Index tracking example question

The table below shows the stock prices and index values over a number of time periods, together with the current tracking portfolio.

Period	A	B	C	D	E	Index
0	39.7	3.1	38.2	72.9	3.6	2655.8
1	73.5	0.5	96.3	60.4	77	5039.7
2	17.3	42.3	62.2	49.7	91.5	4717.7
3	31.6	70.8	57.2	5.2	54.7	4946.7
T=4	64.4	86.9	76.1	27.1	14.3	6506.6
Current tracking portfolio	5	67	8	6	10	

For example in period 3 the stock/share price for stock A is 31.6 and the index value is 4946.7. The current tracking portfolio contains 5 units (shares) of stock A, 67 of B, etc.

Find tracking portfolios containing K=2, 3 and 4 stocks.

How do these tracking portfolios compare with the current tracking portfolio in terms of their tracking performance.

Comment on anything you find noteworthy about your solution.

Index tracking example solution

Utilising the Solver model as constructed in the lecture I get the following tracking portfolios:

	A	B	C	D	E	Error
K=2	0.00	30.77	57.62	0.00	0.00	171.57305
K=3	0.00	35.00	48.49	12.07	0.00	0.43679705
K=4	0.62	35.08	47.90	11.98	0.00	0.378185084

The tracking error associated with the current tracking portfolio is 1588.497173, hence it is clear that (in terms of our measure of tracking performance as discussed in the lecture), the tracking portfolios above would (historically) have been much better than our current tracking portfolio.

Things you may note when you try this example:

- You may find that for a particular K value you have less than K stocks (in terms of the amount of each stock x_i bought). In fact this is implicit in the mathematics we presented. That set of mathematics (formulation of the index tracking problem as a nonlinear mixed-integer program) ensures that $\sum_{i=1}^{N} z_i = K$, where N=5 here. If you look closely at the full formulation we presented in the lecture you will see that (in the absence of any value for the minimum proportion of any stock that must be in the tracking portfolio) there is NOTHING (i.e. no constraints) to prevent $z_i=1$ and $x_i=0$, i.e. nothing to prevent the z variable signifying we have a stock in the tracking portfolio being set equal to one whilst at the same time the x variable signifying how many units of the stock to buy being set equal to zero.

- You may find that your solutions differ from those tabulated above. The essential reason for this is that (for the nonlinear mixed-integer problem we are solving) Solver is using a **heuristic** solution technique (unlike linear/integer programming dealt with previously where we had an optimal solution technique). As such Solver cannot guarantee to find the optimal (best possible) solution. Hence your solutions may (by chance) differ from the ones presented above.

- One way you may notice the heuristic nature of the solutions given by Solver is to look at the error value you have as K increases. Logically as K increases we would expect to decrease (or leave unchanged) the error found (recall we are seeking to minimise tracking error). If your error value goes up at any point as K increases then this is a clear indication of the heuristic nature of the solutions given by Solver.