A guide to postgraduate research in mathematical sciences

The Higher Education Academy

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Written by Martin Greenhow, Brunel University.
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For further copies contact the Network at
info@mathstore.ac.uk

Introduction

Right now you are probably thinking "My undergraduate studies went well, so why on earth do I need to read this?" Many of the study skills and attitudes that enabled you to succeed at undergraduate level will remain valuable. However, a good first degree is a good example of a 'necessary but not sufficient condition' for success at postgraduate level. There are significant differences. Exams play a far less significant role, but nobody spells out any longer what you have to do. You need to take far more responsibility for the strategy as well as the minutiae of the work. This guide, based on http://people.brunel.ac.uk/~mastmmg/ssguide/sshome.htm (denoted SSG below), seeks to help you avoid getting lost during what should be a challenging, but immensely enjoyable, experience.

Many of the issues raised here are explored in detail in "The research student's guide to success" P Cryer, Open University Press and "How to get a PhD" E M Phillips and D S Pugh, Open University Press. See also http://www.engageinresearch.ac.uk/, a useful guide to the structure of a thesis at http://dhost.info/pingke/P-MA-TypicalThesisStructure.htm and http://www.missendencentre.co.uk/links where you will find accounts of doing a PhD written by the students themselves. This guide attempts to filter and map this general advice specifically to mathematics postgraduate research.

Why do postgraduate research?

This is a deadly serious question and you need to answer this honestly. Bad reasons exist, for example, because I want a PhD. The point here is that whilst it may seem like a goal at the start of your studies, at the end you'll be asking "what next?" So it's really important to **enjoy the process** here, and not simply want the product. The best reason is that you are interested in mathematics and want to make a contribution to the field; other reasons may be valid, but if this reason is missing from your list, give this guide to someone else and do something else with your life!

Doing postgraduate research is not a ticket to a job and only academia or high-tech research companies will actually require a PhD. Although the generic skills you develop during your research are sought after by employers, the taught MSc is usually the best way to acquire/enhance specific skills for most jobs, often undertaken when working for the company.

Most PhD positions are advertised at http://www.findaphd.com/. Before applying for postgraduate research you will need to ask yourself why you want to do it in that particular area, in that

particular university and with that particular supervisor. This will involve some research e.g. talking to existing postgrads, academics other than the proposed supervisor (e.g. your undergraduate tutor), sorting out practical arrangements such as accommodation, your family and money commitments, whether or not you can work as a RA (research assistant), demonstrator or GTA (graduate teaching assistant) and if so whether these commitments will allow enough time to carry out your own research.

Money

Think of the PhD as a full-time job and act accordingly. Working your way through a (full-time) PhD or MPhil is not feasible unless you are very disciplined and you work less than 10 hours per week. You do need time to enjoy yourself too, maintaining a sensible work/life balance so you will need either a loan or a bursary for all three years of a PhD. Beware of taking a job at the end of the three years but before you have finished the MPhil or PhD - this is an excellent way to ensure that you never submit your thesis.

On the other hand, teaching undergraduates for a few hours per week during your PhD is very beneficial; you gain much needed money and experience, underpin your own understanding of the basics, and you get a sense of achievement even when your research is temporarily stalled. Take advantage of some teacher training sessions if possible and put them on your c.v. You need to watch out though, since it's easy to be side-tracked by undergraduates, and proper preparation can take as much time as the classes themselves. 4 to 5 contact hours a week is reasonable.

Using English

To write or not to write? In mathematics, one usually needs to prove theorems, write programs and obtain results before beginning serious writing-up: this contrasts strongly with other disciplines, where the act of writing seems to provide a major stimulus to develop the thesis itself. Nevertheless, writing interim reports, summaries of objectives etc will clarify your ideas to yourself and your supervisor, and enable you to develop your writing style to be clear, concise and simple. So don't leave it all to the end!

Your university will run English for Academic Purposes courses. Unless your English is already excellent, attend these. Being able to communicate effectively is essential for your postgraduate studies and subsequent employment.

What is needed?

Much of what needs to be said about coping as an MPhil or PhD student is common to that for MSc and undergraduate students, especially for the project work described in SSG. There are, however, several features which distinguish postgraduate research (MPhil and PhD); principally you are responsible for the content and management of your studies now and nobody will tell you what you need to do next. In collaboration with your supervisor, you set your own objectives and strategies for achieving them.

To gain an MPhil or PhD you need to:

- be knowledgeable of existing work in fairly closely-related areas (for example, your examiners will not be impressed if you cannot explain the standard theories and principal work in your broad area, explain the basis of anything you have quoted or used in your work, or fail to demonstrate that you have actually read all your references),
- be workmanlike, clear and logical in the development of all your ideas and how you express them. Be honest and objective (especially when you do not achieve the results you wanted),
- be original and occasionally inspirational, having real insight into what you are doing. This does not mean being a second Newton; applying existing theories or using known experimental or statistical techniques in a new area/discipline or with new data is generally sufficient,
- have a thesis; that is to say you need to develop a viewpoint which you can defend and which leads logically to a testable conclusion. This is often best specified by writing clearly-specified research questions and detailing the methodology used to answer them,
- have an open mind to others, making the effort to understand what they are doing.

Note that an MPhil or PhD is not simply a critique of existing work or a literature review (though this is part of it). A major difference between an MPhil and a PhD (apart from length and scope of study) is that you will be expected to go to original research papers in a doctorate, whereas quoting from standard texts is acceptable for a masters degree.

Getting fed up!

You may get fed up or even depressed at some stage, probably about half way through, just like everyone else. Don't lose belief in yourself or what you are doing, lean on your friends and family and get help if you need it from others (e.g. counsellors).

You might feel bored with your work, regarding it as trivial. Before you discuss this with your supervisor, try writing up as fully as you can what you have achieved and what is stopping you making further progress. The process of doing this is quite likely to unfreeze you.

You may be unable to make progress. It is sometimes helpful to have several lines of enquiry on the go at the same time. Again writing or making charts, schedules and plans can help you identify precisely what is stopping you. This will then often result in a solution or suggest a new line for your research. See the 'Set work' section at SSG, section 4.22.

You may feel guilty that you are indulging yourself while others are supporting you. Don't; you, your family and society as a whole are making an investment in you that will ultimately benefit everyone.

Preparation for the Oral Examination.

Remember that your supervisor will not have recommended that you submit a thesis which is marginal or will clearly fail. Nevertheless you can still fail by a poor performance in the Viva.

- Prepare for it by reading your thesis again with fresh, critical eyes, writing down possible questions you might get and how you'll answer them. You will be allowed your own copy of the thesis in the Viva, so stick in all questions and answers on post-it notes.
- Find out about the External Examiner's own work; read some of his/her papers if possible.
- Present yourself as logical, organised and honest (otherwise the examiners may not trust your results).
- Be smart; take a pen and paper and board marker pen or chalk.
- Discuss with your supervisor whether you should make an initial 15 minute presentation on what you have done, what your main results are and how they might be used. This is sometimes required, and always useful.

The Oral

The Oral is a defence of your work, not an argument. Your examiners are not 'out to get you' so it should be an enjoyable,

but rigorous, experience. It will usually last 1-3 hours, but will seem significantly shorter.

- Be prepared for the opening question "Please tell me, in your own words, what you have done?" or "What are the strongest/weakest features of your research?" A closing question might be a request to outline where you think future development of your ideas could lead and how this might be done.
- You will probably be asked questions on basic knowledge in areas related to, but outside, your thesis topic. Often these general questions will come from the Internal Examiner.
- Specific questions of detail will be asked by the External Examiner. Failure to answer one or two questions does not mean you will fail the PhD; just say you don't know, or ask for clarification of the question if you don't understand what is being asked.
- It is usually a good idea to have your supervisor present. They cannot answer questions for you (indeed cannot speak unless invited), but can be helpful in clarification of questions, and to prompt you for results etc. which you have forgotten to mention in the heat of the moment. They can also take notes for you, especially if you need to modify the thesis.

At the end, you will normally be told either that you have the PhD, or (more often) that some amendments are required. This can range from substantial re-writing to mere typos, but either way it is essential to do this as soon as possible. You **must** address **all** the points raised; you may not be examined again, so not doing so could mean failure.

In collaboration with your supervisor, you should also write up your work in publication form, which usually involves a re-write in a much more concise format. You can expect to be the principal author of papers based on your work.

What next?

The HERO website http://www.hero.ac.uk, although now closed, does still provide useful links.

Most academic and academic-related jobs are advertised in the Times Higher pages http://www.timeshighereducation.co.uk/

Sign up to: http://jobs.ac.uk/

... and good luck.

A rough view of a PhD

Every PhD is different, but it might be useful for you to compare yours with this scenario. MPhil students will need to map the following 3 years to their time span.

Year 1

- Get straight down to it!
- Read the proposal for your PhD and start reading the background textbooks and papers in your area.
- Attempt model problems as defined by your supervisor.
- Write a mini-report on each and act on the feedback you get from your supervisor (in both content and presentation).
- Find out what other research students in your department are doing.
- If possible, attend taught courses at MSc level, especially if they involve theory or programming skills you will need.
- Attend departmental seminars, even if you don't understand anything.

By the end of year 1 you should have:

> a good idea about what your research will involve. Sounds obvious, but you need to have written down a clear specification of the work and your progress so far,

> read the background, some review/original papers in the same area and related textbooks (not just Google),

> attended courses,

> started to make notes from these sources; keep this focused and don't copy out sections from books, download chunks from the web - this is plagiarism. You will synthesise these notes into a narrative later,

> written down exact references as you find them,

> started a week-by-week log book,

> completed a model problem and written it up,

> given a presentation to other PhD students/staff,

> updated your c.v. and web page.

Year 2

- Tackle the main problem, taking the initiative yourself.
- Develop the main theory, programming and collect and analyse your data or results.
- Write a major report on this work and present it, first in your department, and then at a conference (see 'Giving a talk' in SSG).
- Talk to others in your field at the conference, telling them what you are doing, ask for comments, suggestions, references to published work etc. and find out about their work.
- Keep attending departmental seminars, even if you don't understand everything.

By the end of year 2 you should have:

> a full understanding of the theory and methods to be used,

> read most of the background material and made notes/references,

> tackled **your** main problem,

> made substantial progress on any programming involved. It is not possible to write a decent program without completely understanding what you want/ need to do, so ask your supervisor if anything is not clear. Then draw a flow chart or write 'pseudo code' to specify how the program will work before getting bogged down in language syntax. Back-up all programs and check them using **test cases**,

> written drafts as you go along to aid your thinking; in mathematics this technique is perhaps less useful, but you must keep very detailed notes and back them up with photocopies or notes on your PC (**and** your university's network),

> given a conference talk or poster,

> updated your c.v. and web page.

Year 3

- Plan your career at the start of this year, so you don't have to worry about that as well as your thesis submission and viva.

- Continue with further work as independently from your supervisor as you can, making everything as complete and rigorous as possible.
- Write draft chapters using your earlier reports as a guide – this will take a lot longer than you think.
- Put the whole thing together.
- Submit and defend your thesis and publish your results.
- Give a seminar at another university and sound enthusiastic, even if their PhD students don't understand everything!

Towards the end of year 3 you should have:

> finished all the actual project content,

> structured the main sections and typed them up. This will take longer than you think,

> acted on your supervisor's comments on your draft chapters,

> produced all the graphs and diagrams,

> developed a career plan… this de-stresses you and is one less thing to think about,

> updated your c.v. and web page,

> swapped drafts with other PhD students for comment. If they can't understand what you have done in broad terms, it's **your** fault,

> written the abstract, introduction, conclusions and recommendations sections. Give these to your supervisor for comment in good time,

> finalised any appendices, references and program discs,

> acted on your supervisor's comments,

> put all sections together, check spelling and grammar, and included headers, footers and page numbers.

Some departments have 'perpetual' students hanging around who don't actually want to finish or refuse to submit until the thesis is perfect (it never will be). Plan to submit in three years and stick to it if at all possible.

Getting the best out of your supervisor

Postgraduate research in mathematics requires far more active collaboration between student and supervisor than many other disciplines. You should aim to see your supervisor at least once every two weeks, even if only for a short time.

Your supervisor is likely to be busy, so make sure you maximise the benefit of your contact time by:

- making and keeping appointments,
- preparing both yourself and your supervisor for the meeting by submitting well-written work in advance (if this is a poorly-written first draft you will spend your time correcting the sense, or even the English, of what you have written). Be prepared to update your supervisor on what you are doing now at the start of each meeting,
- being proactive. You are not an automaton to be programmed with the next task; you should tell your supervisor what you intend to try next and ask what they think,
- being objective and not trying to cover up difficulties,
- following their advice or giving explicit reasons why not, or why your ideas are better,
- discussing the 'big picture' (i.e. time management, overall progress, thesis plan etc.) as well as the detail. If you are a Research Assistant working on a project, you will need to discuss the balance between that project and your thesis work,
- setting objectives for both you and your supervisor (with dates),
- setting the date, time and place for the next meeting. Ask for comments on your written work to be returned to you before the next meeting so you can read and act on them beforehand.

You can expect your supervisor to be reasonably accessible, open and friendly to you and enthusiastic and supportive about your work and career. During supervision sessions you can expect your supervisor to give you uninterrupted attention (e.g. by telling callers to call back). Your supervisors' experience will enable them to be constructively critical of your work and its presentation, suggest ideas, references and resources, so you would be unwise to ignore their advice. They will be able to tell you how you are progressing against the normal benchmarks for a PhD, introduce you to other workers in your area, take you to conferences (especially when you are

ready to present your own work), tell you when you are ready to submit your thesis and promote your career by introducing you to others in the field and giving you a reference. You need to do everything you can to deserve this type of attention!

If the student/supervisor relationship is not working you will need to act tactfully; changing your supervisor without starting again from scratch is rarely feasible. It is much better to discuss the issues with your departmental Director of Research, or similar academic, who can act to resolve the problems. This person will also provide factual information on registration, progression and specification for the thesis.

The thesis itself

Your thesis should be a 'narrative text' with a coherent story line which flows logically from one section to the next in a coherent manner. The object is to communicate, not impress or blind your reader with science. You should help your reader as much as possible by using sections within each chapter, diagrams, charts etc and ordinary plain English, free of jargon and slang. Stick to conventional and consistent nomenclature.

Note that students often get referred because of poor presentation and/or inadequate abstracts, introductions, and, especially, conclusions and recommendations.

Typically a thesis should comprise the following sections:

> Title page. See your university's Requirements Guide.

> Acknowledgements - but use a formal style.

> Table of contents.

> List of figures.

> Nomenclature, Definitions and Non-dimensionalisation.

> Abstract of about 2-4 sides. The abstract is an important guide to the reader and must describe the main area of the work and your main results. Usually written last.

> Introduction. This give a 'map' of the work undertaken, states why your work is interesting and important (including applications to other areas of mathematics or beyond), a review of previous work, a clear statement of how you have extended the previous work and what your research questions and methodologies are (if appropriate). This is usually written at the end of the project.

> Method. This can be very technical, so help your reader 'navigate' through the work by clearly stating your assumptions, stressing what is most important (move some material to appendices), using logical paragraph,

section and chapter breaks, diagrams, charts, flow charts and equations with all terms defined (consider reminding the reader of earlier definitions). Keep your strategy in mind and do not get too bogged down in detail. The method section is often written early and is revised as the work progresses and the supervisor comments on it.

> Results. Again this can be very technical, so help your reader as before. When presenting results you should state; any limitations on their generality, the accuracy, describe the sensitivities to changes in assumptions and parameter values, reconsider the model assumptions and refine them if possible (a modeling cycle) and explain your results. This section is often written early and redrafted a number of times.

> Conclusions. Draw some conclusions, probably taking several pages of text. If you won't do this, why would you expect the reader to do it for you? Conclusions must not contain new ideas or afterthoughts, but only statements which are supported by your work and follow naturally and logically from it. They **must** be defensible (as in a court of law) on the basis of **only** what you have presented – do not include unsupported ideas no matter how 'obvious'. Conclusions require a lot of attention.

> Recommendations. This section should contain many of the ideas for future work which will have occurred to you as you worked and which were written down in your log book. Imagine yourself starting a similar project… what would you want to read? Be as specific as possible about new applications of the present theories/techniques, application in areas with similar mathematical/ programming structures and extensions which avoid the limitations/weaknesses of your work. Be honest - this is a sign that you know what you are doing and will be rewarded rather than penalised. Recommendations require a lot of attention.

> Appendices can contain program listings and data files (on a disc to help future researchers), a list of software used, and instructions for reading data and/or running any programs on the disc. Remember that the External Examiner may well want to do this so be specific and include everything that is needed unless this infringes licence or copyright laws. It could also include proofs and material which underpins but is not an essential part of the main narrative.

> References – see the 'Set work' section at SSG for the conventions of correct referencing and advice on how to avoid plagiarism.