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LOCAL AND NON-LOCAL APPROACHES TO CREEP CRACK INITIATION AND PROPAGATION

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ABSTRACT

A functional form of local brittle strength conditions for a time- or history-dependent materials is presented. The particular strength condition associated with the Robinson linear damage accumulation rule and the power-type durability diagram is employed to formulation and analysis of creep crack initiation and propagation problem. The problem is reduced to a non-linear integral Volterra equation, which can be transformed to a linear one for the case of a single crack. Analytical solutions of some simple problems for linear viscoelastic materials are presented for that case and shortcomings of the local approach are pointed out. A non-local approach free from the shortcomings is presented along with an example of its implementation.

INTRODUCTION

A common practice of a body durability local analysis includes usually two steps. First, a crack initiation time $t^*(\sigma)$ for a time-dependent process $\sigma_{ij}(\tau, y)$ in an originally non-cracked body Ω is determined from a strength condition expressed in terms of a damage measure, where τ is time, $y \in \Omega$. After the strength condition is violated, on the second step, an equation for the crack propagation rate (e.g. see [1], [8]) is used for evaluation of the time $t^*(\{\sigma\};\Omega)$ to separation of the body into pieces or to unstable crack growth. The trouble is that the initial condition (initial crack length) for the crack propagation rate equation is often not clearly fixed. The values used in the equation are usually characteristics of the stress field only at the crack tip *y* and can describe neither the scale effect for short cracks nor the influence of the creep damage during the previous period of time on the crack propagation rate. Moreover, the material parameters of the strength condition of the first step seem to be completely unrelated to the parameters the equation for the crack propagation rate.

Trying to avoid the shortcomings, we first describe in this paper a local united approach based on an extension of the classical creep strength conditions to the crack propagation stage, and show its limitations, particularly, inability to predict the experimentally observed crack propagation delay for an already existing crack. To avoid those drawbacks, we then give a non-local modification of that approach merging a special form of the general static non-local approach [2] with the functional description of durability and strength [3]. This allows to analyse strength and durability under long-term loading by both homogeneous and highly inhomogeneous stress fields, and predict the crack initiation in a virgin material without cracks and its propagation through the damaged material as a united process. Note that some other particular non-local approaches were used for predicting long-term strength in [1], [7]. Considered examples of the local and non-local approaches applications lead to linear or non-linear Volterra equations of the first or the second kind and some results of their solutions are also presented.

1. LOCAL BRITTLE STRENGTH AND DURABILITY CONDITIONS

To describe fracture, i.e. crack initiation and propagation under loading, we will analyse the brittle strength, that is strength at a particular point *y* along a particular infinitesimal plane with a normal vector $\vec{\zeta}$ at that point. The local brittle strength condition *for a plane* $\vec{\zeta}$ *at a point* $y\in\Omega$ can be taken in the form

$$\underline{\Lambda}(\sigma(\cdot,y);t,y,\vec{\zeta})<1, \quad (1)$$

where $\underline{\Lambda}(\sigma(\cdot,y);t,y,\vec{\zeta})$ is a local brittle *Temporal Normalised Equivalent Stress Functional* (TNESF) defined similar to [3] on the stresses $\sigma_{ij}(\tau,y)$. It is a positively homogeneous in σ and non-decreasing in τ material characteristics. An example of the TNESF associated with the power durability diagram for long-term loading and the Robinson linear accumulation rule can be taken in the following form similar to [3],

$$\underline{\Lambda}(\sigma(\cdot,y);t,y,\vec{\zeta})=\left\{\int_0^t \frac{\left|\vec{\sigma}(\tau,y,\vec{\zeta})\right|^b}{\left[\sigma_1^*(\tilde{\vec{\sigma}}(\tau,y,\vec{\zeta});y,\vec{\zeta})\right]^b}\,d\tau\right\}^{1/b} \quad (2)$$

where $\vec{\sigma}(\tau,y,\vec{\zeta})$ is a traction vector $\sigma_{ij}\zeta_j$ on the plane $\vec{\zeta}$ at the point *y* at the moment τ; $\left|\vec{\sigma}(\tau,y,\vec{\zeta})\right|$ is a length of the vector $\vec{\sigma}(\tau,y,\vec{\zeta})$; *b* is a non-negative material constant, and $\sigma_1^*(\tilde{\vec{\sigma}}(\tau,y,\vec{\zeta});y,\vec{\zeta})$ is a non-negative material function of the normalized stress $\tilde{\vec{\sigma}}(\tau,y,\vec{\zeta})=\vec{\sigma}(\tau,y,\vec{\zeta})/\left|\vec{\sigma}(\tau,y,\vec{\zeta})\right|$, depending also on *y* and $\vec{\zeta}$ for inhomogeneous and anisotropic materials.

Let us return to the general case. Let a body occupy at an instant *t* an open domain $\Omega(t)$ with the boundary $\Gamma(t)=\Gamma(0)\bigcup Y^*(t)$ consisting of an initial body boundary $\Gamma(0)$ and a new crack surface $Y^*(t)$ occurring and growing during the loading process. Generally, the long-term fracture process (the creep crack initiation and its propagation through the damaged material) can be described by using a brittle non-local TNESF $\underline{\Lambda}(\sigma(\cdot,y);t,y,\vec{\zeta})$ as follows. First, there is no fracture in a body $\Omega(0)$ if inequality (1) is satisfied on all infinitesimal planes $\vec{\zeta}$ at all points $y\in\Omega$. Then a crack or cracks appear at a moment t_0^* at points y^* on planes $\vec{\zeta}^*(y^*)$ where inequality (1) is violated and becomes equality, that is, the points y^* constitute a crack set $Y^*(t_0^*)$, which becomes a part of the body boundary $\Gamma(t_0^*)=\Gamma(0)\bigcup Y^*(t_0^*)$, with the normal vector $\vec{\zeta}^*(y^*)$ and with zero boundary tractions. Taking into account that $\underline{\Lambda}$ is non-decreasing in *t*, we have that the crack initiation instant t_0^*, the crack initiation set $Y^*(t_0^*)$ ($y^*(t_0^*)\in Y^*(t_0^*)$) and the crack initiation planes $Z^*(t_0^*)$ ($\vec{\zeta}^*(y^*)\in Z^*(t_0^*)$) are determined from the following equation and inequality,

$$t_0^*=\sup\{t:\sup_y\sup_{\vec{\zeta}}\underline{\Lambda}(\{\sigma(\cdot;\Gamma(0),y)\};t,y,\vec{\zeta})<1\},\qquad \underline{\Lambda}(\{\sigma(\cdot;\Gamma(0),y)\};t_0^*,y_0^*,\vec{\zeta}(y_0^*))\geq 1 \quad (3)$$

If the sets $Y^*(t_0^*)$ and $Z^*(t_0^*)$ are empty, then t_0^* is an instability instant and $Y^*(t)$ and $Z^*(t)$ will be not empty for any $t>t_0^*$.

The crack set $Y^*(t)$ grows in time. Remembering that $\Omega(t)=\Omega(0)\setminus Y^*(t)$, we thus have the following conditions for the creep crack propagation,

$$\underline{\Delta}(\{\sigma(\cdot;\Gamma(\cdot),y)\};t,y,\vec{\zeta})<1, \qquad y\in\Omega(t) \quad \forall\vec{\zeta}, \tag{4}$$

$$\underline{\Delta}(\{\sigma(\cdot;\Gamma(\cdot),y)\};t,y,\vec{\zeta}^*(y))=1, \qquad y\in Y^*(t) \tag{5}$$

$$\sigma_{ij}(t;\Gamma(t),y)\zeta^*_j(y)=0, \qquad y\in Y^*(t) \tag{6}$$

where $\vec{\zeta}^*(y)$ is the normal to $Y^*(t)$ at $y\in Y^*(t)$ in (5)-(6) if the normal does exist.

Assuming a smooth dependence of $\underline{\Delta}(\{\sigma(\cdot;\Gamma(\cdot),y)\};t,y,\vec{\zeta})$ on $\vec{\zeta}$ and using (4) and (5), the fracture plane with a unit normal $\vec{\zeta}^*(y)$ can be determined from the equations

$$\left.\frac{\partial\underline{\Delta}(\{\sigma(\cdot;\Gamma(\cdot),y)\};t,y,\vec{\zeta})}{\partial\zeta_j}\right|_{\vec{\zeta}=\vec{\zeta}^*(y)}=0, \qquad \left|\vec{\zeta}^*(y)\right|=1, \qquad \forall t,\forall y\in Y^*(t) \tag{7}$$

If there is an analytical or numerical method of the stress field calculation for any crack set Y^*, relations (4)-(7) allow to describe the crack propagation for any instant t. If the direction of crack growth is a priori known then there is no need to determine $\vec{\zeta}^*$

2. EXAMPLE OF LOCAL DURABILITY ANALYSIS

Symmetric plane problem for creep crack initiation and propagation in a linear viscoelastic material. Let us analyse the case when a fracture plane $\vec{\zeta}^*$ is known or can be easily predicted. Let us consider a 2D-problem for a linear viscoelastic homogeneous body that is symmetric with respect to axis x_1 and symmetrically loaded. Let the body have one edge crack of a length $a(\tau)$ or one central crack of a length $2a(\tau)$ or two symmetric edge cracks of a length $a(\tau)$ each along the x_1 axis (in the last two cases the symmetry with respect the axis orthogonal to x_1 is also supposed), already existing or appearing during the process. Thus the geometry change is described by only one parameter $a(\tau)$, i.e. $\Gamma(\tau)=\Gamma(a(\tau))$, and the creep crack propagation path is straight with a normal vector $\vec{\zeta}^*=\{0,1\}$.

Let the body be loaded by boundary traction $q(\tau,x)=q_0(\tau)\hat{q}(x)$ symmetric w.r.t. x_1-axis, where $q_0(\tau)$ is a scalar function. Then according to [6, Chapter 5] the stress field for the viscoelastic problem with a crack propagating along x_1-axis is the same as in the corresponding elastic problem, $\sigma_{ij}(\tau,y)=q_0(\tau)\hat{\sigma}_{ij}(a(\tau),y)$, where $\hat{\sigma}_{ij}(a(\tau),y)=\hat{\sigma}_{ij}(\hat{q}(\tau);a(\tau),y)$. Evidently, $\sigma_{ij}(\tau,y)=q_0(\tau)\hat{\sigma}_{ij}(a_0,y)$, when the geometry does not change.

Let us take TNESF in the form (2), then the equation for the crack initiation moment t_0^* according to (3) is

$$\left|\hat{\sigma}_{22}(a_0,y^*)\right|^b\int_0^{t_0^*}\left|q_0(\tau)\right|^b d\tau=\left(\sigma_1^*\right)^b, \tag{8}$$

where $a_0 = 0$ if there is no crack initially in the body, y^* is the tip of an already existing crack or the stress concentration point where the crack will initiate, σ_1^* and *b* are constants of the power durability diagram. If there exists an initial crack, $a_0 \neq 0$, then (8) implies $t_0^* = 0$ due to the stress singularity at the crack tip, $|\hat{\sigma}_{22}(a_0, a_0)| = \infty$, i.e. the crack starts to propagate without any delay after the load application.

Let the origin of the coordinate system be in the middle of the central crack or at the open end of the edge crack or at the point where the crack will appear. Then the coordinate of the crack tip is $y_1^* = a(t)$ and the dependence $a(\tau)$ for the developing crack length is to be obtained from (5), that is reduced to the following Volterra non-linear integral equation of the first kind,

$$\int_{t_0^*}^{t} |\hat{\sigma}_{22}(a(\tau), a(t))|^b |q_0(\tau)|^b d\tau = (\sigma_1^*)^b - |\hat{\sigma}_{22}(a_0, y^*)|^b \int_0^{t_0^*} |q_0(\tau)|^b d\tau. \tag{9}$$

We can change variables in (9) similar to Zobnin and Rabotnov (see [6] where a solution of the problem below is presented for *b*=1). Taking into account (8), we then arrive at the following non-convolution *linear* Volterra equation of the first kind for $g(a) = [q_0(\tau(a))/\sigma_1^*]^b d\tau(a)/da$,

$$\int_{a_0}^{a(t)} |\hat{\sigma}_{22}(a, a(t))|^b g(a) da = 1 - \frac{|\hat{\sigma}_{22}(a_0, a(t))|^b}{|\hat{\sigma}_{22}(a_0, a_0)|^b} \tag{10}$$

Crack in an infinite plane under uniform loading. Consider now a more particular example of a straight crack with a length $2a(\tau)$ in an infinite plate. The origin of the Cartesian coordinate system $\{x_1, x_2\}$ coincides with the centre of the crack. Let a uniform traction with $q(\tau, x) = q_0(\tau)$ is applied parallel to the x_2-axis at infinity. For an elasticity body the normal stress $\sigma_{22}(\tau, y_1)$ near the crack tip can be approximated asymptotically (e.g. [9]) by the expression

$$\sigma_{22}(\tau; a(\tau), y_1) = \frac{K_1(\tau; a(\tau))}{\sqrt{2\pi(y_1 - a(\tau))}}, \qquad K_1(\tau; a(\tau)) = q(\tau)\sqrt{2\pi a(\tau)}, \tag{11}$$

where $K_1(\tau)$ is the mode 1 stress intensity factor. On the other hand, an exact expression for $\sigma_{22}(\tau, x_1)$ ahead of the crack in an infinite isotropic or anisotropic plate has the form (e.g. [9])

$$\sigma_{22}(\tau; a(\tau), y_1) = \frac{K_1(\tau; a(\tau)) y_1}{\sqrt{\pi a(\tau)(y_1^2 - a^2(\tau))}}, \tag{12}$$

For a linear viscoelastic body the expressions (11) and (12) remain valid (see [6]) and hence can be used for the following durability analysis.

For tensile traction $q(\tau) = q_0 = const$, the durability problem can be solved analytically. Equation (8) implies the fracture time for an infinite plane without crack is $t_\infty^* = (\sigma_1^*/q_0)^b$ under the considered loading. As was mentioned above, $t_0^* = 0$ if there exists an initial crack.

Let $\tilde{t} = t/t_\infty^*$ be the normalised time. After substituting asymptotic stress (11) into (10), the equation can be solved using the Laplace transform *under the assumption b*<2, giving

$$\frac{da(\tilde t)}{d\tilde t} = \frac{K_1^{2-b}(a_0)K_1^b(a(\tilde t))}{q_0^2 2^{b/2}\sin(b\pi/2)}\left[\frac{K_1^2(a(\tilde t))}{K_1^2(a_0)}-1\right]^{1-b/2}. \qquad (13)$$

After using the expression for the stress intensity factor (11) and integration, this gives

$$\tilde t(a) = \frac{\sin(b\pi/2)}{2^{-b/2}\pi}\left[\ln\left(\frac{a}{a_0}\right) - (2-b)\frac{a_0}{2a}\,{}_3F_2\left(\{1,1,2-b/2\},\{2,2\},\frac{a_0}{a}\right) - \gamma - \psi(0,b/2)\right] \qquad (14)$$

Here ${}_3F_2$ is the hypergeometric function, ψ is the digamma function and γ is the Euler constant. The results are presented on Fig. 1 and 2 for different values of *b* by dashed lines.

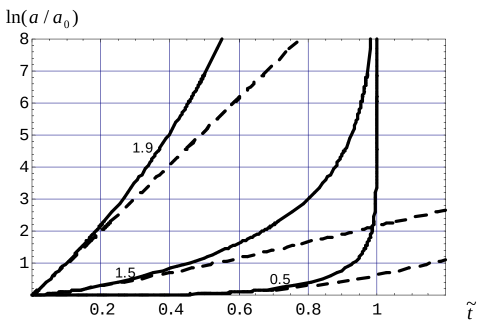

Figure1. Creep crack length vs. time for different b (local approach)

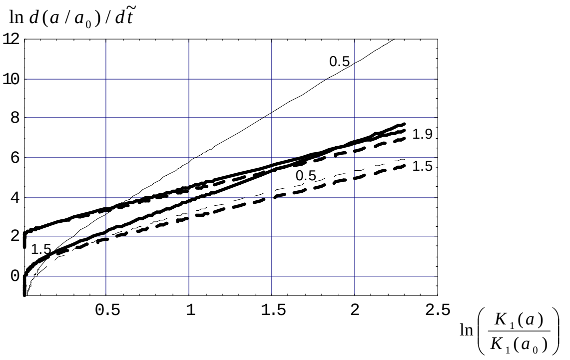

Figure 2. Creep crack growth rate vs. stress intensity factor for different b (local approach).

If we use exact stress distribution (12), after solving the corresponding Volterra equation *under the assumption* $b < 2$,we arrive at the relations

$$\frac{da(\tilde t)}{d\tilde t} = \frac{K_1^2(a(\tilde t))}{q_0^2\sin(b\pi/2)}\left[\frac{K_1^4(a(\tilde t))}{K_1^4(a_0)}-1\right]^{1-b/2} \qquad (15)$$

$$\tilde t(a) = 1 - \left(\frac{a_0}{a}\right)^{2-b}\frac{2\sin\left(\frac{(b-2)\pi}{2}\right)}{(b-2)\pi}\,{}_2F_1\left(1-\frac{b}{2},1-\frac{b}{2},2-\frac{b}{2};\frac{a_0^2}{a^2}\right) \qquad (16)$$

presented on Fig.1 and 2 for different *b* by solid lines.

One can see both from expression (16) and Fig.1 that the durability of the *infinite* plane with any crack is the same as its durability t_∞^* without crack. On the contrary, expression (14) based on approximate (asymptotic) stress representation (11) predicts unrealistic infinite durability for the infinite plane with a crack. The crack growth rates given by expressions (13), (15) look like the Paris type law for fatigue problem.

The both solutions are valid only for $b < 2$ and blow up (predicting instant infinite crack propagation) when $b \to 2$, that is, they are not able to describe the creep crack propagation for common structural materials with experimentally determined values for durability diagram constants (usually $b \geq 4$). The local approach does not also predict the creep crack start delay observed experimentally. A way to overcome those shortcomings is an application of a non-local approach.

3. NON-LOCAL BRITTLE STRENGTH AND DURABILITY CONDITIONS

We will suppose that strength at a point $y\in\Omega$ *on a plane* $\vec{\zeta}$ *depends not only on the stress history at that point,* $\sigma_{ij}(\tau, y)$ *but also on the stress history in its neighbourhood and generally, in the whole of the body,* $\sigma(\tau, x)$, $x\in\Omega$.. A non-local brittle temporal normalised equivalent stress functional $\underline{\Lambda}^{\Theta}(\sigma;t,\Gamma,y,\vec{\zeta})$, which is positively homogeneous in σ and non-decreasing in *t*, can be introduced. It is considered as a material characteristics implicitly reflecting influence of material microstructure. Then the non-local strength condition *for a plane* $\vec{\zeta}$ *at a point* $y\in\Omega$ takes the form $\underline{\Lambda}^{\Theta}(\sigma;t,\Gamma,y,\vec{\zeta})<1$.

The simplest examples of the non-local brittle TNESFs and strength conditions are obtained by replacing the local stress $\sigma_{ij}(\tau, x)$ by its non-local counterpart $\sigma_{ij}^{\Theta}(\tau;\Gamma,y,\vec{\zeta})$ in the corresponding local brittle TNESFs described in Section **2**,

$$\underline{\Lambda}^{\Theta}(\sigma;t,\Gamma,y,\vec{\zeta}) = \underline{\Lambda}(\sigma^{\Theta}(\cdot,\Gamma(\cdot),y,\vec{\zeta});t,\Gamma,y,\vec{\zeta}), \qquad (17)$$

Similar to the non-local analysis ([2] and references therein) and [1], [7], the non-local stress $\sigma_{ij}^{\Theta}(\tau;\Gamma(\tau),y,\vec{\zeta})$ can be taken particularly as a weighted average of $\sigma_{ij}(\tau,x)$,

$$\sigma_{ij}^{\Theta}(\tau;\Gamma(\tau),y,\vec{\zeta}) = \int_{\Omega^{\Theta}(y,\vec{\zeta};\Gamma)} w_{ijkl}(y,x,\vec{\zeta};\Gamma)\sigma_{kl}(\tau;x)dx \qquad (18)$$

where the weight function *w* and the non-locality zone Ω^{Θ} (some neighbourhood of *y*) are characteristics of material point and plane and generally of the body shape Γ, such as $\int_{\Omega^{\Theta}(y,\vec{\zeta};\Gamma)} w_{ijkl}(y,x,\vec{\zeta};\Gamma)=\delta_{ik}\delta_{jl}$. Under this condition, the normalised equivalent stress will be uniform for any body point under any uniform stress field.

For example, $\Omega^{\Theta}(y,\vec{\zeta};\Gamma)$ can be taken as a 2D disc of a diameter 2δ in a 3D body Ω(*t*) or as a 1D segment of a length 2δ for a 2D body Ω(*t*), in the plane $\vec{\zeta}$ with the centre at *y*, where δ is considered as a material parameter. Near the boundary Γ(*t*), $\Omega^{\Theta}(y,\vec{\zeta};\Gamma)$ should be taken as an intersection of the disc/segment with Ω(*t*).

Using the introduced the brittle non-local TNESF $\Lambda^{\Theta}(\sigma;t,\Gamma,y,\vec{\zeta})$, the long-term fracture process (the creep crack initiation and its propagation through the damaged material) can be described as in Section **1** after replacement there the stress tensor σ by its non-local counterpart σ^{Θ}.

4. EXAMPLE OF NON-LOCAL DURABILITY ANALYSIS

Let us consider the 2D problem from Section **2** using the non-local durability analysis with particular non-local TNESF (17), (18) where the crack propagation plane $\vec{\zeta}^{*}$ is prescribed by the problem symmetry, $\Omega^{\Theta}(y,\vec{\zeta};\Gamma)$ is the interval $(y_1-\delta_{-}(y_1), y_1+\delta)$ for *y* ahead of the crack *a*(*t*) and not close to an opposite body boundary, $\delta_{-}(y_1)=\min(\delta,|y_1-a(t)|$ and δ is a material constant. Let $w_{ijkl}(y,x,\vec{\zeta})=w(y,x)\delta_{ij}\delta_{kl}$, where *w*(*y*,*x*) is a bounded function, which is considered as a material characteristics to be identified. As possible approximations, one can choose e.g. $w(y,x)$ constant w.r.t $x\in\Omega(y)$ arriving at the Neuber stress averaging, cf [5], a piece-wise linear or a more smooth hat-shaped dependence on x.

Repeating the same reasoning as in Section **2** but now for the non-local stress $\sigma^{\Theta}_{ij}(\tau;a,y_1)$, we arrive at the same equations (8)-(10) where $\hat{\sigma}_{22}(a,y_1)$ must be replaced by

$$\hat{\sigma}^{\Theta}_{22}(a,y_1)=\int_{y_1-\delta_-(y_1)}^{y_1+\delta} w(y_1,x_1)\hat{\sigma}_{22}(a,y_1)dx_1 \qquad (19)$$

For a problem with initially existing crack, the crack propagation start instant t_0^* obtained from the non-local counterpart of (8) is non-zero since $\left|\sigma^{\Theta}_{22}(a_0,a_0)\right|<\infty$ at the crack tip in spite $\left|\sigma_{22}(a_0,a_0)\right|=\infty$. For example, the start delay for a constant q_0 is $t_0^*=t_\infty^*\left|\hat{\sigma}^{\Theta}_{22}(a_0,a_0)\right|^{-b}$.

We can differentiate the non-local counterpart of (10) w.r.t. $a(t)$ and arrive at the following linear non-convolution Volterra equation of the second kind for the unknown function $g(a)$

$$g(a(t))+\int_{a_0}^{a(t)}K(a(t),a)g(a)da=Y(a(t)), \qquad (20)$$

$$K(a(t),a)=\left|\sigma^{\Theta}_{22}(a(t),a(t))\right|^{-b}\frac{\partial}{\partial a(t)}\left|\sigma^{\Theta}_{22}(a,a(t))\right|^{-b},$$

$$Y(a(t))=-\left|\sigma^{\Theta}_{22}(a_0,a_0)\right|^{-b}K(a(t),a_0).$$

Let us consider the non-local version of the particular problem from Section **2**, with the piece-wise linear weight $w(y_1,x_1)=\dfrac{2(\delta-|y_1-x_1|)}{\delta^2+2\delta\delta_-(y_1)-\delta_-^2(y_1)}$ for $x_1\in(y_1-\delta_-(y_1),y_1+\delta)$

Substituting (12) into (19) for $y_1=a_0$ and $\delta_-(y_1)=0$, we obtain after integration the corresponding non-local stress at the crack tip. It can be used to estimate the material parameter $\delta=\dfrac{32}{9\pi}\left(\dfrac{K_{1c}}{\sigma_r}\right)^2$ from the experimental data on the monotonous tensile strength σ_r for a smooth sample and the critical stress intensity factor K_{1c} for a sample with a long crack. The non-local stress can be also used in the above formula for t_0^* to calculate the crack start delay $t_0^*=t_\infty^*\left[\dfrac{(a_0+\delta)\sqrt{2a+\delta}}{\delta^{3/2}}+\dfrac{a_0^2}{\delta^2}\ln\dfrac{a_0}{a_0+\delta+\sqrt{\delta(2a+\delta)}}\right]^{-b}$ under a uniform tensile traction $q(\tau,x)=q_0(\tau)$.

Results of the numerical solution of Volterra equation (20) with $\delta=0.5a_0$ are presented on Fig. 3 and Fig. 4 for different b. The specific non-monotonous and non-smooth dependence of the crack growth rate $\dfrac{d(a/a_0)}{d\tilde{t}}$ on the stress intensity factor $\dfrac{K_1(a)}{K_1(a_0)}$ at the beginning, Fig. 4, can be perceived as a signature of the particular weight $w(y,x)$ and employed for simulation of the short crack retardation near inter-grain boundaries. Such curves may be useful for experimental identification of $w(y,x)$.

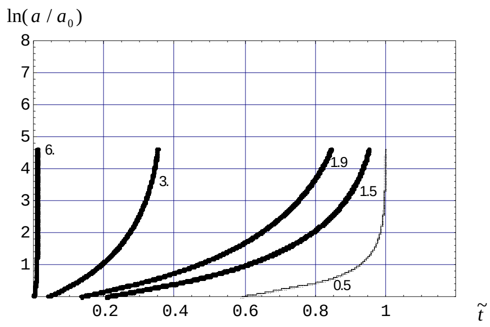

Figure 3. Creep crack length vs. time for different b (non-local approach)

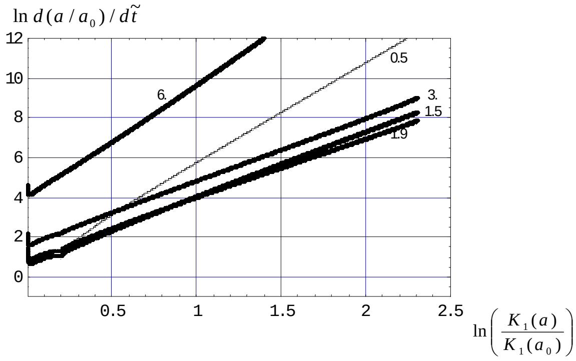

Figure 4. Creep crack growth rate vs. stress intensity factor for different b (non-local approach).

CONCLUSIONS

A united description of creep crack initiation and propagation is principally possible using the local as well as the non-local approach, however the local approach in the considered examples can be applied only for a limited range of material long-term parameters and cannot describe the crack start delay. The non-local approach seems to be free of the drawbacks. When the stress fields are available analytically or numerically and the strength conditions are associated with the linear accumulation rule, the 2D problem in both approaches can be reduced to non-linear Volterra equation(s) for the unknown crack geometry. It can be transformed for a single crack to a linear non-convolution Volterra equation in the case of a material with a power-type durability diagram

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