POPULATION HEURISTICS

A population heuristic can be described as an "intelligent" probabilistic search algorithm and is based on the evolutionary process of biological organisms in nature. During the course of evolution, natural populations evolve according to the principles of natural selection and "survival of the fittest".

Individuals who are more successful in adapting to their environment will have a better chance of surviving and reproducing, whilst individuals who are less fit will be eliminated. This means that the *genes* from highly fit individuals will spread to an increasing number of individuals in each successive generation.

The combination of good characteristics from highly adapted parents may produce even more fit offspring. In this way, species evolve to become increasingly better adapted to their environment.

A population heuristic simulates these processes in a computer by taking an initial population of individuals and applying genetic operators in each reproduction. In optimisation terms, each individual in the population is encoded into a string or *chromosome* which represents a possible *solution* to a given problem.

The *fitness* of an individual is evaluated with respect to a given objective function. Highly fit individuals (solutions) are given opportunities to reproduce by exchanging pieces of their genetic information, in a *crossover* procedure, with other highly fit individuals. This produces new "offspring" solutions (i.e. *children*), who share some characteristics taken from both parents. Mutation is often applied after crossover by altering some genes in the strings.

The offspring replace some or all of the current population and the process repeats.

A SIMPLE POPULATION HEURISTIC

Generate an initial population of individuals (solutions)

Evaluate fitness (solution quality) of individuals in the population

repeat

- Select parents from the population
- Recombine (mate) parents to produce children
- Mutate the children
- Evaluate fitness of the children
- Replace some or all of the population by the children

until you decide to stop, whereupon report the best solution encountered

FORMULATION

minimise $\sum_{t=1}^{T} (r_t - R_t)^2/T$

subject to

$$\sum_{i=1}^{N} z_i = K$$

$\varepsilon_i z_i \leq V_{iT} x_i / C \leq \delta_i z_i \quad i=1,...,N$

$$C_{trans} = \sum_{i=1}^{N} \text{transaction cost}(X_i \rightarrow x_i \text{ at time T})$$

$C_{trans} \leq \gamma C$

$$\sum_{i=1}^{N} V_{iT} x_i = C - C_{trans}$$

$x_i \geq 0 \quad i=1,...,N$

$z_i \in [0,1] \quad i=1,...,N$

INCOMPLETE DETAILS

We have developed a population heuristic (PH) for solving the formulation of the problem presented above.

In terms of representing a TP in our population heuristic we use variables $y_i \geq 0$ (i=0,1,...,N) where:

y_0 represents the fraction of C associated with transaction cost, so that $y_0=C_{trans}/C$

y_i (i=1,...,N) represents the fraction of C associated with the total value of stock i in the new TP at time T, so that $y_i=V_{iT}x_i/C$

and we must have $\sum_{i=0}^{N} y_i=1$

Suppose we have K=3 and N=10. Then one example individual (TP) would be:

0	0.10
4	0.30
7	0.20
8	0.40

HAVING A CHILD

Parent 1			Parent 2	
0	0.10	and	0	0.05
4	0.30		4	0.20
7	0.20		7	0.20
8	0.40		8	0.55

come together to have a child:

0	?
4	?
7	?
8	?

Complications:

- elements of the child need to sum to one
- minimum and maximum proportions
- having exactly K stocks in the child

UNFITNESS

One subtlety here is that our representation provides two ways to calculate the transaction cost for any TP:

- the value of y_0 directly relates to the total transaction cost incurred (via $y_0=C_{trans}/C$).
- the y_i values (i=1,...,N) implicitly imply how many units of each stock to hold (via $y_i=V_{iT}x_i/C$). Taken over all stocks and compared with the current TP $[X_i]$ this leads to the total transaction cost incurred in moving from $[X_i]$ to $[x_i]$.

We need to try and ensure that total transaction cost calculated via y_0 and total transaction cost calculated via $[y_i]$ are the same (since if not how can a TP as represented in our PH have meaning?)

In our PH we evolve TPs in which the total transaction cost calculated via y_0 agrees with the total transaction cost calculated via $[y_i]$ by using "*unfitness*" together with an appropriate population replacement scheme.