STRESS SINGULARITY IN THE NEIGHBORHOOD OF A RIB IN A COMPOSITE INHOMOGENEOUS ANISOTROPIC BODY, AND SOME APPLICATIONS TO COMPOSITES

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The article considers the edge effect resulting from the presence of a line of intersection of the free surface of the body and the surface of junction of its inhomogeneous anisotropic elastic parts. It is shown that the problem of the degree of stress singularity in the neighborhood of this line can be reduced to a two-dimensional problem for an infinite composite wedge. A number of examples are calculated for composites made up of layers with various arrangements of fibers, and also for layers reinforced by isotropic material. The degree of stress singularity is plotted as a function of the parameters of the materials and the geometry of the junction.

There have been a number of studies in recent years of stress singularities at a singular point of a boundary (point on a rib or on a line of intersection of a free surface and the surface of junction of several bodies, which will also be called a rib). The studies have basically employed two methods: either they have sought solutions of a certain form satisfying homogeneous equations and boundary conditions near the singular point [1-5], or, by means of the Mellin transformation and residue theory, the problem has been solved for an infinite wedge [6-12]. In the former case, there is a question about the completeness of the resultant system of functions and the effect of certain right sides of the equations and boundary conditions on the asymptotic form of the solution near the singular point; while in the latter case it is difficult to obtain a solution for a finite region with nonrectilinear boundaries.

For elliptic boundary-value problems with continuous coefficients in noncompound finite two-dimensional regions with corner points, or multidimensional regions with conical points, the asymptotic form of the solution was given in [13]. This asymptotic form was obtained by reduction (using the truncating function of the initial problem in a finite region) to a problem in an infinite wedge (or cone), which was solved by means of integral transforms. The shortcomings of the approaches described above were thus eliminated. Paper [14] considered the problem for a compound two-dimensional region with a junction that emerged onto a free surface. In this paper the initial problem was reduced to a system of singular integral equations, which was then solved by means of the Mellin transformation. Results for the asymptotic form in a multidimensional region with smooth ribs were obtained in [15].

In what follows, we will use these results to consider a continuously inhomogeneous composite anisotropic elastic solid body that is subjected to mass forces and a temperature field with arbitrary boundary conditions.

1. Assume that D' and D" are parts of a composite body, each of which obeys a generalized Hooke's law with allowance for thermal stresses, and the equilibrium equations

$$\sigma_{ij}=A_{ijkl}\varepsilon_{kl}-\beta_{ij}T \qquad (1)$$

$$\sigma_{ij,j}=F_i \qquad (2)$$

The index after the comma denotes the derivative with respect to the corresponding coordinate; summation is assumed over repeating indexes. Substituting (1) into (2), expressing the deformations in terms of the displacements, and using the fact that tensor A_{ijkl} is symmetrical, we obtain the basic system of equations in the displacements for each part:

$$(A_{ijkl}u_{k,l})_{,j}=F_i+(\beta_{ij}T)_{,j}\equiv f_i^\circ \quad (i,j,k,l=1,2,3) \tag{3}$$

We introduce the Cartesian coordinate system x_1, x_2, x_3, with its origin coinciding with the singular point in question, its x_3 axis directed along the tangent to the rib, and the x_1 axis directed along the tangent to the surface of junction. We also introduce the cylindrical system r, φ, x_3, where the angle φ is reckoned from x_1 in the x_1x_2 plane. Assume that the boundary conditions on the free surface from each side of the rib in the neighborhood of the coordinate origin are supplied by three suitable conditions of the six given below:

$$A_{ijkl}u_{k,l}n_j=p_i^\circ+\beta_{ij}Tn_j\equiv t_i^\circ, \quad u_i=u_i^\circ \quad (i=1,2,3) \tag{4}$$

Thus, we can encompass problems with displacements specified on a free boundary, with specified stresses, the problem of contact with a rigid contour, and also the mixed problem with line of change of the boundary conditions coinciding with the rib in the neighborhood of the point in question. On the surface of junction near the point in question, the boundary conditions will be provided by six suitable conditions out of the ten that follow:

$$(A'_{ijkl}u'_{k,l}-A''_{ijkl}u''_{k,l})n_j=(\beta_{ij}'T'-\beta_{ij}''T'')n_j\equiv t_i^\circ, \; u_i'-u_i''=u_i^\circ \quad (i=1,2,3)$$
$$A'_{i2kl}u'_{k,l}=\beta_{i2}'T'\equiv t_i', \; A''_{i2kl}u''_{k,l}=\beta_{i2}''T''\equiv t_i'' \quad (i=1,3) \tag{5}$$

Thus, we can encompass rigid connection with "tightness," and contact without friction. Thus, we have elliptical conjugacy problem (3)-(5).

In [15], expressions for the asymptotic behavior of the solution of an elliptic boundary-value problem in a noncompound region with ribs were given under the assumption that the solution is infinitely differentiable along a rib with derivatives that belong to some functional space with weight. This guarantees that the initial problem can be reduced to a two-dimensional one. Only solutions for which all derivatives encountered in the equation were defined were considered.

For the equations of elasticity, however, it is of interest to consider the solution with displacements belonging to Sobolev space $u_i\in W^1(D'\cup D'')$, W^1 is a space with integrable squares of the function and its first derivatives (energy space). Making integral estimates for $u_{i,3}$ as done in [16,17] for equations in multidimensional noncompound regions with ribs, we can show that when certain conditions are imposed on the right side of the basic system and the boundary conditions, we have $u_{i,3}\in W^1$. This guarantees that the initial problem can be reduced to a two-dimensional one for almost all x_3. Furthermore, if we make integral estimates for $u_{i,33}$, we obtain that $u_{i,33}\in W^1(D'\cup D'')$, and for all x_3 the solution $u_i\in W^1(\pi)$ of initial problem (3)-(5) satisfies a two-dimensional boundary-value problem obtained from (3)-(5) by discarding terms with derivatives with respect to x_3 and with small derivatives, whose right side belongs to $L_2(\pi)$. Here π is a section of a sufficiently small neighborhood of the singular point in question by a plane perpendicular to the rib.

Furthermore, if we apply the methods of [13] or [14] to the resultant two-dimensional problem, i.e., if we use the truncating function to reduce this problem to one with constant coefficients equal to their values at the singular point under consideration in an infinite composite two-dimensional wedge, and solving it by means of the Mellin transformation, we can show that the solution near the singular point has the form

$$u_i=\sum_{\alpha<\operatorname{Re}s_k<1} r^{1-s_k}\sum_{n=0}^{N_k-1}\Phi_{ikn}(\varphi)(\ln r)^n+u_i^{\sim}$$
$$\sigma_{ij}=\sum_{\alpha<\operatorname{Re}s_k<1} r^{-s_k}\sum_{n=0}^{N_k-1}\psi_{ijkn}(\varphi)(\ln r)^n+\sigma_{ij}^{\sim} \tag{6}$$

Here s_k are the zeros of the analytic function $\Delta(s)$, while N_k is their multiplicity; $\Delta(s)$ is independent of the right sides of the fundamental system of equations and the boundary conditions, and its zeros are involved in a similar representation of the solution of some model problem in an infinite composite two-dimensional wedge with the same aperture angle as in the initial problem at the singular point in the local coordinate system. The equations of the model problem are the principal part of the equations obtained from the initial ones by discarding terms with derivatives with respect to x_3 in the principal system and the boundary conditions. The coefficients of these equations are constant and equal to their values in the initial equations near the singular point, while the right sides are arbitrary and nonzero in finite regions not containing a corner point, and integrable. The quantities α, $u_i^{\sim}$, $\sigma_{ij}^{\sim}$ in (6) are determined by the smoothness of the right sides of initial problem (3)-(5) near the singular point; in particular, if $f_i^{\circ}=o(r^{-1})$, $u_{i,r}^{\circ}\to 0$, $t_i\to 0$ as $r\to 0$, then $\alpha<0$, $u_i^{\vee}<\infty$, $\sigma_{ij}^{\vee}\to 0$; Φ_{ikn}, ψ_{ijkn} are certain functions of the polar angle φ.

In the case under consideration, the equations of the model problem will be Eqs. (3)-(5) with certain right sides, if we set $A_{ijkl}=$ const ($l, j=1, 2$; $i, k=1, 2, 3$). Let us recall that in the local coordinate system the x_3 axis is directed along the rib, and hence $n_3 = 0$ at the singular point.

It is not hard to see that these equations coincide with those obtained for a homogeneous composite two-face wedge of infinite cross section in which the stress field is independent of the coordinate x_3. The difference is that additional conditions are imposed on the right sides for the realization of this state in the wedge: they should note give the total stress and moment. But, as already noted, the right sides do not affect $\Delta(s)$. Thus, $\Delta(s)$ in expansion (6) for an arbitrary composite body will be the same as in the corresponding problem for a wedge in which the stresses do not vary along the rib.

If at the singular point, tensor A_{ijkl} has a plane of symmetry perpendicular to the rib for both sides of the body, then system (3)-(5) for the model problem breaks down into two unconnected systems: for u_1 and u_2 for $i = 1$ and 2, and for u_3 for $i = 3$. In the general case of loading, expansion (6) will contain terms generated by the zeros of Δ for both systems (let us call them Δ_1 and Δ_2, respectively), and the resulting function $\Delta=\Delta_1\Delta_2$. But if the initial problem is posed for a prismatic composite body and the stress field in it is independent of x_3, then in the absence of torsion and antiplane deformation, i.e., when there is tension, bending, plane deformation, and a two-dimensional temperature field that does not cause torsion, the equation for u_3 will be identically satisfied by the function

$$u_3=Ax_1+Bx_2+C$$

Then expansion (6) will contain only terms generated by Δ_1. For pure torsion or antiplane deformation, only terms generated by Δ_2 will remain in (6).

Let us consider the problem of stress singularity. It can be generated either by the first term in (6), i.e., by the presence of zeros of $\Delta(s)$ in the strip $0\leq \mathrm{Re}\, s<1$ and hence by the form of the differential operators of the principal system and the boundary conditions and the geometry of the body near the singular point; and by the second term in (6), $\sigma_{ij}^{\sim}$, i.e., by insufficient smoothness of the right sides near the singular point.

Let

$$u_i=O(r^{1+\varepsilon}),\quad p_i^{\circ}=O(r^{\varepsilon}),\quad \beta_{ij}T=O(r^{\varepsilon}),\quad F_i=O(r^{-1+\varepsilon})\quad \text{as}\quad r\to 0 \tag{7}$$

$f_1=O(f_2)$ as $r\to 0$ means that $\lim|f_1/f_2|<\infty$ as $r\to 0$.

For an energy solution to exist, it is sufficient that $\varepsilon>-1$. For the behavior of the right sides near the rib not to generate singular terms, i.e., for $\sigma_{ij}^{\sim}$ to be a bounded function, it is sufficient that $\varepsilon>0$; if $\varepsilon=0$ and $s=0$ is a root of $\Delta(s)$, then the appearance of a stress singularity of logarithmic form is possible; $\sigma_{ij}^{\sim}=O(r^{\varepsilon})$ and the stresses will contain terms of the form $\sigma_{ij}=O(r^{\varepsilon})$ for $-1<\varepsilon<0$.

Let us consider, furthermore, the boundary effect in composites. This issue has been taken up in a number of studies, in which the equations of anisotropic elasticity for layers at different angles have been solved by the method of finite differences [18,19] or the method of finite elements [20]. It has been noted that infinite stresses may arise in the neighborhood in which the free surface intersects the surface of connection of the layers. Their asymptotic form has not been investigated. Let us investigate the way in which the degree of stress singularity depends on the reinforcement factor and the local geometry of connection in orthogonally reinforced glass and carbon plastics, and also in composites joined to a layer of aluminum.

Each layer will be regarded as homogeneous and isotropic, with its characteristics being determined by the parameters of the binder and reinforcement and the reinforcement factor. Rigid adhesion is assumed between the layers; stresses are specified in the neighborhood of the rib on the free surfaces. Otherwise the composite is arbitrarily loaded and subjected to a temperature field. Conditions (7) are assumed to be satisfied for $\varepsilon \geqslant 0$. As indicated above, the stresses will have the form (6), and if in $0<\mathrm{Re}\, s<1$ there are roots s_k of the corresponding function $\Delta(s)$, then singular terms of the form $\sigma \sim r^{-s_k}$ will appear in (6).

We will consider only those packings of layers which yield tensors A_{ijkl} near the rib that possess a plane of symmetry perpendicular to the rib. Then $\Delta(s)$ breaks down into $\Delta_1(s)$, corresponding to plane deformation, and $\Delta_2(s)$, corresponding to torsion (or antiplane deformation). Function $\Delta_1(s)$ was obtained in [12], where a solution was constructed for an infinite composite anisotropic wedge under plane deformation.

The characteristic equation for the plane problem of anisotropic elasticity has the form

$$b_{11}\mu^4-2b_{16}\mu^3+(2b_{12}+b_{66})\mu^2-2b_{26}\mu+b_{22}=0 \tag{8}$$

where b_{ij} are material constants [12,21]. A material for which all roots of (8) are different will be assigned to type A. A material having pairwise different roots (8) will be assigned to type B. Type B includes isotropic materials and transversely isotropic materials, with axis of isotropy perpendicular to the plane under consideration. The plane problem for a composite wedge, both parts of which belong to type A will be called problem A-A; when one part belongs to type A and the other to type B we will speak of problem A-B; and when both parts belong to type B we will speak of problem B-B.

Now we require $\Delta_1(s)$ for problem A-B; in this case $\Delta_1(s)=\det(B_{1ij})$, and matrix B_{1ij} has the form

$$\left\|\begin{matrix}
a_1^{2-s} & a_2^{2-s} & a_3^{2-s} & a_4^{2-s} & 0 & 0 & 0 & 0\\
a_1^{1-s}b_1 & a_2^{1-s}b_2 & a_3^{1-s}b_3 & a_4^{1-s}b_4 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & a_5^{2-s} & a_6^{2-s} & a_5^{1-s}a_6(2-s) & a_6^{1-s}a_5(2-s)\\
0 & 0 & 0 & 0 & a_5^{1-s}b_5 & a_6^{1-s}b_6 & a_5^{-s}\omega_{56} & a_6^{-s}\omega_{65}\\
1 & 1 & 1 & 1 & -1 & -1 & s-2 & s-2\\
\mu_1 & \mu_2 & \mu_3 & \mu_4 & -\mu_5 & -\mu_6 & (s-1)\mu_5-\mu_6 & (s-1)\mu_6-\mu_5\\
p_1 & p_2 & p_3 & p_4 & -h_5 & -h_5 & (s-1)h_5-h_6 & (s-1)h_5-h_6\\
q_1 & q_2 & q_3 & q_4 & -\mu_5 h_5 & -\mu_6 h_5 & (s-1)\mu_5 h_5-\mu_6 h_6 & (s-1)\mu_6 h_5-\mu_5 h_6
\end{matrix}\right\|$$

Here μ_j are the roots of (8) for the two parts of the wedge. For simplicity we assume that j = 1,...,4 for type A material and j = 5, 6 for type B material in matrix B_{1ik};

$$a_j=\cos\varphi+\mu_j\sin\varphi,\qquad b_j=\mu_j\cos\varphi-\sin\varphi$$
$$p_j=b_{11}\mu_j^2+b_{12}-b_{16}\mu_j,\qquad q_j=b_{12}\mu_j+b_{22}\mu_j^{-1}-b_{26}$$
$$h_5=b_{12}''-\mu_5\mu_6 b_{11}'',\qquad h_6=h_5-b_{11}''(\mu_5-\mu_6)^2$$
$$\omega_{ij}(s)=(1-s)a_j b_i+a_i b_j$$

φ is the angle of inclination of the free boundary to the line of junction; in the first material $\varphi=\varphi_1>0$, while in the second $\varphi=\varphi_2<0$. For case A-A, the right side of B_{1ij} is altered and will have a form analogous to the left side with its own μ_j, a_j, b_j, p_j, q_j, φ of course (matrix B_{1ij} is written out in [12] for this case). For problem B-B, the left half of the matrix is altered; it becomes analogous to the right. Function $\Delta_2(s)$ was obtained in [5], where self-similar solutions for the torsion problem were sought. In the notation employed here, $\Delta_2(s)=\det(B_{2ij})$.

Matrix B_{2ij} has the form

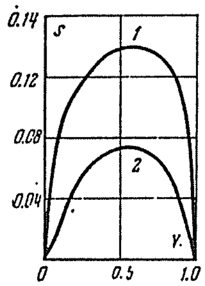

Fig. 1

$$\begin{Vmatrix} a_9^{1-s} & a_{10}^{1-s} & 0 & 0 \\ 0 & 0 & a_{11}^{1-s} & a_{12}^{1-s} \\ 1 & 1 & -1 & -1 \\ r_9 & r_{10} & -r_{11} & -r_{12} \end{Vmatrix}$$

where the a_j are represented in the same form as above; the index j = 9, 10 refers to the first material, j = 11, 12 to the second; μ_j are roots of the equation $b_{55}\mu^2-2b_{45}\mu+b_{44}=0$, $r_j=b_{45}-b_{44}\mu_j^{-1}$.

In calculating the degree of singularity as a function of the reinforcement factor, it was assumed that the reinforcing fibers and the matrix are isotropic and have the following characteristics [22]: $E=8.4\cdot10^3$ kg·mm^{-2}, $\nu=0.22$, for glass fiber; $E=42\cdot10^3$ kg·mm^{-2}, $\nu=0.16$ for carbon fiber; and E = $0.35\cdot10^3$ kg·mm^{-2}, $\nu=0.35$, for epoxy binder. To obtain the anisotropy constants of the layer in terms of the reinforcement factor and the elasticity constants of the components, the relationships given in [23] were employed. In calculations using these formulas, the layer turns out to be transversely isotropic with isotropy axis parallel to the direction of reinforcement. To investigate the stress singularity of an orthogonally reinforced composite made up of such layers, with a rib perpendicular to one of the directions of reinforcement and parallel to the other, we obtain case A-B and the corresponding matrix B_{1ij} written out above.

Figure 1 shows the degree of singularity of the stresses arising in orthogonal reinforced glass and carbon plastics as a function of the volume fiber content V. The rib is parallel to one of the directions of reinforcement at the point under consideration, while the free surface is perpendicular to the surface of junction of the layers ($\varphi_1=-\varphi_2=90°$). Curve 1 refers to carbon plastic, curve 2 to glass plastic. The maximum value of the degree of singularity s occurs in the neighborhood of V = 0.5.

As is to be expected, for isotropic components with reinforcement V = 0 and V = 1, the degree of singularity s = 0, since both layers become identical and isotropic. If the reinforcing fibers are anisotropic, then this symmetry will not occur, and for V = 1 the degree of singularity may be nonzero. Figure 1 shows the roots $\Delta_1(s)$. There are no roots $\Delta_2(s)$ in the interval under consideration for this composite, i.e., torsion of such composites about an axis parallel to the rib does not yield a stress singularity.

In [18-20], calculations were made for a composite with the following layer characteristics: Young's modulus in the direction of reinforcement $E_L=14.6\cdot10^3$ kg·mm^{-2}; transverse Young's modulus $E_z=E_T=1.48\cdot10^3$ kg·mm^{-2}; shear moduli G_{LT} = $G_{Tz}=G_{Lz}=0.598\cdot10^3$ kg·mm^{-2}; Poisson coefficients $\nu_{LT}=\nu_{Tz}=\nu_{Lz}=0.21$. The Z axis was perpendicular to the plane of the layer. Calculations showed that the degree of stress singularity in composites orthogonally reinforced by such layers was 0.033.

Combinations of reinforced plastics and metals (e.g., carbon plastics and aluminum) have come into use of late. Let us consider the stress singularity that may arise in such materials. In our calculations we employed a high-modulus carbon plastic with the following characteristics [22]: $E_L=20\cdot10^3$ kg·mm^{-2}, $E_T=E_z=2.1\cdot10^3$ kg·mm^{-2}, $G_{LT}=G_{Lz}=G_{Tz}=0.85\cdot10^3$ kg·mm^{-2}, $\nu_{LT}=\nu_{Lz}=\nu_{Tz}=0.21$. The characteristics of the aluminum were: $E=7.2\cdot10^3$ kg·mm^{-2}, $\nu=0.3$.

Figures 2 and 3 show the root $\Delta_1(s)$ as a function of the angle of inclination of the free surface of the carbon plastic to the surface of junction for fixed angles of inclination of the free aluminum surface (Fig. 2 refers to the case in which the rib is perpendicular to the direction of reinforcement of the plastic, while Fig. 3 is for the case in which it is parallel). The numbers next to the curves give the angle (in degrees) between the free surface of aluminum and the surface of junction. it can be seen that, beginning with some angle, a second singular root arises in the strip $0<\mathrm{Re}\, s<1$.

Figures 4 and 5 also pertain to cases in which the rib is perpendicular and parallel, respectively, to the direction of reinforcement. The curves represent two families: 1) s as a function of φ_1 (angle of inclination of carbon plastic), when the materials locally comprise a half-plane ($\varphi_1-\varphi_2=$ 180°); 2) plane with a cut $\varphi_1-\varphi_2=360°$). The dashed line gives Re s, when two real roots go over into a pair of complex-conjugate ones. It turned out in calculations that $|\mathrm{Im}\, s|<$ 0.1 in these cases.

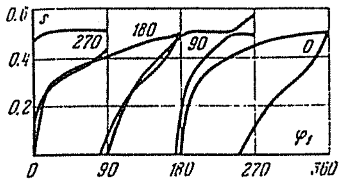

Fig. 2

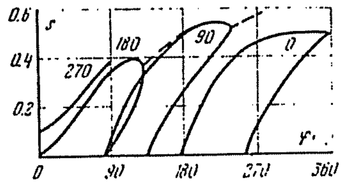

Fig. 3

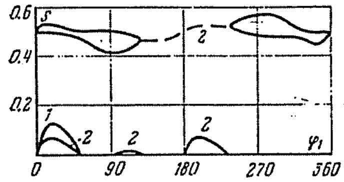

Fig. 4

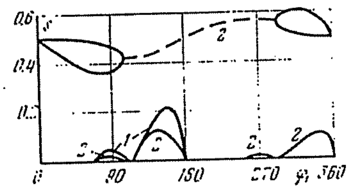

Fig. 5

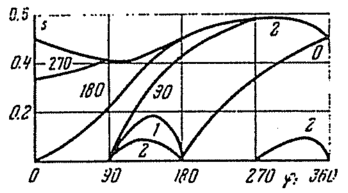

Fig. 6

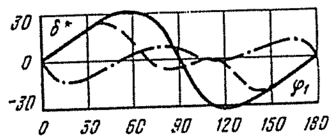

Fig. 7

Since $G_{LT}=G_{TZ}=G_{LZ}$, in the layer of composite under consideration, $\Delta_2(s)$ will be the same for cases in which the rib is parallel and perpendicular to the direction of reinforcement.

Figure 6 shows the roots $\Delta_2(s)$ in the strip $0<\mathrm{Re}\,s<1$ as functions of φ_1 for four fixed values of φ_2, and also for composite half-plane (1) and plane (2) with a cut.

Let us define the regions of φ_1, φ_2, values for which there is no stress singularity, and the critical values of these angles such that the maximum root $\Delta(s)$ in the strip $0\leq\mathrm{Re}\,s<1$ is zero. Assume that $\delta=\varphi_1-\varphi_2-180°$ is the local deviation from the half-plane of the section of the body perpendicular to the rib. Figure 7 shows the critical value δ^* as a function of φ_1. The solid line gives δ_2^* for $\Delta_2(s)$; the dashed line gives δ_{11}^* for $\Delta_1(s)$, if the rib is parallel to the direction of reinforcement; and the dot-dash line gives δ_{12}^* for $\Delta_1(s)$, if the rib is perpendicular to the direction of reinforcement. Under arbitrary loading, there will be no stress singularity when the rib is parallel to the direction of reinforcement for $\delta<\min(\delta_{11}^*, \delta_2^*)$, and when the rib is perpendicular to the direction of reinforced for $\delta<\min(\delta_{12}^*, \delta_2^*)$. If $\varepsilon=0$ in estimates (7), then a stress singularity of logarithmic form may arise for $\delta=\delta^*$.

We should note that $\Delta_1(s)=\det(\bar{B}_{1ij})$ has the root s = 0 for all angles φ_1, φ_2 for case A-B, but, as we can see from (13) and (14), from [12], and from the form of B_{1ij}, this root does not generate the corresponding term of representation (6). In other words, in this case $\Delta_1(s)$ should be taken to be the function $(B_{1ij})s^{-1}$.

Analyzing the graphs in Figs. 2-7, we can conclude that a stress singularity can arise in aluminum in combination with carbon plastic, even if the composite body is bounded by a smooth surface $(\varphi_1-\varphi_2=180°)$. In this case, for arbitrary loading, there will be no singularity in the range $54°<\varphi_1<90°$, when the rib is perpendicular to the direction of reinforcement, and in the range $0°<\varphi_1<70°$, when the rib is parallel to the direction of reinforcement. For large angles φ_1, $|\varphi_2|$; several singular terms of expansion (9) appear.

The same method can be used to consider problems with other boundary conditions.

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