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ASYMPTOTIC BEHAVIOR OF THE SOLUTIONS OF SOME INTEGRAL EQUATIONS AND PLANE PROBLEMS OF ELASTICITY NEAR ANGULAR CORNERS, WITH FORCES SPECIFIED ON THE BOUNDARY

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In this paper, we consider the boundary integral equations of the simple-layer elastic potential and Weyl-Khatsirevich potential to which it is possible to reduce plane boundary value problems of linear elasticity for compressible and incompressible isotropic media, as well as linearized problems of hydrodynamics of incompressible viscous fluids, in domains with a piecewise-smooth boundary and under forces specified on the boundary [1]. Within the framework of the approach of [2], using the Mellin integral transform, as in [3] for the integral equations of antiplane strain, we obtain explicit representations for the asymptotic forms of the densities of the integral equations up to bounded terms inclusive. We also give the asymptotic forms for critical angles, when logarithmic terms appear in addition to power-law ones; and we indicate the relationship between the intensity coefficients of the densities for critical and subcritical angles.

We also give the asymptotic form of the stresses in the neighborhood of angular points all the way to bounded terms, and we provide expressions for the stress intensity coefficients in terms of the intensity coefficients of the densities of the integral equations.

1. INTRODUCTION

In [1], problems of elasticity for non-simply-connected domains with angular points, and with forces specified on the boundary ($\sigma_{ij}n_j|_{\partial D}=g_i(s)$), were reduced to a boundary integral equation of type I (BIE I) by means of a simple-layer potential of the first kind:

$$Q_i^{I}(s_0)-\int_{\partial D}K_{ij}^{I}(s,s_0)Q_j^{I}(s)\,ds=h_i^{I}(s_0)$$
$$K_{ij}^{I}(s,s_0)=[\pi r^2(1+\varkappa)]^{-1}\{[4z_iz_jr^{-2}+(\varkappa-1)\delta_{ij}]z_ln_l(s_0)+ \\ +(\varkappa-1)[n_j(s_0)z_i-n_i(s_0)z_j]\} \qquad (1.1)$$

or to a boundary integral equation of type III (BIE III) by means of the Weyl-Khatsirevich potential:

$$Q_i^{III}(s_0)-\int_{\partial D}K_{ij}^{III}(s,s_0)Q_j^{III}(s)\,ds=h_i^{III}(s_0)$$
$$K_{ij}^{III}(s,s_0)=(2/\pi)r^{-4}z_iz_jz_ln_l(s_0)$$

Here $z_i=x_i(s_0)-x_i(s)$, s is the arc coordinate of the contour; $r^2=z_iz_i$, $n_i(s_0)$ is the outward normal to ∂D; the constant of plane elasticity $\kappa > 1$; $h_i^{I}(s)=g_i(s)$, $h_i^{III}(s)=g_i(s)+\sigma_{0ij}n_j(s)$, where σ_{0ij} is an additional stress field that is infinitely smooth all the way to the boundary ∂D, which appears for non-simply-connected domains D. Summation from 1 to 2 is understood over repeating indexes; δ_{ij} is the Kronecker delta.

The kernels of both equations have strong stationary singularities at angular points of ∂D; in addition, the kernel of BIE I has a Cauchy-type singularity, while the kernel of BIE III is continuous at sufficiently smooth points of the contour. These equations were studied in [1] in Lebesgue spaces $L_p(\partial D)$ for two-dimensional domains D. Paper [4] examined BIE I in three-dimensional domains with smooth edges.

The stresses can be expressed in terms of the densities Q_i^{I}, Q_i^{III} as follows [1]:

$$\sigma_{il}(Q^{I}) = -\frac{1}{\pi(1+\varkappa)}\int_{\partial D}\frac{1}{r^2}\left\{\left[4\frac{z_i z_j}{r^2}+(\varkappa-1)\delta_{ij}\right]z_l+(\varkappa-1)\left[\delta_{jl}z_i-\delta_{il}z_j\right]\right\}Q_j(s)\,ds \tag{1.2}$$

$$\sigma_{il}(Q^{III}) = -\frac{2}{\pi}\int_{\partial D}\frac{z_i z_j z_l}{r^4}Q_j(s)\,ds \tag{1.3}$$

We should note that representation (1.3) also yields a solution of the Stokes equations for an incompressible viscous fluid, and the equivalent equations for an incompressible elastic medium, and constitutes the simple-layer hydrodynamic potential that reduces problems with specified forces for these media to the same BIE III [1].

2. ASYMPTOTIC FORM OF SOLUTION OF BIE

Let us first consider the case in which s^* is the arc coordinate of an angular point, while D is an infinite wedge $\{x_i=\rho(\delta_{1i}\cos\theta+\delta_{2i}\sin\theta),\ 0<\rho<\infty,\ -\omega/2<\theta<\omega/2\},\ 0<\omega<2\pi$.

Then BIE I and III transform as follows:

$$Q_{ip}(\rho_0)-\int_0^\infty K_{ijpm}(\rho,\rho_0)Q_{jm}(\rho)\,d\rho=h_{ip}(\rho_0) \tag{2.1}$$

$$\begin{gathered}Q_{jm}(\rho):=Q_j(s^*+(-1)^m\rho),\quad h_{ip}(\rho_0):=h_i(s^*+(-1)^p\rho_0)\\ K_{ijpm}(\rho,\rho_0):=K_{ij}(s^*+(-1)^m\rho,\ s^*+(-1)^p\rho_0)\end{gathered} \tag{2.2}$$

Assume that $L_{2,\alpha}(0,\infty)$ is a weighting space of functions with norm

$$\|f\|_{L_{2,\alpha}}:=\left\{\int_0^\infty[f(\rho)\rho^\alpha]^2\,d\rho\right\}^{1/2}<\infty,\quad L^{\vee}_{2,\alpha}:=\bigcap_{\alpha<\beta<1/2}L_{2,\beta}$$

i.e., the set $L^{\vee}_{2,\alpha}$ consists of functions that are square-integrable with any weight whose index $\beta\in(\alpha,1/2)$.

Let $h_{ip}(\rho_0)\in L^{\vee}_{2,\alpha_0},\ \alpha_0<1/2$. We will seek a solution Q_{ip} of BIE (2.1), that belongs to $L^{\vee}_{2,\alpha}$ with the largest possible $\alpha < 1/2$, i.e., to the set

$$M_{1/2}:=\bigcup_{\alpha<1/2}L^{\vee}_{2,\alpha}\equiv\bigcup_{\alpha<1/2}\left(\bigcap_{\alpha<\beta<1/2}L_{2,\beta}\right)$$

In particular, functions that satisfy the following estimates for any $\alpha < 1/2$ belong to this set:

$$\begin{gathered}Q=O(r^{-1/2-\alpha+\varepsilon})\ (r\to 0),\quad Q=O(r^{-1-\varepsilon})\ (r\to\infty),\ \forall\varepsilon>0\\ Q\in L_2(a_0,a_\infty),\ \forall a_0,a_\infty\ (\infty>a_\infty>a_0>0)\end{gathered}$$

and these are usually employed in determining the asymptotic forms of the solutions of boundary value problems (see, e.g., [5]).

Having in mind that $h_{ip}{}^{\vee}\in L^{\vee}_{2,\alpha_0},\ Q_{ip}\in M_{1/2}$, and following [6], we will solve (2.1) using the Mellin transform; after applying it to (2.1), we arrive at the following system of linear algebraic equations:

$$\langle Q_{ip}\rangle(\gamma)-\hat{K}_{ijpm}(\gamma)\langle Q_{jm}\rangle(\gamma)=\langle h_{ip}\rangle(\gamma),\quad 1/2+\alpha<\gamma<1 \tag{2.3}$$

$$\langle Q_{ip}\rangle(\gamma):=\int_0^\infty Q_{ip}(\rho)\rho^{\gamma-1}\,d\rho,\quad \hat{K}_{ijpm}(\gamma):=\int_0^\infty K_{ijpm}(1,u)\,u^{\gamma-1}\,du \tag{2.4}$$

which obtains on the interval $1/2+\alpha<\mathrm{Re}\,\gamma<1$ for some $\alpha < 1/2$. Here function $Q_{ip}(\rho)$ can be represented as follows in terms of the transform $\langle Q_{ip}\rangle(\gamma)$:

$$Q_{ip}(\rho)=\int_{c-i\infty}^{c+i\infty}\langle Q_{ip}\rangle(\gamma)\rho^{-\gamma}\,d\gamma,\quad 1/2+\alpha<c<1 \tag{2.5}$$

In deriving (2.3) and (2.4), we took into account that for an infinite wedge we have $K_{ijpm}(\rho,\rho_0)=\rho^{-1}K_{ijpm}(1,\rho_0/\rho)$ by virtue of (2.1).

Substituting (1.1) into (2.4) and performing a number of manipulations,

$$K^{I\wedge}_{ijpm}(\gamma)=-a^I(\gamma)\chi_{ij}\Omega_{pm}-b^I(\gamma)\Omega_{ij}e_{pm}+c^I(\gamma)\delta_{ij}\Omega_{pm}- \\ -d^I(\gamma)e_{ij}e_{pm}+f^I(\gamma)e_{ij}\chi_{pm} \tag{2.6}$$

$$\begin{gathered}
e_{ij}:=\delta_{1i}\delta_{2j}-\delta_{2i}\delta_{1j},\quad \chi_{ij}:=-\delta_{1i}\delta_{1j}+\delta_{2i}\delta_{2j},\quad \Omega_{ij}:=\delta_{1i}\delta_{2j}+\delta_{2i}\delta_{1j}\\
a^I:=2(\gamma-1)\sin\varphi\cos[(\gamma-1)\varphi]\,[(1+\varkappa)\sin(\gamma\pi)]^{-1}\\
b^I:=-2(\gamma-1)\sin\varphi\sin[(\gamma-1)\varphi]\,[(1+\varkappa)\sin(\gamma\pi)]^{-1}\\
c^I:=-\sin[(\gamma-1)\varphi]/\sin\gamma\pi\\
d^I:=(\varkappa-1)\cos[(\gamma-1)\varphi]\,[(\varkappa+1)\sin(\gamma\pi)]^{-1}\\
f^I:=-(\varkappa-1)(\varkappa+1)^{-1}\operatorname{ctg}(\gamma\pi),\quad \varphi:=\omega-\pi
\end{gathered} \tag{2.7}$$

Matrix $K^{III\wedge}_{ijpm}(\gamma)=K^{I\wedge}_{ijpm}(\gamma)|_{\varkappa=1}$.

The solution of system (2.3) has the form

$$\langle Q_{ip}\rangle(\gamma)=G^{\wedge}_{ijpm}(\gamma)\langle h_{jm}\rangle(\gamma)/\Delta(\gamma) \tag{2.8}$$

Here $G^{\wedge}_{ijpm}(\gamma)$ are the algebraic complements of the elements of the 4×4 matrix of system (2.3); $\Delta(\gamma)$ is its determinant. They can be written explicitly as follows:

$$\begin{gathered}
G^{\wedge}_{1111}=G^{\wedge}_{1122}=1-K^{\wedge 2}_{2212}+K^{\wedge 2}_{1211}+K^{\wedge}_{1212}K^{\wedge}_{2112}-K^{\wedge}_{1211}K^{\wedge}_{2212}(K^{\wedge}_{2112}+K^{\wedge}_{1212})\\
G^{\wedge}_{2111}=-G^{\wedge}_{2122}=K^{\wedge}_{1211}+K^{\wedge}_{2112}(K^{\wedge}_{1112}-K^{\wedge}_{2212})+\\
+K^{\wedge}_{1211}(K^{\wedge 2}_{1211}-K^{\wedge 2}_{2112}-K^{\wedge}_{2212}K^{\wedge}_{1112})\\
G^{\wedge}_{1211}=G^{\wedge}_{1112}=-K^{\wedge}_{1112}+K^{\wedge}_{1211}(K^{\wedge}_{1212}+K^{\wedge}_{2112})+\\
+K^{\wedge}_{2212}(-K^{\wedge 2}_{1211}+K^{\wedge}_{1112}K^{\wedge}_{2212}-K^{\wedge}_{1212}K^{\wedge}_{2112})\\
G^{\wedge}_{2121}=-G^{\wedge}_{2112}=K^{\wedge}_{2112}+K^{\wedge}_{1211}(-K^{\wedge}_{2212}+K^{\wedge}_{1112})-\\
-K^{\wedge 2}_{1211}K^{\wedge}_{1212}-K^{\wedge}_{1112}K^{\wedge}_{2112}K^{\wedge}_{2212}+K^{\wedge}_{1212}K^{\wedge 2}_{2112}\\
G^{\wedge}_{2211}=-G^{\wedge}_{1222}=-K^{\wedge}_{1211}+K^{\wedge}_{1212}(-K^{\wedge}_{1112}+K^{\wedge}_{2212})+\\
+K^{\wedge}_{1211}(-K^{\wedge 2}_{2211}+K^{\wedge}_{1112}K^{\wedge}_{2212}+K^{\wedge 2}_{1212})\\
G^{\wedge}_{2211}=G^{\wedge}_{2222}=1+K^{\wedge 2}_{1211}-K^{\wedge 2}_{1112}+K^{\wedge}_{1212}K^{\wedge}_{2112}+\\
+K^{\wedge}_{1112}K^{\wedge}_{1211}(K^{\wedge}_{2112}+K^{\wedge}_{1212})\\
G^{\wedge}_{1222}=-G^{\wedge}_{1212}=K^{\wedge}_{1212}+K^{\wedge}_{1211}(-K^{\wedge}_{2222}+K^{\wedge}_{1112})-\\
-K^{\wedge 2}_{2211}K^{\wedge}_{2112}-K^{\wedge}_{1212}K^{\wedge}_{1112}K^{\wedge}_{2212}+K^{\wedge 2}_{1212}K^{\wedge}_{2112}\\
G^{\wedge}_{2212}=G^{\wedge}_{2221}=-K^{\wedge}_{2212}-K^{\wedge}_{1211}(K^{\wedge}_{1212}+K^{\wedge}_{2112})+\\
+K^{\wedge}_{1112}(-K^{\wedge 2}_{1211}+K^{\wedge}_{1112}K^{\wedge}_{2212}-K^{\wedge}_{2112}K^{\wedge}_{1212})
\end{gathered} \tag{2.9}$$

To obtain $G^{\wedge I}_{ijpm}$ we take elements $K^{\wedge I}_{ijpm}$ in (2.9), while for $G^{\wedge III}_{ijpm}$ we take elements $K^{\wedge III}_{ijpm}$:

$$\begin{gathered}
\Delta^I(\varkappa,\omega,\gamma)=16[(\varkappa+1)\sin\gamma\pi]^{-4}\Delta_*(1,\omega,\gamma)\Delta_*(-1,\omega,\gamma)\times\\
\times\Delta_*(\varkappa,2\pi-\omega,\gamma)\Delta_*(-\varkappa,2\pi-\omega,\gamma)
\end{gathered} \tag{2.10}$$

$$\Delta^{III}(\omega,\gamma)=\Delta^I(1,\omega,\gamma),\quad \Delta_*(\varkappa,\omega,\gamma):=\varkappa\sin[(\gamma-1)\omega]+(\gamma-1)\sin\omega$$

Since $h_{jm}(\rho)\in L^{\vee}_{2,\alpha_*}$, in accordance with [6], $\langle h_{jm}\rangle(\gamma)$ are regular analytic functions in the strip $\alpha_*+{}^1/_2<\operatorname{Re}\gamma<1$, in which, in addition, the norm of $\langle h_{jm}\rangle(\gamma)$ in L_2 on the straight line $\operatorname{Re}\gamma=c$ is expressed in terms of the norm of $h_{jm}(\rho)$ in $L_{2,c-1/2}$. Then, in this strip, the right side of (2.8) is a meromorphic function with poles of finite multiplicity at the zeros (with respect to γ) of functions $\Delta_*(\pm1,\omega,\gamma)$, $\Delta_*(\pm\varkappa,2\pi-\omega,\gamma)$ for Q_{ip}^I and of functions $\Delta_*(\pm1,\omega,\gamma)$, $\Delta_*(\pm1,2\pi-\omega,\gamma)$ for Q^{III}_{ip}. The zeros of these functions were studied earlier in [7-11] in particular, the zeros of $\Delta_*(1,\omega,\gamma)$ yield the degrees of singularity of the stresses in an elastic wedge with internal angle ω that is loaded by symmetrical edge forces, while the zeros of $\Delta_*(-1,\omega,\gamma)$ yield the analogous degrees of singularity in the antisymmetrical case. The zeros of $\Delta_*(-\varkappa,\omega,\gamma)$ yield the degrees of singularity in a wedge with angle ω loaded by symmetrical boundary displacements, while the zeros of $\Delta_*(\varkappa,\omega,\gamma)$ yield the degrees of singularity for the corresponding antisymmetrical case. In the strip $0<\operatorname{Re}\gamma<1$ these zeros are real and simple, and the strip in question contains not more than one zero of each of these functions in the range $0<\omega<2\pi$; the root γ_1 of function $\Delta(\varkappa, \omega, \gamma)$ is not less than root γ_2 of $\Delta(-\varkappa, \omega, \gamma)$, $\varkappa\geqslant 1$ and $\gamma_1(\varkappa, \omega), \gamma_2(\varkappa, \omega)<{}^1/_2$.

It can be seen from this that $\langle Q_{i\nu}\rangle(\gamma)$ is a regular analytic function in the strip $\alpha_*+{}^1/_2<\operatorname{Re}\gamma<1$, where $\alpha_*=\max(0, \alpha_0)$ and hence $Q_{i\nu}(\rho)\in L_{2,\alpha_*}^{\vee}$ [6], i.e., as was assumed, $Q_{i\nu}\in M_{1/2}$.

Now it remains to substitute (2.8) into (2.5) in order to obtain the solution of (2.1). To obtain the asymptotic form of the solution as $\rho \to 0$, as in [2,12], we shift the contour of integration in (2.5) in the γ plane to the left up to the line $\operatorname{Re}\gamma=\alpha_*+{}^1/_2$, taking the residues at points of the poles of $\langle Q_{i\nu}\rangle(\gamma)$. The fact that this operation is permissible, as well as the embedding for the remainder term

$$Q_{i\nu}^*(\rho):=\int\limits_{c-i\infty}^{c+i\infty} \langle Q_{i\nu}\rangle'(\gamma)\rho^{-\gamma}\,d\gamma\in L_{2,\alpha_0},\quad {}^1/_2+\alpha_*<c<\min\nolimits_{1/2+\alpha_*<\operatorname{Re}\gamma_i}\operatorname{Re}\gamma_i$$

where γ_i are the zeros of Δ^{I}, Δ^{III}, follows from the form of functions $G_{ij\nu m}(\gamma)$, $\Delta(\gamma)$ and embedding $h_{lm}\in L_{2,\alpha_0}^{\vee}$. Of interest is a particular case in which $h_{lm}(\rho)\in L_{2,-1/2}^{\vee}$ and, in addition, for small ρ:

$$h_{lm}(\rho)=h_{lm}(0)+h_{lm}^*(\rho),\quad h_{lm}^*(\rho)\in M_{-1/2}(0,a_0):=\bigcup_{a<-1/2} L_{2,a}(0,a_0),\quad \forall a_0>0$$

In this case, the preceding reasoning can be continued, by shifting the contour of integration in (2.5) to the left of point $\gamma = 0$ and taking the residue of the integrand in it.

Determining the residues explicitly, and allowing for the fact that elements $G_{ij\nu m}^{\vee}(\gamma)$ at points γ_i of the zeros of $\Delta(\gamma)$ are linearly dependent, we will have the following representations for the solution $Q_i(s)$ in the left and right neighborhoods $Q_i^{\mathrm{I}-}(s)=Q_{i1}(|s-s^*|)$ and $Q_i^{\mathrm{I}+}(s)=Q_{i2}(|s-s^*|)$ respectively of angular point s^*

$$\begin{gathered}
Q_i^{\mathrm{I}\mp}(s)=\sum_{k=1}^{4} A_k^{\mathrm{I}}(\omega,\varkappa)\,d_{ki}^{\mathrm{I}\mp}(\omega,\varkappa)\,|s-s^*|^{-\gamma_k^{\mathrm{I}}}-\\
-\pi(\varkappa+1)[2\sin\omega(\sin\omega-\omega\cos\omega)]^{-1}B_4(\omega)\,d_{4i}^{\mathrm{I}\mp}(\omega,\varkappa)+O_i^{\mathrm{I}*\mp}(s)\quad (2.11)\\
Q_i^{\mathrm{I}*\mp}(s)\in M_{-1/2}(0,a_0),\quad \forall a_0>0\\
d_{0i}^{\mathrm{I}\mp}:=e_{ij}t_{j\nu}^{\mp}(\omega,0,\varkappa)n_\nu^{\mp},\ d_{1i}^{\mathrm{I}\mp}:=t_{i\nu}^{\mp}(\omega,\gamma_1^{\mathrm{I}}(\omega),\varkappa)n_\nu^{\mp}\\
d_{2i}^{\mathrm{I}\mp}:=e_{ij}t_{j\nu}^{\mp}(\omega,\gamma_2^{\mathrm{I}}(\omega),\varkappa)n_\nu^{\mp},\ d_{3i}^{\mathrm{I}\mp}:=q_{i\nu}^{*}(2\pi-\omega,\gamma_3^{\mathrm{I}}(\omega))n_\nu^{\mp}\\
d_{4i}^{\mathrm{I}\mp}:=\pm e_{ij}q_{j\nu}^{*}(2\pi-\omega,\gamma_4^{\mathrm{I}}(\omega))n_\nu^{\mp}\\
t_{i\nu}^{\mp}(\omega,\gamma,\varkappa):=\{\delta_{ij}\sin[(1-\gamma)(\omega-\pi)]\mp e_{ij}(\varkappa-1)(\varkappa+1)^{-1}\times\\
\times\cos[(1-\gamma)(\omega-\pi)]\}q_{j\nu}^{\mp}(\omega,\gamma)\\
q_{i\nu}^{\mp}(\omega,\gamma):=-\delta_{i\nu}\sin(\gamma\omega/2)\pm e_{i\nu}\cos(\gamma\omega/2)
\end{gathered}\tag{2.11}$$

where $\gamma_i^{\mathrm{I}}(\omega)$ $(i=1-4)$ are the maximum roots for $\operatorname{Re}\gamma_i^{\mathrm{I}}<1$ of the following equations respectively:

$$\begin{gathered}
\Delta_*(1,\omega,\gamma_1^{\mathrm{I}})=0,\quad \Delta_*(-1,\omega,\gamma_2^{\mathrm{I}})/\gamma_2^{\mathrm{I}}=0\\
\Delta_*(\varkappa,2\pi-\omega,\gamma_3^{\mathrm{I}})=0,\quad \Delta_*(-\varkappa,2\pi-\omega,\gamma_4^{\mathrm{I}})=0
\end{gathered}\tag{2.12}$$

It is evident from this that $\gamma_1^{\mathrm{I}}(\omega)<0$ for $\omega<\pi$; $\gamma_2^{\mathrm{I}}(\omega)<0$ for $\omega<\omega_0=\operatorname{tg}\omega_0\approx 4{,}493$; $\gamma_3^{\mathrm{I}}(\omega)$, $\gamma_4^{\mathrm{I}}(\omega)<0$ for $\omega > \pi$. On these intervals of ω for the corresponding γ_i^{I} the cofactors $|s-s^*|^{-\gamma_i^{\mathrm{I}}}\in M_{-1/2}(0,a_0)$ and the terms that contain them can be combined with the remainder term $Q_i^{\mathrm{I}*\pm}(s)$, i.e., we can omit them in the explicit rendition of (2.11). In addition, for $0<\omega<2\pi$ the parameters $\gamma_i<{}^1/_2$ $(i=1-4)$.

The density intensity coefficients $A_i^{\mathrm{I}}(\omega,\varkappa)$ are analytic functions of ω, and, in the ranges $0\leqslant\gamma_i(\omega)<1$ of interest to us, they have simple poles for $\gamma_i^{\mathrm{I}}(\omega)=0$ (for finite $h_m(s^*)$); for $A_1^{\mathrm{I}}, A_3^{\mathrm{I}}, A_4^{\mathrm{I}}$ this occurs for $\omega = \pi$, while for A_2^{I} it occurs for $\omega = \omega_0$. Removing these singularities, we can write the coefficients as follows:

$$\begin{gathered}
A_1^{\mathrm{I}}(\omega,\varkappa)=A_1^{\mathrm{I}0}(\omega,\varkappa)+(\varkappa+1)^2[2(3\varkappa-1)\sin\omega]^{-1}B_1(\omega)\\
A_2^{\mathrm{I}}(\omega,\varkappa)=A_2^{\mathrm{I}0}(\omega,\varkappa)-\pi(\varkappa+1)\{2(\sin\omega-\omega\cos\omega)\times\\
\times\sin[(1-\gamma_2(\omega))(\omega-\pi)]\}^{-1}B_4(\omega)\\
A_3^{\mathrm{I}}(\omega,\varkappa)=A_3^{\mathrm{I}0}(\omega,\varkappa)-(\varkappa-1)(2\sin\omega)^{-1}B_4(\omega)
\end{gathered}\tag{2.13}$$

$$A_4^{\mathrm{I}}(\omega,\varkappa)=A_4^{\mathrm{I}0}(\omega,\varkappa)-(\varkappa^2-1)[2(3\varkappa-1)\sin\omega]^{-1}B_4(\omega) \tag{2.13}$$

Functions $A_i^{\mathrm{I}0}(\omega,\varkappa)$ are continuous with respect to ω for $0<\gamma_i<1/2$, if the specified boundary forces $g_i(s)$ on which they depend vary continuously with respect to ω at almost all points $s\in\partial D$:

$$\begin{gathered}B_k(\omega):=[g_i^-(s^*)y_i^{(k)-}(\omega)+g_i^+(s^*)y^{(k)+}(\omega)]/2\\ y_i^{(1)\mp}(\omega):=-(\delta_{ij}\sin\omega\mp e_{ij}\cos\omega)n_j^{\mp}(\omega),\quad y_i^{(2)\mp}(\omega):=\pm y_i^{(1)\mp}(\omega)\\ y_i^{(3)\mp}(\omega):=\mp e_{ij}y_j^{(1)\mp}(\omega),\ y_i^{(4)\mp}(\omega):=\pm y_i^{(3)\mp}(\omega)\end{gathered} \tag{2.14}$$

Note that, since $B_4(\omega;\sigma_{\alpha ij}n_j^{\mp})=0$, $B_1(\pi;\sigma_{\alpha ij}n_j^{\mp})=0$, we have $B_4(\omega;h_i^{\mp})=B_4(\omega;g_i^{\mp})$, $B_1(\pi;h_i^{\mp})=B_1(\pi;g_i^{\mp})$, i.e., the parameters $B_4(\omega)$ and $B_1(\pi)$ are generated only by the parts of the specified boundary conditions that are not self-consistent at the angular point (and cannot be the result of any continuous stress field), and hence $B_4(\omega)$ and $B_1(\pi)$ remain unchanged when self-consistent forces are applied.

Let us also write out the asymptotic form for the critical values of the angles $\omega = \omega_0$, π, when some of the coefficients in (2.11) become unbounded.

For $\omega = \omega_0 \approx 257.5°$ we have $\gamma_2^{\mathrm{I}}(\omega_0)=0$; $\gamma_1^{\mathrm{I}}(\omega_0)>0$; $\gamma_3^{\mathrm{I}}(\omega_0)$, $\gamma_4^{\mathrm{I}}(\omega_0)<0$,

$$\begin{gathered}Q_i^{\mathrm{I}\mp}(s)=A_1^{\mathrm{I}}(\omega_0,\varkappa)d_{1i}^{\mathrm{I}\mp}(\omega_0,\varkappa)|s-s^*|^{-\gamma_1^{\mathrm{I}}}+\pi(\varkappa+1)\times\\ \times(\omega_0\sin\omega_0)^{-1}B_4(\omega_0)\{-d_{2i}^{\mathrm{I}\mp}(\omega_0,\varkappa)\ln|s-s^*|\mp\\ \mp(\omega_0/2)e_{ij}d_{2j}^{\mathrm{I}\mp}(\omega_0,\varkappa)\mp(\omega_0-\pi)(\varkappa-1)[(\varkappa+1)\sin\omega_0]^{-1}q_{ip}^{\mp}(\omega_0,0)n_p^{\mp}\}+\\ +A_2^{\mathrm{I}0}(\omega_0,\varkappa)d_{2j}^{\mathrm{I}\mp}(\omega_0,\varkappa)+Q_i^{\mathrm{I}*\mp}(s)\end{gathered} \tag{2.15}$$

For $\omega = \pi$ we have

$$\begin{gathered}\gamma_1(\pi)=\gamma_3(\pi)=\gamma_4(\pi)=0,\quad \gamma_2(\pi)<0\\ Q_i^{\mathrm{I}\mp}(s)=(\varkappa^2-1)(4\pi\varkappa)^{-1}e_{ij}[g_j^-(s^*)-g_j^+(s^*)]\ln|s-s^*|\pm\\ \pm(\varkappa+1)^2(8\varkappa)^{-1}[g_i^-(s^*)-g_i^+(s^*)]+[(\varkappa-1)(\varkappa+1)^{-1}A_1^{\mathrm{I}0}(\varkappa,\pi)+\\ +A_4^{\mathrm{I}0}(\varkappa,\pi)]n_i-A_3^{\mathrm{I}0}(\pi,\varkappa)e_{ij}n_j+Q_i^{\mathrm{I}**}(s)\end{gathered} \tag{2.16}$$

Like (2.11), expressions (2.15) and (2.16) were obtained from (2.8)-(2.10) by calculating the residues $\langle Q_{ip}\rangle(\gamma)\rho^{-\gamma}$ with respect to γ for the corresponding angles ω; however, they can also be obtained from (2.11)-(2.13) by passages to the limit as $\gamma_1 \to 0$ with allowance for the following relations that derive from (2.12):

$$\begin{gathered}\left.\frac{d\gamma_2^{\mathrm{I}}}{d\omega}\right|_{\omega=\omega_0}=\frac{2}{\omega_0},\quad \left.\frac{d\gamma_1^{\mathrm{I}}}{d\omega}\right|_{\omega=\pi}=\frac{2}{\pi}\\ \left.\frac{d\gamma_3^{\mathrm{I}}}{d\omega}\right|_{\omega=\pi}=-\frac{\varkappa+1}{\pi\varkappa},\quad \left.\frac{d\gamma_4^{\mathrm{I}}}{d\omega}\right|_{\omega=\pi}=-\frac{\varkappa-1}{\pi\varkappa},\quad \left.\frac{d^2\gamma_1}{d\omega^2}\right|_{\omega=\pi}=-\frac{8}{\pi^2}\end{gathered} \tag{2.17}$$

Now, considering the asymptotic behavior of the solution of BIE III, we note that it cannot be obtained from (2.11)-(2.16) by direct substitution of $\kappa = 1$, since the function $\gamma_3^{\mathrm{I}}(\omega,\varkappa)$ is not simultaneously continuous with respect to both its arguments.

Making the substitution $\kappa = 1$ in (2.6), (2.7), and (2.10), and only then calculating the residues $\langle Q_{ip}^{\mathrm{III}}\rangle(\gamma)\rho^{-\gamma}$, we obtain the asymptotic form

$$\begin{gathered}Q_i^{\mathrm{III}\mp}(s)=\sum_{k=1}^{4}A_k^{\mathrm{III}}(\omega)d_{ki}^{\mathrm{III}\mp}(\omega)|s-s^*|^{-\gamma_k^{\mathrm{III}}}\mp\pi(\sin\omega-\omega\cos\omega)^{-1}\times\\ \times B_4(\omega)n_i^{\mp}(\omega)+\pi[\sin\omega+(2\pi-\omega)\cos\omega]^{-1}A_0(\omega)n_i^{\mp}(\omega)+Q_i^{\mathrm{III}*\mp}(s)\ (2.18)\end{gathered} \tag{2.18}$$

$$\begin{gathered}d_{1i}^{\mathrm{III}\mp}:=q_{ip}^{\mp}(\omega,\gamma_1^{\mathrm{III}}(\omega))n_p^{\mp},\quad d_{2i}^{\mathrm{III}\mp}:=e_{ij}q_{jp}^{\mp}(\omega,\gamma_2^{\mathrm{III}}(\omega))n_p^{\mp}\\ d_{3i}^{\mathrm{III}\mp}:=\pm q_{ip}^{\pm}(2\pi-\omega,\gamma_3^{\mathrm{III}}(\omega))n_p^{\mp},\quad d_{4i}^{\mathrm{III}\mp}:=\pm e_{ij}q_{jp}^{\pm}(2\pi-\omega,\gamma_4^{\mathrm{III}}(\omega))n_p^{\mp}\end{gathered}$$

Here $Q_i^{\mathrm{III}*\mp}(s)\in M_{-1/2}(0,a_0)$, $\forall a_0>0$; $\gamma_i^{\mathrm{III}}(\omega)$ $(i=1\div4)$ are the maximum roots for $\mathrm{Re}\,\gamma_i^{\mathrm{III}}<1$ respectively of the equations

$$\begin{gathered}\Delta_*(1,\omega,\gamma_1^{\mathrm{III}})=0,\quad \Delta_*(-1,\omega,\gamma_2^{\mathrm{III}})/\gamma_2^{\mathrm{III}}=0\\ \Delta_*(1,2\pi-\omega,\gamma_3^{\mathrm{III}})=0,\quad \Delta_*(-1,2\pi-\omega,\gamma_4^{\mathrm{III}})/\gamma_4^{\mathrm{III}}=0\end{gathered} \tag{2.19}$$

From this we have $\gamma_1^{III}(\omega)<0$ for $\omega<\pi$; $\gamma_2^{III}(\omega)<0$ for $\omega<\omega_0 = \mathrm{tg}\,\omega\approx 4{,}493$; $\gamma_3^{III}(\omega)<0$ for $\omega>\pi$; $\gamma_4^{III}(\omega)<0$ for $\omega>\omega_{00}=2\pi+\mathrm{tg}\,\omega_{00}\approx 1{,}790$. On these intervals of the angle ω, for the corresponding γ_i^{III} the cofactors $|s-s^*|^{-\gamma_i^{III}}\in M_{-\frac{1}{2}}(0, a_0)$ and the terms that contain them can be combined with the remainder term $Q_i^{III**}(s)$, i.e., we can omit them in the explicit rendition of (2.18). In addition, for $0<\omega<2\pi$ the parameters $\gamma_i^{III}<1/2$ $(i=1\div 4)$.

As for $Q_1{}^{I}$, the density intensity coefficients $A_i^{III}(\omega)$ are analytic functions of ω. In the range $0<\gamma_i(\omega)<1$ the corresponding coefficients $A_1^{III}(\omega)$ and $A_3^{III}(\omega)$ are continuous, while the coefficients $A_2^{III}(\omega)$ and $A_4^{III}(\omega)$ have simple poles for $\omega=\omega_0(\gamma_2^{III}(\omega)=0)$ and $\omega=\omega_{00}(\gamma_4^{III}(\omega)=0)$ respectively. Removing these singularities, we can write the coefficients A_2^{III}, A_4^{III} as follows:

$$A_2^{III}(\omega)=A_2^{III*}(\omega)-\pi(\sin\omega-\omega\cos\omega)^{-1}B_4(\omega)$$
$$A_4^{III}(\omega)=A_4^{III*}(\omega)-\pi[\sin\omega+(2\pi-\omega)\cos\omega]^{-1}A_0(\omega) \qquad (2.20)$$

Functions $A_2^{III*}(\omega)$, $A_4^{III*}(\omega)$ are already continuous for $0<\gamma_i^{III}<1/2$. Note also that $\gamma_1^{III}(\pi)=0$ and $A_1^{III}(\pi)=-B_1(\pi)=1/2(g_i^{-}-g_i^{+})e_{i\gamma}n_{\gamma}$. In addition, despite the above stipulations, an analogy can be pursued between the terms of the asymptotic form Q^{I} and Q^{III} if we set $A_i^{III}(\omega)=A_i^{I}(1,\omega)\sin[(1-\gamma_i^{III})(\omega-\pi)]$ $(i=1,2)$; $A_i^{III}(\omega)=A_i^{I}(1,\omega)$ $(i=3,4)$. In this case, however, the parameter $A_3^{I}(1,\omega)=A_3^{1*}(1,\omega)$ will no longer be continuous at point $\omega = \omega_{00}$.

As above for $Q_1{}^{I}$, we also write out the asymptotic form for $Q_1{}^{III}$ for the case of critical angle values. For $\omega = \omega_0 \approx 257.5°$ we will have

$$\gamma_2^{III}(\omega_0)=0;\quad \gamma_1^{III}(\omega_0)>0;\quad \gamma_3^{III}(\omega_0)\quad \gamma_4^{III}(\omega_0)<0$$
$$Q_i^{III\mp}(s)=A_1^{III}(\omega_0)d_{1i}^{III\mp}(\omega_0)|s-s^*|^{-\gamma_1^{III}}\mp$$
$$\mp[2\pi(\omega_0^2\sin\omega_0)^{-1}B_4(\omega_0)\ln|s-s^*|+A_2^{III*}(\omega_0)]n_i^{\mp}+$$
$$+\pi(\omega_0\sin\omega_0)^{-1}B_4(\omega_0)e_{ij}n_j^{\mp}+(2\cos\omega_0)^{-1}A_0(\omega_0)n_i^{\mp}+Q_i^{III*\mp}(s) \qquad (2.21)$$

For $\omega = \pi$ we will have

$$\gamma_1^{III}(\pi)=\gamma_2^{III}(\pi)=0,\quad \gamma_3^{III}(\pi),\quad \gamma_4^{III}(\pi)<0$$

$$Q_i^{III\mp}(s)=\pm(g_i^{-}-g_i^{+})/2-A_0(\pi)n_i-A_3(\pi)e_{i\gamma}n_{\gamma}+Q_i^{III**\mp}(s)$$

For $\omega = \omega_{00} \approx 102.5°$ we will have $\gamma_4^{III}(\omega_{00})=0$;

$$\gamma_3^{III}(\omega_{00})>0;\quad \gamma_1^{III}(\omega_{00})\quad \gamma_2^{III}(\omega_{00})<0$$

$$Q_i^{III}(s)=A_3^{III}(\omega_{00})d_{3i}^{III\mp}(\omega_{00})|s-s^*|^{-\gamma_3^{III}}+$$
$$+[2\pi(2\pi-\omega_{00})^{-2}(\sin\omega_{00})^{-1}A_0(\omega_{00})\ln|s-s^*|+$$
$$+A_4^{III*}(\omega_{00})]n_i^{\mp}\pm(2\cos\omega_{00})^{-1}B_4(\omega_{00})n_i^{\mp}\pm\pi[(2\pi-$$
$$-\omega_{00})\sin\omega_{00}]^{-1}A_0(\omega_{00})e_{ij}n_j^{\mp}+Q_i^{III*\mp}(s) \qquad (2.22)$$

Expressions (2.21)-(2.22), like the corresponding expressions for Q^{I}, can be obtained both by direct calculation of the residues $\langle Q_{ip}^{III}\rangle(\gamma)\rho^{-\gamma}$ for critical angles ω, and also by passage to the limit in (2.18) with allowance for the following relations that derive from (2.19):

$$\left.\frac{d\gamma_2^{III}}{d\omega}\right|_{\omega=\omega_0}=\frac{2}{\omega_0},\quad \left.\frac{d\gamma_1^{III}}{d\omega}\right|_{\omega=\pi}=-\left.\frac{d\gamma_2^{III}}{d\omega}\right|_{\omega=\pi}=\frac{2}{\pi}$$
$$\left.\frac{d\gamma_4^{III}}{d\omega}\right|_{\omega=\omega_{00}}=-\frac{2}{2\pi-\omega_{00}} \qquad (2.23)$$

In representations (2.11)-(2.22), the coefficients A_1 and B_k are functionals of the specified right sides h_{ip} on the sides of the wedge.

In analyzing the asymptotic behavior of the solution of a BIE on an arbitrary contour ∂D, made up of segments of Lyapunov curves, we can write the equation in the neighborhood of the angular point s* under consideration in the form (2.1) with some finite upper limit a_0, where the kernel K_{ijpm} is generated on the basis of the tangent to ∂D at the angular point. Then, the resultant BIE on the segment $[0, a_0]$ must be continued, using the same formulas for the kernel, onto the semiaxis $[0, \infty]$, setting $Q_{jm}(\rho)=0,\ \rho>a_0$. The additional integral terms that are generated in the process must be transferred to the right side. Then this right side $h_{im}^0(\rho)$ becomes conditionally specified, but, as before, it will belong to $L_2V_{-1/2}(0, \infty)$, while for ρ we have $h_{im}^0(\rho)=h_{im}^0(0)+h_{im}^{0*}(\rho),\ h_{im}^{0*}\in M_{-1/2}(0, a_0)$, if the original right side h_{im} belonged to these classes on $[0, a_0]$.

Thus, it is possible to convert to the already-investigated system (2.1) on a semiaxis with a conditionally specified right side $h_{im}{}^0$. In addition, since in the vicinity of the angular point s* the original right side g_i and the conditionally specified ones differ only in terms of the self-consistent forces, the parameters $B_4(\omega)$ and $B_1(\omega)$ will be defined on the basis of the same fommulas (2.14) in terms of the initial forces g_i. This cannot be said regarding the coefficients A_i, which are not expressed a priori in terms of the right sides for a body with an arbitrary boundary (without solving the equation completely on this boundary).

Furthermore, if in the neighborhood of angular point s* the right side $g_i^*(s)$ is a Holder function, then it can be shown that the remainder term $Q_i^{**}(s)$ in asymptotic forms (2.11)-(2.22) also belongs to a Holder space and $Q_i^{**}(s^*)=0$.

We should note that the representations of asymptotic forms (2.11)-(2.22) are invariant with respect to the position of a Cartesian coordinate system.

Paper [13] offered a representation of the asymptotic form of the density $Q_i{}^I(s)$ of BIE I, containing only high-order terms, corresponding to terms with coefficients $A_1{}^I$ and $A_3{}^I$ in (2.11). This asymptotic form was obtained by reducing the integral equations to ancillary boundary value problems, and by using the asymptotic forms for the solutions of the latter, which were known in advance. The degrees of singularity given in [13] coincide with $\gamma_1{}^I$ and $\gamma_3{}^I$, whereas the form of the eigenvectors given there is different.

In [14], a technique similar to that employed here was used to analyze the asymptotic form of the density of the direct boundary integral equations of plane problems for the Laplace equation.

3. ASYMPTOTIC FORM OF STRESSES

Representations (1.2) and (1.3) yield representations for the transforms of the stresses $\langle\sigma_{ij}\rangle(\gamma,\theta)$ in a wedge-shaped domain, in terms of the transforms of the densities $\langle Q_{ip}\rangle(\gamma)$. From this, using (2.8) and direct calculation of the residues upon displacement, as above, of the contour of integration in the γ plane in the inverse Mellin transform, we obtain the asymptotic form of the stresses near the angular point s*. In the local polar (ρ, θ) coordinate system with origin at angular point s* and with angle θ that is reckoned counterclockwise from the bisectrix of the angle ω, for $\pi<\omega<2\pi,\ \omega\neq\omega_*$, the stresses have the form

$$\sigma_{ij}(\rho,\theta)=\sum_{m=1}^{2}K_m\sigma_{ij}^{(m)}(\theta)\rho^{-\gamma_m}+\sigma_{ij}{}^0(\theta)+\sigma_{ij}{}^*(\rho,\theta) \qquad (3.1)$$

$$\begin{aligned}
\sigma_{\rho\rho}^{(1)}&:=(2-\gamma_1)\cos(\gamma_1\omega/2)\{\cos[(2-\gamma_1)\omega/2]\}^{-1}\cos[(2-\gamma_1)\theta]+\\
&\quad+(\gamma_1+2)\cos(\gamma_1\theta)\\
\sigma_{\theta\theta}^{(1)}&:=-(2-\gamma_1)\cos(\gamma_1\omega/2)\{\cos[(2-\gamma_1)\omega/2]\}^{-1}\cos[(2-\gamma_1)\theta]+\\
&\quad+(2-\gamma_1)\cos(\gamma_1\theta) \qquad (3.2)\\
\sigma_{\rho\theta}^{(1)}&:=-(2-\gamma_1)\cos(\gamma_1\omega/2)\{\cos[(2-\gamma_1)\omega/2]\}^{-1}\sin[(2-\gamma_1)\theta]+\gamma_1\sin(\gamma_1\theta)\\
\sigma_{\rho\rho}^{(2)}&:=-\gamma_2\cos(\gamma_2\omega/2)\{\cos[(2-\gamma_2)\omega/2]\}^{-1}\sin[(2-\gamma_2)\theta]-(\gamma_2+2)\sin(\gamma_2\theta)\\
\sigma_{\theta\theta}^{(2)}&:=-\gamma_2\cos(\gamma_2\omega/2)\{\cos[(2-\gamma_2)\omega/2]\}^{-1}\sin[(2-\gamma_2)\theta]-(2-\gamma_2)\sin(\gamma_2\theta)
\end{aligned}$$

$$\sigma_{\rho\theta}^{(2)} := -\gamma_2 \cos(\gamma_2\omega/2)\{\cos[(2-\gamma_2)\omega/2]\}^{-1}\cos[(2-\gamma_2)\theta]+\gamma_2\cos(\gamma_2\theta)$$

$$\begin{aligned}
&\sigma_{\rho\rho}^{0}(\theta) = -B_1(\cos\omega\cos 2\theta+1)/\sin\omega+B_2\sin 2\theta-B_3\cos 2\theta- \\
&\quad -B_4[2\theta+(\cos\omega+\omega\sin\omega)\sin 2\theta]/(\sin\omega-\omega\cos\omega) \\
&\sigma_{\theta\theta}^{0}(\theta) := -B_1(\cos\omega\cos 2\theta-1)/\sin\omega-B_2\sin 2\theta+B_3\cos 2\theta- \\
&\quad -B_4[2\theta-(\cos\omega+\omega\sin\omega)\sin 2\theta]/(\sin\omega-\omega\cos\omega) \\
&\sigma_{\rho\theta}^{0}(\theta) := -[B_1\,\mathrm{ctg}\,\omega+B_3]\sin(2\theta)+B_2\cos 2\theta+B_4[1-(\cos\omega+\omega\sin\omega)\cos 2\theta]/ \\
&\quad /(\sin\omega-\omega\cos\omega) \qquad (\gamma_k=\gamma_k^{I}(\omega)=\gamma_k^{III}(\omega),\ k=1,2)
\end{aligned} \tag{3.2}$$

The stress intensity coefficients K_i can be expressed in terms of the density intensity coefficients A_i^{I} or A_i^{III} of BIE I or III respectively:

$$K_i = A_i^{I} 2(\varkappa+1)^{-1}\{\mathrm{ctg}(\gamma_i\pi)\sin[(1-\gamma_i)(\omega-\pi)]+ +(\varkappa-1)(\varkappa+1)^{-1}\cos[(1-\gamma_i)(\omega-\pi)]\} \tag{3.3}$$

$$K_i = A_i^{III}\,\mathrm{ctg}(\gamma_i\pi) \tag{3.4}$$

The coefficients $B_k(\omega)$ in the terms $\sigma_{ij}^{0}(\theta)$ are independent of the radius are given by (2.14), i.e., they are expressed explicity in terms of the limiting values of the applied forces $g_i^{\mp}(s^*)$. In obtaining these expressions for σ_{ij}^{0}, in addition to the direct and inverse Mellin transformation, substitution of (3.1) into the boundary conditions $\sigma_{ij}n_j|_{\partial D}=g_i$ was also employed.

Returning to boundary value problems in domains with an arbitrary boundary made up of segments of Lyapunov curves, we can readily observe that reasoning similar to that given at the end of §2 leads to representation of the stresses in the form $\sigma_{il}=\sigma_{il}^{(a)}+\sigma_{il}^{(b)}$. Here the term $\sigma_{il}^{(a)}$ is given by formulas (1.2)-(1.3), in which ∂D is replaced by the boundary of the wedge formed by the tangents to the initial boundary at angular point s*, while the density Q_i coincides with the initial density in some zone near s* and is equal to zero away from this zone. The asymptotic form of this term is given by formulas (3.1), (3.2), in which the constants B_i (i = 1, 2, 3) in σ_{ij}^{0} are replaced by indeterminate constants. The remainder term $\sigma_{ij}^{(b)}$ generated by Q_i values on the part of the original boundary ∂D remote from s*, as well as by the curvature of ∂D near s*, is a continuous function all the way to s*, and thus affects only the indeterminacies of the constants B_i (i = 1, 2, 3). Substituting σ_{il} into the original boundary conditions $\sigma_{ij}n_j|_{\partial D}=g_i$, we obtain for B_i their former values yielded by (2.14).

The remainder term $\sigma_{il}^{*}(\rho,\theta)\to 0$ as $\rho\to 0$, if $g_i^{\mp}(s)$ is a Holder function in the neighborhood of point s*. For $\pi<\omega<\omega_0$ we have $\rho^{-\gamma_2}\to 0$ as $\rho\to 0$, and the terms for m = 2 can be attached to σ_{ij}^{*}, i.e., they can be omitted in the explicit rendition of (3.1).

Taking account of (2.13) and (2.20), we obtain from (3.3) and (3.4) that for $\omega=\pi$, ω_0 the coefficients $K_1(\omega)$, $K_2(\omega)$ become unbounded, and can be represented as follows:

$$K_1(\omega)=K_1^{0}(\omega)+B_1(\omega)/(2\sin\omega)$$

$$K_2(\omega)=[K_2^{0}(\omega)-\pi B_4(\omega)/(\sin\omega-\omega\cos\omega)]\,\mathrm{ctg}(\gamma_2\pi)$$

Now the functions $K_1^{0}(\omega)$, $K_2^{0}(\omega)$ are continuous for critical angles $\omega=\pi$, $\omega=\omega_0$ respectively, if, as ω varies, the boundary ∂D (and the functions g_i specified on it) varies continuously. Their values for these angles can be expressed via the terms of the asymptotic forms of the densities of BIE I or BIE III:

$$K_1^{0}(\pi)=(3\varkappa-1)(\varkappa+1)^{-2}A_1^{I0}(\pi,\varkappa)$$

$$K_2^{0}(\omega_0)=-2(\varkappa+1)^{-1}\sin\omega_0 A_2^{0}(\omega_0,\varkappa)-2\pi^2(\varkappa-1)[(\varkappa+ +1)\omega_0^{3}\sin\omega_0]^{-1}B_4(\omega_0)$$

$$K_1^{0}(\pi)=-\lim_{\omega\to\pi}\frac{A_1^{III}(\omega)+B_1(\omega)}{2\sin\omega}-B_1(\pi), \qquad K_2^{0}(\omega_0)=A_2^{III\,0}(\omega_0)$$

We will now write out the asymptotic form of the stress for the critical angles. For $\omega=\omega_0\approx 257.5°$ we have

$$\sigma_{\rho\rho}=K_1\sigma_{\rho\rho}^{(1)}(\theta)\rho^{-\gamma_1}+[-2(\omega^2\sin\omega)^{-1}B_4(\omega)\ln\rho-$$
$$-K_2^0(\omega)/\pi][2\theta+\sin(2\theta)/\cos\omega]-B_1(\omega)[\cos\omega\cos(2\theta)+$$
$$+1]/\sin\omega+B_2(\omega)\sin(2\theta)-B_3(\omega)\cos(2\theta)+2(\omega^2\sin\omega)^{-1}\times$$
$$\times B_4(\omega)[\theta(1-\cos(2\theta)/\cos\omega)-(\omega/2)\sin\omega\sin(2\theta)]+\sigma_{\rho\rho}^*(\rho,\theta)$$

$$\sigma_{\theta\theta}=K_1\sigma_{\theta\theta}^{(1)}(\theta)\rho^{-\gamma_1}+[-2(\omega^2\sin\omega)^{-1}B_4(\omega)\ln\rho-$$
$$-K_2^0(\omega)/\pi][2\theta-\sin(2\theta)/\cos\omega]+B_1(\omega)(\cos\omega\cos\theta-1)/\sin\omega-$$
$$-B_2(\omega)\sin(2\theta)+B_3(\omega)\cos(2\theta)-2(\omega^2\sin\omega)^{-1}B_4(\omega)[\theta(1-$$
$$-\cos(2\theta)/\cos\omega)-(\omega/2)\sin\omega\sin 2\theta]+\sigma_{\theta\theta}^*(\rho,\theta) \qquad (3.5)$$

$$\sigma_{\rho\theta}=K_1\sigma_{\rho\theta}^{(1)}(\theta)\rho^{-\gamma_1}+[-2(\omega^2\sin\omega)^{-1}B_4(\omega)\ln\rho-$$
$$-K_2^0(\omega)/\pi][\cos(2\theta)/\cos\omega-1]+[B_1(\omega)\operatorname{ctg}\omega+$$
$$+B_3(\omega)]\sin 2\theta+B_2(\omega)\cos 2\theta+2(\omega^2\sin\omega)^{-1}B_4(\omega)[\theta\sin(2\theta)/\cos\omega-$$
$$-(\omega/2)\sin\omega\cos 2\theta]+\sigma_{\rho\theta}^*(\rho,\theta)$$

For $\omega = \pi$:

$$\sigma_{\rho\rho}=[(2/\pi)B_1(\pi)\ln\rho+2K_1^0(\pi)][1-\cos(2\theta)]+$$
$$+\pi^{-1}B_1(\pi)[2\theta\sin(2\theta)-1-\cos 2\theta)+B_2(\pi)\sin 2\theta-$$
$$-B_3(\pi)\cos 2\theta-\pi^{-1}B_4(\pi)[2\theta-\sin(2\theta)]+\sigma_{\rho\rho}^*(\rho,\theta)$$
$$\sigma_{\theta\theta}=[(2/\pi)B_1(\pi)\ln\rho+2K_1^0(\pi)][1+\cos(2\theta)]-$$
$$-\pi^{-1}B_1(\pi)[2\theta\sin(2\theta)-1-\cos(2\theta)]-B_2(\pi)\sin 2\theta+$$
$$+B_3(\pi)\cos 2\theta-\pi^{-1}B_4(\pi)[2\theta+\sin(2\theta)]+\sigma_{\theta\theta}^*(\rho,\theta) \qquad (3.6)$$

$$\sigma_{\rho\theta}=[(2/\pi)B_1(\pi)\ln\rho+2K_1^0(\pi)]\sin(2\theta)+$$
$$+\pi^{-1}B_1(\pi)[2\theta\cos(2\theta)+\sin(2\theta)]+B_2(\pi)\cos(2\theta)+$$
$$+B_3(\pi)\sin(2\theta)+\pi^{-1}B_4(\pi)[1+\cos(2\theta)]+\sigma_{\rho\theta}^*(\rho,\theta)$$

The asymptotic form of the stresses (3.5), (3.6) for singular angles $\omega = \omega_0$, π can be obtained both directly using the technique described above for the remaining angles, or by passage to the limit from (3.1) and (3.2) as $\omega \to \omega_0$, π with allowance for (2.17) and (2.22).

It can be seen from (3.5) and (3.6) that for $\omega = \omega_0$, π a logarithmic singularity appears in the representation for the stresses for $\omega = \pi$ (i.e., at a point of smoothness of contour ∂D) it is the principal one. The coefficient for the logarithm can be expressed explicitly in terms of the right sides of the boundary conditions: for $\omega = \omega_0$ in terms of the non-self-consistent part of the specified forces (i.e., the part that cannot be represented as a result of the pairing conditions for the stresses in terms of the stress field which is continuous all the way to s*), and for $\omega = \pi$ (point of smoothness of the boundary) in terms of the discontinuity of the tangential forces. We should note that all the coefficients in the bounded stress terms can also be expressed explicitly in terms of the limiting values of the specified forces $g_i^\pm(s^*)$, except for $K_1^0(\pi)$, $K_2^0(\omega_0)$, which can be expressed in terms of the coefficients of the asymptotic form of the density of the BIE. The representations for the stresses near re-entrant angles that were obtained in [9], using a modification of the technique of [7], agree with the corresponding terms in (3.2), (3.5), and (3.6) after some manipulations.

For salient angles $(0<\omega<\pi)$ the stresses are finite and have the form $\sigma_{ij}(\rho,\theta)=\sigma_{ij}^0(\theta)+\sigma_{ij}^*(\rho,\theta)$, where σ_{ij}^0 are given by expressions (3.2), while $\sigma_{ij}^*(\rho,\theta)\to 0$, $\rho\to 0$ if $g_i^\pm(s)$ are Holder functions in the left and right neighborhoods of point s*.

We should also note that, in the case of numerical solution, the coefficients of asymptotic forms (2.15), (2.16), (2.21), (2.22), (3.5), and (3.6) for the critical angles ω_0, π, ω_{00} can be obtained not only by direct approximation of the densities or stresses near angular corners, containing logarithmic terms, but also (with allowance for the relationship between the coefficients of the critical and subcritical asymptotic forms) on the basis of the values of the coefficients for sufficiently close subcritical angles ω.

Thus, we have obtained the asymptotic forms of the densities of BIE and stress fields in the neighborhood of angular corners in compressible and incompressible elastic media, and also in the Stokes problem for an incompressible viscous fluid. In the first place, this information can be further used to clarify the behavior of the solution near angular corners. In particular, it is evident from the results that for angles $\omega < \pi$ the principal terms of the asymptotic form of the stresses are completely determined by the behavior of the specified forces near the angular corners, and can be calculated without solving the boundary value problem completely. Second, this information is required to refine numerical solution methods for problems with angular points, through more precise approximation of the solution near angular corners, based on the asymptotic forms obtained in this paper, and also for direct calculation of the stress intensity coefficients on the basis of the density intensity coefficients of the integral equation.

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5 January 1988 Moscow