1',F_2',F_3',F_4',F_5',F_6',F_1'',F_2'',G_1'',G_2'',F_5'',F_6''\|$$

$$F_j=\int_0^\infty D_j^2f_j(z_j)z_j^{s-1}\,dz_j,\qquad G_j=\int_0^\infty D_jg_j(z_j)z_j^{s-1}\,dz_j$$

where the asterisk denotes transposition.

Matrix B_{ij} for case A-B is given on page 30.

Here it is assumed that the indexes in the first part of the wedge (A) range from 1 to 6, while in the second (B), they run from 7 to 12. Also (and only here) it was assumed that $a_j=a_j(\varphi_1)$, $b_j=b_j(\varphi_1)$, if $j=1,2,\ldots,6$, while $a_j=a_j(\varphi_2)$, $b_j=b_j(\varphi_2)$, if j = 7, 8, ..., 12; $\omega_{ij}=(1-s)a_jb_i+a_ib_j$.

$$
\left\|\begin{array}{cccccccccccc}
\delta_1 a_1^{2-s} & \delta_2 a_2^{2-s} & \delta_3 a_3^{2-s} & \delta_4 a_4^{2-s} & \delta_5 a_5^{2-s} & \delta_6 a_6^{2-s} & 0 & 0 & 0 & 0 & 0 & 0 \\
\delta_1 a_1^{1-s} b_1 & \delta_2 a_2^{1-s} b_2 & \delta_3 a_3^{1-s} b_3 & \delta_4 a_4^{1-s} b_4 & \delta_5 a_5^{1-s} b_5 & \delta_6 a_6^{1-s} b_6 & 0 & 0 & 0 & 0 & 0 & 0 \\
\lambda_1 a_1^{1-s} & \lambda_2 a_2^{1-s} & \lambda_3 a_3^{1-s} & \lambda_4 a_4^{1-s} & \lambda_5 a_5^{1-s} & \lambda_6 a_6^{1-s} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & a_7^{2-s} & a_8^{2-s} & a_7^{1-s} a_8 (2-s) & a_8^{1-s} a_7 (2-s) & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & a_7^{1-s} b_7 & a_7^{-s} \omega_{78} & a_8^{-s} \omega_{87} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & a^{1-s} b_7 & 0 & 0 & a_{11}^{1-s} & a_{12}^{1-s} \\
-\delta_1 & -\delta_2 & -\delta_3 & -\delta_4 & -\delta_5 & -\delta_6 & 1 & 1 & 2-s & 2-s & 0 & 0 \\
\delta_1 \mu_1 & \delta_2 \mu_2 & \delta_3 \mu_3 & \delta_4 \mu_4 & \delta_5 \mu_5 & \delta_6 \mu_6 & -\mu_7 & -\mu_8 & (s-1)\mu_7 - \mu_8 & (s-1)\mu_8 - \mu_7 & 0 & 0 \\
-\lambda_1 & -\lambda_2 & -\lambda_3 & -\lambda_4 & -\lambda_5 & -\lambda_6 & 0 & 0 & 0 & 0 & 1 & 1 \\
p_1 & p_2 & p_3 & p_4 & p_5 & p_6 & -h_7 & -h_7 & (s-1) h_7 - h_8 & (s-1) h_7 - h_8 & 0 & 0 \\
q_1 & q_2 & q_3 & q_4 & q_5 & q_6 & -\mu_7 h_7 & -\mu_8 h_7 & (s-1)\mu_7 h_7 - \mu_8 h_8 & (s-1)\mu_8 h_7 - \mu_7 h_8 & 0 & 0 \\
r_1 & r_2 & r_3 & r_4 & r_5 & r_6 & 0 & 0 & 0 & 0 & -r_{11} & -r_{12}
\end{array}\right\|
$$

For case A-A, matrix B_{ij} is altered such that its right part will be analogous to the left—naturally, with its own $a_j, b_j, \mu_j, \lambda_j, \delta_j, p_j, q_j, r_j$.

Assume that $\Delta = \det(B_{ij})$, A_{ij} are the algebraic complements of B_{ij}, $a_j = a_j(\varphi)$, $b_j = b_j(\varphi)$. Solving system (1.7) and substituting the resultant values of F_j and G_j into the formulas for the transforms of the stresses and displacements, we obtain a solution in Mellin transforms. As an example, let us write out the transform of the stress $\langle \sigma_r \rangle$ for case A-B. In type A material,

$$\langle \sigma_r' \rangle = \sum_{i=1}^{12} T_i \sum_{j=1}^{6} a_j^{-s} b_j^2 A_{ij} \Delta^{-1} \tag{1.8}$$

In type B material ($m = 15 - j$):

$$\langle \sigma_r'' \rangle = \sum_{i=1}^{12} T_i \sum_{j=7}^{8} a_j^{-s} b_j^2 [A_{ij} + (2b_j^{-1} b_m - s a_j^{-1} a_m) A_{ij+2}] \Delta^{-1} \tag{1.9}$$

For case A-A, the transforms of the stresses in both parts of the wedge will be analogous to (1.8). Applying the inverse Mellin transformation to the expressions for the transforms of the voltages and displacements, using residue theory, and assuming that for large and for small r the t_i can be represented as sums (possibily infinite) of different powers of r, we obtain (see [5] for more detail) in type A material

$$\sigma_r' = \sum_{s_k < 1} \operatorname{res}_{s_k} \left\{ \sum_{i=1}^{12} T_i \sum_{j=1}^{6} a_j^{-s} b_j^2 A_{ij} \Delta^{-1} r^{-s} \right\} = \sum_{s_k < 1} r^{-s_k} \sum_{n=0}^{N_k - 1} \Psi'_{rkn}(\varphi) (\ln r)^n \tag{1.10}$$

while in type B material

$$\sigma_r'' = \sum_{s_k < 1} \operatorname{res}_{k} \left\{ \sum_{i=1}^{12} T_i \sum_{j=7}^{8} a_j^{-s} b_j^2 [A_{ij} + (2b_j^{-1} b_m - s a_j^{-1} a_m) A_{ij+2}] \Delta^{-1} r^{-s} \right\} = \sum_{s_k < 1} r^{-s_k} \sum_{n=0}^{N_k - 1} \Psi''_{rkn}(\varphi) (\ln r)^n \tag{1.11}$$

Satisfaction of conditions (1.6) requires that the path of integration in the invers Mellin transformation lie in the strip

$$\max_{\operatorname{Re} s_k < 1} (\operatorname{Re} s_k) < c < \min_{\operatorname{Re} s_k > 1} (\operatorname{Re} s_k)$$

This implies the constraint $s_k<1$ in (1.10) and (1.11). Here s_k are the poles of $\langle\sigma_{ij}\rangle$; N_k are their multiplicity; and Ψ_{rkn} are smooth functions of the angle φ. If $t_i\to 0$ as $r\to 0$, then there are no poles T_i in the strip $0\le \mathrm{Re}\, s<1$, and all the s_k in this strip are among the zeros of $\Delta(s)$. In other words, in this case the singular terms in (1.10) and (1.11) are determined only by the zeros of $\Delta(s)$ in the strip $0\le \mathrm{Re}\, s<1$.

2. It was established in [1] that in the neighborhood of the line of intersection of the free surface of a three-dimensional anisotropic elastic solid, and the junction surface of its different parts (which we will call the edge), the stresses have the form

$$\sigma_{ij}=\sum_{\alpha<\mathrm{Re}\, s_k<1} r^{-s_k}\sum_{n=0}^{N_k-1}\Psi_{ijkn}(\varphi)(\ln r)^n+\sigma_{ij}^{\vee} \tag{2.1}$$

Here r, φ are local polar coordinates introduced in the plane perpendicular to the edge, with origin at the point of the edge under investigation; s_k are the zeros of some analytic function $\Delta(s)$; N_k are their multiplicity; and Ψ_{ijkn} are smooth functions. If in the neighborhood of the point of the edge under consideration there are specified stresses t_i on the free surface, and $t_i\to 0$ as $r\to 0$, then $\alpha<0$, while $\sigma_{ij}^{\vee}$ is a bounded function. In this case the stress singularity is determined only by the form of $\Delta(s)$, which, as noted in [1], coincides with a function whose zeros are present in the analogous representation of the stress field in the plane model problem for a compound wedge. The equations of the model problem can be obtained from the initial ones by discarding terms with derivatives with respect to x_3 (the coordinate x_3 is directed along the tangent to the edge at the point in question), as well as the lower derivatives, and by "freezing" the coefficients at the point in question. The right sides of the fundamental system of equations and boundary conditions of the model problem are integrable and nonzero in finite regions that do not contain the coordinate origin. Thus, the problem solved in §1 for a compound anisotropic wedge will be a model problem if in the initial problem the stresses are specified on the free surfaces near the point in question, and there is rigid adhesion (possibly with interference) on the junction surface, while the function $\Delta(s)$ as found there will determine the singular terms in expansion (2.1).

3. The edge effect in laminar composites was considered in [8-10]. For layers at different angles, the equations of anisotropic elasticity were solved by the finite-element or finite-difference method. It was noted that the stresses can be large or even infinite near the edge (the numerical methods employed could not pin down this infinity in pure form), but their asymptotic form was not investigated. Paper [1] examined the degree of stress singularity as a function of the elastic and geometrical parameters of the layers for composites for which the directions of fiber packing were parallel or perpendicular to the edge.

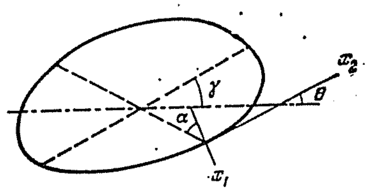

Fig. 1

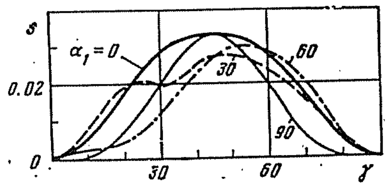

Fig. 2

Using the form of $\Delta(s)$, obtained above, let us consider the degree of singularity of the stresses arising in carbon plastic made up of layers at different angles, and in carbon plastic bonded to an aluminum plate.

A carbon-plastic specimen is shown schematically in Fig. 1. The packing plane coincides with the plane of the figure. The dashed lines indicate the direction of packing, while the dot-dash line indicates the bisectrix of the angle between the directions of packing in adjacent layers. The coordinate origin is located at the point of the edge under investigation between adjacent layers.

As in [8-10], we will consider a layer of carbon plastic with the following characteristics: Young's modulus in the direction of reinforcement $E_L=14.6\cdot 10^3$ kg·mm^{-2}, $E_z=E_T=$1 kg·mm^{-2}, shear moduli $G_{LT}=G_{Tz}=G_{Lz}=0.598\cdot10^3$ kg·mm^{-2}, Poisson's ratios $\nu_{LT}=\nu_{Lz}=\nu_{Tz}=0.21$. The z axis is perpendicular to the plane of the layer. The relationships of [7] were used to obtain the elastic characteristics a_{km} of the layer under rotation in its plane. The layers under consideration are not transversely isotropic, and hence they constitute t A material for all angles α, θ, γ (see Fig. 1). Thus, $\Delta=\det(B_{ij})$, where B_{ij} refers to case A. The free surface is perpendicular to the junction surface ($\varphi_1=-\varphi_2=90°$).

Figure 2 depicts the degree of stress singularity s as a function of the angle γ f fixed angle α_1 between one of the directions of reinforcement and the normal to the edg In the general case, there is one absolute maximum of the degree of singularity as the difference in orientation in adjacent layers varies from 0 to ±90°. When the edge is pe pendicular or parallel to one of the directions of reinforcement, the maximum occurs wh the reinforcement becomes orthogonal, and the graph is symmetrical relative to this poi In the remaining cases, symmetry disappears and the maximum shifts, while remaining in the neighborhood of orthogonal reinforcement. Near the position in which the second direction of reinforcement becomes parallel to the edge, a slight local minimum may ari

Figure 3 shows s as a function of the angle θ of inclination of the axis of symmet of the packings to the edge for three fixed values of $\gamma\leqslant45°$. In effect, this represents the change in degree of singularity in going around the contour of the specimen (see Fi 1).

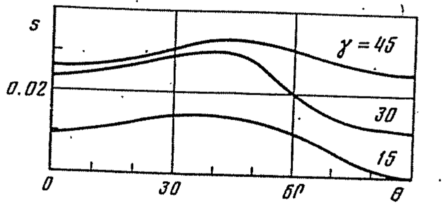

Fig. 3

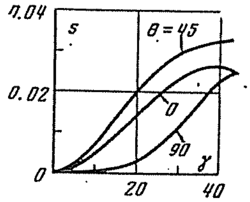

Fig. 4

Figure 4 shows s as a function of the angle of orientation difference for three fi angles θ. It follows from Fig. 1 that $s(\gamma, \theta)=s(\gamma, -\theta)=s(90°-\gamma, 90°-\theta)$. For constant θ, the deg of singularity increases almost monotonically on the interval $0<\gamma<45°$.

It follows from Figs. 2-4 that in the carbon plastic under consideration a stress singularity can occur for any α, γ, θ, with the exception of $\gamma=0$, when the degree of singul ity vanishes, since in this case the jump in the elastic parameters on the junction sur disappears. The degree of singularity is at a maximum when the edge is parallel to one the directions of reinforcement and perpendicular to the other.

Let us investigate the degree of singularity of the stresses that arise when a lay of carbon plastic is bonded to a layer of aluminum. Calculations were made for a layer high-elastic-modulus composite with characteristics $E_L=20\cdot10^3$ kg·mm^{-2}, $E_z=E_T=2.1\cdot10^3$ kg·mm^{-2} $G_{LT}=G_{Tz}=G_{Lz}=0.85\cdot10^3$ kg·mm^{-2}, $\nu_{LT}=\nu_{Lz}=\nu_{Tz}=$ = 0.21 (see, e.g., [11]) and aluminum with paramet $E=7.2\cdot10^3$kg·mm^{-2}, $\nu=0.3$. Now we have case A-B and the corresponding matrix B_{ij} and functio $\bar{\Delta}(s)$.

Figure 5 shows the degree of stress singularity s as a function of the angle α betw the direction of reinforcement of the carbon-plastic layer and the normal to the edge, three fixed values of the angles of inclination of the surface of the carbon plastic (φ_1) and aluminum (φ_2) to the junction surface ($\varphi_1=90°$, $\varphi_2=-90°$, solid line; $\varphi_1=180°$, $\varphi_2=-90°$ dashed line; $\varphi_1=90°$, $\varphi_2=-180°$, dot-dash line). We should note that, for sufficiently large angles φ_1, $|\varphi_2|$ up to three singular roots appear. In the general case, s is not a monotonic fu tion of α, but if $\varphi_1=-\varphi_2=90°$, the degree of singularity increases monotonically when the e is rotated from a position parallel to the direction of reinforcement to a position perp dicular to it.

Assume that $\delta=180°-(\varphi_1-\varphi_2)$ is the local deviation of the free surface around the edge from the compound half-plane. Figure 6 shows the critical value δ*, for which the maxim root of Δ(s) in the strip $0\leqslant\mathrm{Re}\,s<1$ becomes zero, as a function of φ_1 for five fixed α value (α=0, thin solid line; 30°, dashed line; 60°, dot-dash line; 75°, open circles; 90°, hea

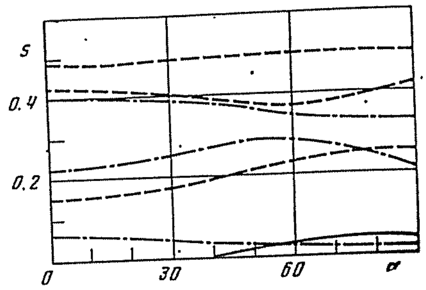

Fig. 5

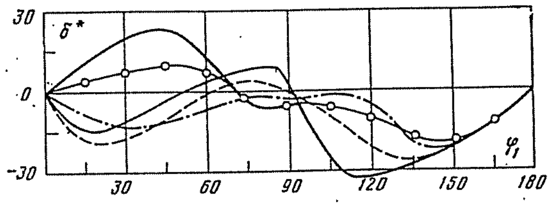

Fig. 6

solid line). For a continuous (noncompound) body, $\delta^*=0$. For $\delta<\delta^*$, there will be no singularity, while for $\delta>\delta^*$ it may appear.

The curves for $\alpha=90°$ and 0 corresponding to the position of the edge parallel and perpendicular to the degree of reinforcement are taken from [1]. As α varies, the variation of δ* is fairly complex. This is evidently to be explained by the fact that the Young's modulus of aluminum lies between the minimum and maximum Young's modulus of carbon plastic, while the tangential modulus of aluminum is always greater than that of carbon plastic.

For specified materials, the availability of graphs of the type shown in Fig. 6 makes it possible to choose the local geometry near the edge in such a way that there is no stress singularity in the junction.

The proposed method can be readily used to construct matrices B_{ij}, i.e., the corresponding functions $\Delta(s)$, and to determine the degrees of singularity for other types of boundary conditions as well as for a larger number of parts; in particular, it can be used to examine a crack that goes beyond the interface of a composite arbitrarily anisotropic body.

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