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EDGE EFFECT IN LAMINATE COMPOSITE MATERIALS*

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This study deals with a singularity in the stress distribution in a composite material which an analysis within the linear elastic range may reveal on the line of intersection between the free surface of a specimen and a surface joining its adjacent layers. This line will henceforth be called the edge.

In other studies dealing with the edge effect in composite materials [1-5] each layer was regarded as an anisotropic homogeneous one, all layers rigidly joined together and the entire specimen in a state of generalized plane strain under tension or flexure. The equations of anisotropic elasticity were solved by the finite-difference method [1, 2], by the finite-element method [3, 4], or by expansion into a double series of Legendre polynomials [5]. In the way these methods have been used, however, they hardly yield a true description of the edge effect with respect to stresses, because they are efficacious only when a bounded function will be the solution and not when the stresses can have singularities, as in the case of composite materials. In the application of these methods, the edge stresses remain bounded, but their maximum increases with increasing accuracy of the method (e.g., with an increasing number of terms retained in the series of orthogonal polynomials [5]) and this indirectly confirms the existence of a singularity. Here it will be demonstrated that in composite materials with characteristics close to real, there may appear a stress singularity, then there will be given examples of calculating the degree of such a singularity as a function of the reinforcement factor and of the fiber lay angle in adjacent layers of the composite

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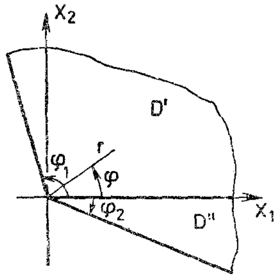

Fig. 1

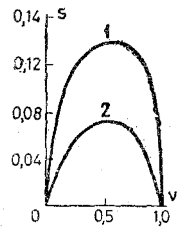

Fig. 2

specimen. A stress singularity is also often found in the case of a high-modulus composite material joined to aluminum. The data obtained here provide a theoretical basis for modification of those methods so that they will take into account a stress singularity in an explicit manner. For instance, one can use singular finite elements or expand the stresses into orthogonal polynomials in a series multiplied by the term containing the singularity.

We consider a multilayer composite consisting of identical layers stacked at different angles and a composite joined to an aluminum layer. Each layer is assumed to be homogeneous and anisotropic, its characterisitcs either are given or can be determined from the parameters of the binder and the reinforcement including the reinforcement factor.

We introduce a Cartesian system of coordinates X_1, X_2, X_3 and a cylindrical system of coordinates r, φ, X_3, their origins at the point of interest on the edge, with the X_3 axis along the tangent to the edge and the X_1 axis along the tangent to the surface joining the layers at the given point. A section through the region around the edge ($X_3 = 0$) is shown in Fig. 1, where D' denotes one layer and D'' denotes another layer ($\varphi_2 \leqslant 0 \leqslant \varphi_1$.) The layers are joined rigidly with an interference fit and given forces act at the free surfaces in the vicinity of the given point on the edge. Beyond this vicinity the composite is loaded arbitrarily and subject to thermal action in a temperature field. Let the boundary forces P_i°, the temperature field θ, the displacement jump u_i° at the joint surface, and the body forces F_i in the vicinity of the given point also satisfy the conditions

$$P_i^0=0(r^{\varepsilon});\quad \theta=0(r^{\varepsilon});\quad u_i=0(r^{1+\varepsilon});\quad F_i=0(r^{-1+\varepsilon}). \tag{1}$$

with some $\varepsilon \geqslant 0$. Then, according to the results of an earlier study [6], the stresses around the given point on the edge can be represented as

$$\sigma_{ij}=\tilde{\sigma}_{ij}+\sum_{0\leqslant s_k<1} r^{-s_k}\sum_{n=0}^{N_k-1}\psi_{ijkn}(\varphi)(\ln r)^n, \tag{2}$$

where s_k are zeros of the analytic function $\Delta(s) = \det B_{ij}$; B_{ij} are matrices [7-9] involved in the solution of the model problem of a compound wedge, $\tilde{\sigma}_{ij} < \infty$; N_k is the multiplicity of zeroes s_k, taken as larger by one for $s_k = 0$ when $\varepsilon = 0$; and $\psi_{ijkn}(\varphi)$ are bounded smooth functions.

Expression (2) indicates that the stresses in the vicinity of the edge are bounded when $\varepsilon > 0$ and function $\Delta(s)$ has no zeroes in the $0 \leqslant \mathrm{Re}\ s < 1$ interval, but a power-law stress singularity near the given point is possible when this function has zeroes in that interval. When function $\Delta(s)$ has a second-order zero at $s = 0$ or when it has a simple zero and $\varepsilon = 0$, then a logarithmic type of stress singularity at the edge is possible. In this way, the behavior of the zeros of function $\Delta(s)$ yields information as to whether the stresses in the vicinity of the edge will be finite or infinite and, in the latter case, will also yield the degree of singularity.

It must be noted that, as has been shown in the latest studies of elliptical boundary-value problems for regions with irregular boundaries and, specifically, in the study of compound anisotropic bodies [6], representation (2) will also remain valid in the case of a curvilinear (but sufficiently smooth) edge. Then function $\Delta(s)$ and thus also the degree of singularity s_k in expression (2) will depend neither on the curvature of the edge nor on the intensity of the applied loads. They will be determined solely by the kind of boundary conditions around the given point on the edge, on the elasticity constants of the materials around this point, and by the local geometry, namely by the angles between tangents to the joint

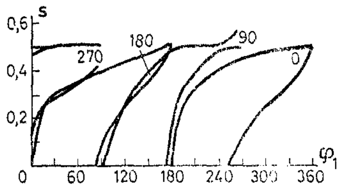

Fig. 3

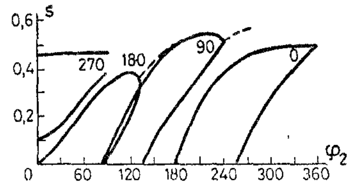

Fig. 4

surface and the free surface at that point. A smooth bounding of functions $\psi_{ijkn}(\varphi)$ can be expressed as the sum of several known functions which depend on nothing except angle φ and have constant unknown coefficients. The latter ones can be regarded as the generalized stress intensity coefficients, and only they will depend on the boundary conditions and on the boundary geometry at all other points of the body.

There will be shown here graphs depicting the dependence of the degree of singularity on the joint geometry, on the lay of fibers in adjacent layers, and on the reinforcement factor. It is noteworthy that these data provide a basis for modifying the local geometry so as to avoid a stress singularity in the elastic mode of the solution to the problem or simply high stresses in an actual product. This seems to be entirely feasible, at least in the case of composite materials joined to metals.

1. Stress Singularity in a Laminate Composite Material with an Edge Parallel or Perpendicular to the Direction of Reinforcement. Here will be considered only fiber lay patterns yielding elasticity tensors A_{ijkl} of adjacent layers near the edge which have a plane of symmetry perpendicular to it. According to the discussion in an earlier study [6], function $\Delta(s)$ whose zeros in the interval $0 \leqslant \mathrm{Re}\ s < 1$ determine the degree of singularity splits into function $\Delta_1(s)$ corresponding to plane deformation and function $\Delta_2(s)$ corresponding to antiplane deformation (or twist): $\Delta = \Delta_1(s)\Delta_2(s)$. Function $\Delta_1(s)$ has been determined in another study [7] and the form of function $\Delta_2(s)$ has been established elsewhere [8] during a search for self-adjoint solutions to the problem of twisting.

The problem of calculating the degree of singularity thus reduces to determining the zeros of $\Delta(s)$ or, which is easier, of $\Delta_1(s)$ and $\Delta_2(s)$ separately in the interval $0 \leqslant \mathrm{Re}\ s < 1$, where Δ_1 and Δ_2 are the determinants of complex (8×8) and (4×4) matrices. The roots were found numerically by the Mueller method with the aid of a BÉSM-6 high-speed computer. Finding the roots for any fixed set of parameters required 30 sec of machine time in the central processor.

In calculations of the degree of singularity as a function of the reinforcement factor, both the reinforcing fibers and the binder were assumed to be isotropic, with the following characteristics [10]: $E = 8.4 \cdot 10^3$ kgf/mm^2 and $\nu = 0.22$ for glass fiber, $E = 42 \cdot 10^3$ kgf/mm^2 and $\nu = 0.16$ for carbon fiber, $E = 0.35 \cdot 10^{-3}$ kgf/mm^2 and $\nu = 0.35$ for epoxide binder.

Relations given in another study [11] were used for expressing the anisotropy constants of a layer in terms of the elasticity constants of the material components and the reinforcement factor. Calculations based on these relations have yielded a transversely isotropic layer with the axis of isotropy in the direction of the lay.

The graph in Fig. 2 indicates how the degree of stress singularity in orthogonally reinforced glass-plastics and carbon-plastics depends on the volume fraction of fibers, with the edge parallel to one of the directions of reinforcement at the given point and the free surface perpendicular to the surface joining the layers ($\varphi_1 = -\varphi_2 = 90°$). Curve 1 refers to a carbon-plastic and curve 2 refers to a glass-plastic. The maximum degree of singularity is reached with an about $v = 0.5$ reinforcement factor. As is to be expected in the case of isotropic components, the degree of singularity is $s = 0$ with a reinforcement $v = 0$ or $v = 1$ corresponding to identical isotropic two layers. When the reinforcing fibers are anisotropic, then there will be no such symmetry and the degree of singularity cannot be zero with $v = 1$. The zeros of function $\Delta_1(s)$ are shown in Fig. 2. Function $\Delta_2(s)$ for the given composite materials does not have zeros in the said interval, which means that antiplanar deformation or twisting of specimens about an axis parallel to the edge will not produce a stress singularity.

In those studies [1-3] calculations were made for a composite material with the following characteristics of a layer:

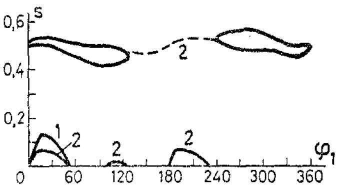

Fig. 5

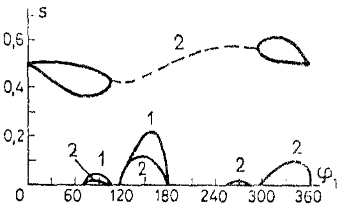

Fig. 6

Young's modulus in the direction of reinforcement $E_L = 14.6 \cdot 10^3$ kgf/mm²;
Young's modulus in the transverse directions $E_Z = E_T = 1.48 \cdot 10^3$ kgf/mm²; (3)
Shear modulus $G_{LT} = G_{TZ} = G_{LZ} = 0.598 \cdot 10^3$ kgf/mm²;
Poisson coefficient $\nu_{LT} = \nu_{LZ} = 0.2$.

The calculations in this study have yielded a degree of stress singularity $s = 0.033$ in such orthogonally reinforced composite materials.

Since recently reinforced plastics are often joined to metals as, e.g., carbon-plastics to aluminum, let us examine the stress singularity which can appear in such materials. We consider a high-modulus composite material with the characteristics [9]

$$E_L = 20 \cdot 10^3 \text{ kgf/mm}^2 \;;\; E_Z = E_T = 2.1 \cdot 10^3 \text{ kgf/mm}^2,$$
$$G_{LT} = G_{TZ} = G_{LZ} = 0.85 \cdot 10^3 \text{ kgf/mm}^2 \;,\; \nu_{LT} = \nu_{LZ} = \nu_{TZ} = 0.21. \tag{4}$$

and aluminum with the elasticity constants

$$E = 7.2 \cdot 10^3 \text{ kgf/mm}^2,\; \nu = 0.3.$$

The graphs in Figs. 3 and 4 indicate how the roots of $\Delta_1(s)$ depend on the angle at which the free surface of a composite specimen is inclined to the joint surface, at various fixed sloping angles of the free aluminum surface. The graph in Fig. 3 and in Fig. 4 refer to edges, respectively, perpendicular or parallel to the direction of reinforcement. The number next to each curve is the corresponding sloping angle (in degree) of the aluminum surface. The dashed lines represent Re s when two real roots become a pair of complex conjugate ones. Calculations have yielded $|\text{Im } s| < 0.1$ for these cases.

The graphs in Fig. 5 and in Fig. 6 refer to edges, respectively, perpendicular or parallel to the direction of reinforcement. The curves here fall into two families depicting the dependence of s on φ_1 (sloping angle of the free composite surface): curve 1 when the materials form locally a half plane ($\varphi_1 - \varphi_2 = 180°$) and curves 2 when the materials form a plane with a slit ($\varphi_1 - \varphi_2 = 360°$).

Since in the given layer of composite material $G_{LT} = G_{TZ} = G_{LZ}$, hence $\Delta_2(s)$ will be the same in both cases of an edge perpendicular and parallel to the direction of reinforcement. The roots of $\Delta_2(s)$ in the interval $0 \leqslant \text{Re } s < 1$ and thus the degree of singularity due to twisting about an axis parallel to the edge is shown in Fig. 7 as a function of φ_1, for four different fixed angles φ_2 as well as for a compound half plane and a compound plane with a slit.

It is interesting to find the ranges of angles φ_1 and φ_2 where no stress singularity occurs, also the critical values of these angles at which the largest root of $\Delta(s)$ in the interval $0 \leqslant \text{Re } s < 1$ is zero. Let $\delta = \varphi_1 - \varphi_2 - 180°$ be the local deviation from the half plane of the body section perpendicular to the edge ($\varphi_2 < 0$ by definition, Fig. 1). The critical deviation δ^* is shown in Fig. 8 as a function of the angle φ_1. The solid line represents δ^*_2 for $\Delta_2(s)$, the dash line represents δ^*_{11} for $\Delta_1(s)$ when the edge is parallel to the direction of reinforcement, and the dash-dot line represents δ^*_{12} for $\Delta_1(s)$ when the edge is parallel to the direction of reinforcement.

We note that $\delta^* = 0$ for a homogeneous (not compound) body. Under an arbitrary load (with the forces at the free surface in the vicinity of the point of interest given) there will be no stress singularity with the edge parallel to the direction of reinforcement if $\delta < \min(\delta^*_{11}, \delta^*_2)$ and with the edge perpendicular to the direction of reinforcement if $\delta < \min(\delta^*_{12}, \delta^*_2)$. When $\varepsilon = 0$ in expression (1), which means that the applied forces are bounded in the vicinity of the given point on the edge and have there a discontinuity of the first kind, and when analogous constraints are imposed on the other remaining actions, then a stress singularity of the logarithmic type can occur if $\delta(\varphi_1) = \delta^*(\varphi_1)$.

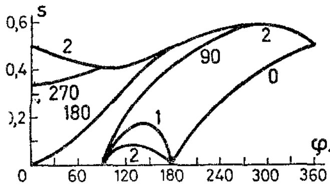

Fig. 7

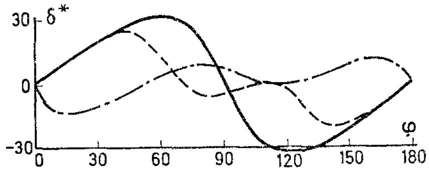

Fig. 8

An analysis of the graphs in Figs. 3-8 leads to the conclusion that a stress singularity can exist at joints of aluminum with a composite material, even when the body is bounded by a smooth surface ($\varphi_1 - \varphi_2 = 180°$). Under an arbitrary load (with the forces in the vicinity of the point of interest given) in the case of a composite material with the characteristics (3), however, no stress singularity will be found either within the range $54° < \varphi_1 < 90°$ with the edge perpendicular to direction of reinforcement or within the range $0° < \varphi_1 < 70°$ with the edge parallel to the direction of reinforcement. At sufficiently large angles φ_1 and $|\varphi_2|$, moreover, there will appear several singular terms in the stress representation.

2. Stress Singularity in an Arbitrarily Reinforced Laminate Composite Material. Now we will consider a composite material with an arbitrary orientation of fibers in adjacent layers and a composite material joined to aluminum. A laminate composite specimen is shown schematically in Fig. 9. The plane of fiber lay coincides with the plane of the paper. The dash lines represent the lay angles and the dash-dot line represents the bisector of the angle between the directions of reinforcement in adjacent layers.

We consider a layer of carbon-plastic material with the same characteristics (3) [1-3]. For determining the elastic characteristics of this layer rotated in its plane we will use the relations in [12].

As before, the problem of determining the degree of singularity reduces to a numerical determination of the zeros of function $\Delta(s)$ in the interval $0 \leqslant \mathrm{Re}\ s < 1$. The form of this function has been established in an earlier study [9]. Here the calculation of its zeros by the Mueller method for a fixed set of parameters already requires ≈1.5 min of machine time on a BÉSM-6 high-speed computer.

The degree of stress singularity s has been plotted in Fig. 10 as a function of the angle γ, at various fixed angles α_1 (these angles indicated next to the curves) between one of the directions of reinforcement and the normal to the edge. In the general case the degree of singularity passes through one absolute maximum as the divergence between fiber orientations in adjacent layers changes from 0 to ±90°. With the edge either perpendicular or parallel to one of the directions of reinforcement, the maximum degree of singularity is reached when the reinforcement becomes orthogonal and the $s(\gamma)$ curve is then symmetric with respect to that s_{max}. In other cases the symmetry vanishes and the maximum shifts somewhat from its location in the orthogonal case. Near the location at which the second direction of reinforcement is parallel to the edge, furthermore, the degree of singularity may pass through a mild local minimum.

The degree of singularity s as a function of the angle θ which the axis of symmetry of the lay pattern makes with the edge has been plotted in Fig. 11 for three different fixed angles $\gamma \leqslant 45°$ (these angles are indicated next to the curves). This is, in fact, how the degree of singularity varies as one moves along the edge around the periphery of the specimen.

The degree of singularity s as a function of the angle of divergence between fiber orientation in adjacent layers has been plotted in Fig. 12 for three different fixed angles θ (these angles are indicated next to the curves). It follows from the graph in Fig. 9 and from symmetry considerations that $s(\gamma, \theta) = s(\gamma, -\theta) = s(90° - \gamma, 90° - \theta)$. At a constant angle θ the degree of singularity s increases monotonically throughout almost the entire range $0° < \gamma < 45°$.

One can conclude from the graphs in Figs. 10-12 that a stress singularity in the given composite material is possible at any angles α, θ, γ except $\gamma = 0$, at which the singularity vanishes together with the jump of elastic characteristics at the joint surface. The degree of singularity is maximum when the edge runs parallel to one direction of reinforcement and perpendicularly to the other.

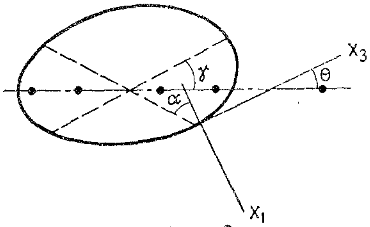

Fig. 9

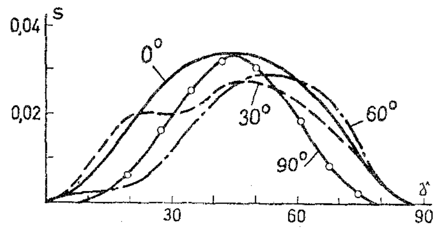

Fig. 10

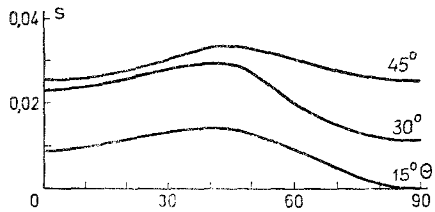

Fig. 11

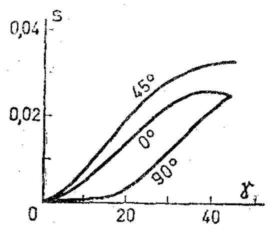

Fig. 12

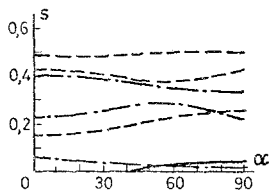

Fig. 13

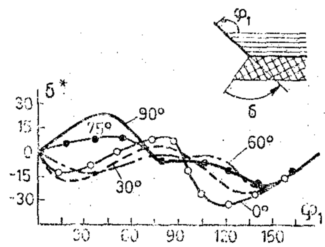

Fig. 14

Let us now examine the degree of stress singularity in the case of a composite layer joined to an aluminum layer. The calculations for this case were made using a high-modulus composite material with the characteristics (4) and aluminum.

The degree of singularity s as a function of the angle α between the direction of reinforcement and the normal to the edge has been plotted in Fig. 13 for three different fixed sloping angles of the composite surface φ_1 and of the aluminum surface φ_2, respectively, relative to the joint surface (solid line $\varphi_1 = 90°$ and $\varphi_2 = -90°$, dash lines $\varphi_1 = 180°$ and $\varphi_2 = -90°$, dash-dot lines $\varphi_1 = 90°$ and $\varphi_2 = -180°$). We note that up to three singular roots exist when angles φ_1 and $|\varphi_2|$ are sufficiently large. Generally s does not vary monotonically with α, but it does so when $\varphi_1 = \varphi_2 = 90°$ and the edge is rotated from the position parallel to the direction of reinforcement to a position perpendicular to it, i.e., as one moves along the edge around the periphery of the specimen.

Let $\delta = \varphi_1 - \varphi_2 - 180°$ denote the local deviation of the free specimen surface near the edge from the compound half plane, just as in Sec. 1. The graph in Fig. 14 depicts the critical angle δ^*, at which the largest root of $\Delta(s)$ in the range $0 \leqslant \mathrm{Re}\, s < 1$ (and thus the maximum degree of stress singularity) becomes zero, as a function of the angle φ_1 (the various fixed angles α are indicated next to the curves). There is no singularity when $\delta < \delta^*$, but there will generally exist one when $\delta > \delta^*$. The curves for $\alpha = 90°$ and $\alpha = 0°$, corresponding to an edge, respectively, parallel and perpendicular to a direction of reinforcement, are taken from Fig. 8. As angle α changes, the variation of δ^* follows a rather intricate trend. An obvious explanation for this is that in our example the Young modulus of aluminum lies between the maximum and the minimum Young modulus of the carbon-plastic, while the shear modulus of aluminum is always higher than that of a composite material.

The availability of graphs such as those shown in Figs. 8 and 14 makes it possible to design the local geometry near the edge for given materials so as to avoid a stress singulairty at the joint. The degree of singularity can be calculated by the same method also for other boundary conditions near a particular point of interest.

Although the results here pertaining to fracture mechanics of composite materials are in a sense qualitative rather than quantitative, inasmuch as the generalized stress intensity factors near a singularity remains unknown, these results nevertheless on the one hand yield some reliable information about the trend of stresses near the edge of a composite and on the other hand allow for calculation of the stress intensity factors by other methods.

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