1. Let Y_1, ..., Y_n denote a random sample drawn from $N(0, \sigma^2)$. Let $\bar{Y} = \frac{1}{n} \Sigma_{i=1}^n Y_i$ and $S_n^2 = \frac{1}{n-1} \Sigma_{i=1}^n (Y_i - \bar{Y})^2$ be the sample mean and variance respectively.

(1). Write S_n^2 as a quadratic form of vector $\boldsymbol{Y} = (Y_1, ..., Y_n)'$, then prove that $\bar{Y}$ and S_n^2 are independent.

(2) Find the distribution of S_n^2.

2. Let Y_1 and Y_2 denote a random sample drawn from $N(0, \sigma^2)$, find the distributions of $(Y_1 + Y_2)^2$ and $(Y_1 - Y_2)^2$, then prove that $Y = (Y_1 + Y_2)^2/(Y_1 - Y_2)^2$ follows a F-distribution $F(1, 1)$.