Payoff tables

Consider the example of a company that can invest in a new product produced by another company. Depending upon how much they want to invest they are entitled to a specified share of the profits made by the product over the next year. If they invest £80m they are entitled to 50% of the profits, but if they invest £35m only 25% of the profits. Of course they could choose not to invest at all. There are three scenarios for the demand for the product, high, medium or low and in these cases the total profit from the product would be £300m, £200m and £50m respectively. What should the company do?

> **Reflect on this problem for five minutes. Would you invest £80m, £35m or choose not to invest? Why or why not?**

Payoff table solution

The standard approach to problems of this type is to construct a **payoff table**, also known as a decision table, indicating for each of the possible decisions the payoff that the company receives. Look at the spreadsheet below:

	A	B	C	D	E	F
1	Choice	Invest	% of profit		Demand	Profit
2	A	***80***	***50***		High	***300***
3	B	***35***	***25***		Medium	***200***
4	C	***0***	***0***		Low	***50***
5						
6					Demand	
7		Payoff	Choice	High	Medium	Low
8			A	70	20	-55
9			B	40	15	-22.5
10			C	0	0	0

Cells A2 to A4 show the three choices, labelled A, B and C, of investing £80m, £35m, or nothing respectively. Cells F2 to F4 show the profit for each of the demand scenarios and cells D8 to F10 show the payoff (profit obtained by the company) given each of their possible decisions (choices) and each of the possible demand scenarios. For example the profit of 15 (£m) shown in cell E9 corresponds with the company making choice B, investing 35 and being entitled to 25% of the profit that occurs when the outcome is the medium demand scenario with a total profit of 200 (so that the payoff/profit the company gains is 25% of 200 less 35 = 15). There are other cells in this spreadsheet (not currently shown above) but these do not concern us for now.

The key points here are:

- the company can only choose one of the three possible choices
- the demand will be one of the three possible scenarios

It is the uncertainty in demand, which scenario will occur, that creates the decision problem here. If we knew which demand scenario was going to occur (what demand was going to be) over the next year then the decision as to which choice to make would be trivial (if high choose A, if medium choose A and if low choose C, why ?)

Assume for the moment that we have no idea of the probabilities associated with each of the demand scenarios. There are a number of standard decision criteria that can be used as considered below.

One thing to note here though is that the standard decision criteria below assume that the payoff table is constructed such that:

- the rows correspond to choices - "things we control"
- the columns correspond to "outcomes" – "things we do not control"

You can see how the payoff table above was of this form – with the rows being our three choices (A, B and C) and the columns being the demand scenarios.

Optimistic – maximax – maximise the maximum payoff for each choice

Find the maximum payoff for each possible decision and then choose the decision that gives the maximum payoff. Here the maximum payoffs associated with A, B and C are 70, 40 and 0 respectively, so the maximum of these is 70 corresponding to choice A. This is an optimistic criteria since we make the choice that will lead to the largest possible payoff provided we have the most favourable demand situation. Note here that the actual outcome given this choice of A will be either 70 (as we hope) or 20 or –55 depending upon the demand that occurs.

Conservative (pessimistic) – maximin - maximise the minimum payoff for each choice

Find the minimum payoff for each possible decision and then choose the decision that gives the maximum payoff. Here the minimum payoffs associated with A, B and C are -55, -22.5 and 0 respectively, so the maximum of these is 0 corresponding to choice C. This is a conservative/pessimistic criteria since we make the choice that will lead to the largest possible payoff even if we have the most unfavourable demand situation.

Regret – minimax – minimise the maximum regret for each choice

For each choice/demand scenario calculate the difference between the payoff for the best choice we could have made if we had known in advance the demand that was going to occur and the payoff for that choice/demand scenario. These values are known as the **regret** values. They represent the **opportunity loss** we have incurred by not making the best possible choice for the demand that did occur. These regret values can also be seen below.

	A	B	C	D	E	F
1	Choice	Invest	% of profit		Demand	Profit
2	A	***80***	***50***		High	***300***
3	B	***35***	***25***		Medium	***200***
4	C	***0***	***0***		Low	***50***
5						
6					Demand	
7		Payoff	Choice	High	Medium	Low
8			A	70	20	-55
9			B	40	15	-22.5
10			C	0	0	0
11						
12					Regret	
13			Choice	High	Medium	Low
14			A	0	0	55
15			B	30	5	22.5
16			C	70	20	0

Here the regret value of 5 for cell E15 is associated with choice B and the medium demand scenario. The best choice we could have made if we had known in advance medium demand was going to occur would have been choice A with a payoff of 20. As in cell E15 we have made choice B we only obtain a payoff of 15 so our regret is 20 - 15 = 5. For each of the demand scenarios columns (D13 to D16, E13 to E15 and F13 to F16) the regret values shown are calculated by taking the maximum payoff for that demand scenario and subtracting from it the particular payoff values for the choice/scenario being considered. Note here that small regret values are better than large regret values.

You will see there is always a zero in each column – at least one choice must lead to the maximum payoff possible in that column and so we have no regrets (or in the words of Edith Piaf, "Non, Je ne Regrette Rien").

Having calculated the regret values we calculate the maximum regret for each choice, so here 55 for A, 30 for B and 70 for C. The logic here is that we make the choice that minimises this maximum regret – so here choice would be B with a value of 30. The reasoning here is that if we make choice B then we will not ‘miss out’ too much by having made a wrong choice irrespective of the demand that occurs. If we make choice B then if high demand occurs we are 'missing out' on 30, if medium demand occurs we are 'missing out' on 5 and if low demand occurs we are 'missing out' on 22.5. Note here that the actual outcome given this choice of B will be either 40 or 15 or –22.5 depending upon the demand that occurs.

All of the above have not taken any probability information into account. There are two standard decision criteria approaches to introducing probability into the situation. and we consider these below.

Equally likely – maximise average payoff

Here we assume that each demand scenario is equally likely (so they all have equal probability). We calculate the average payoff for each choice and take the maximum of these values. Here the average payoffs for A, B and C are 11.67, 10.83 and 0 respectively and the maximum of these is 11.67 associated with choice A.

Expected monetary value – maximise the probability weighted payoff

Here we assume we have information as to the probability of each demand scenario. We calculate the expected monetary value (EMV) which is a probability weighted average for each choice and take the maximum of these values. Examining the spreadsheet as below:

	A	B	C	D	E	F	G	H
1	Choice	Invest	% of profit		Demand	Profit	Probability	
2	A	***80***	***50***		High	***300***	***0.2***	
3	B	***35***	***25***		Medium	***200***	***0.3***	
4	C	***0***	***0***		Low	***50***	***0.5***	
5								
6					Demand			
7		Payoff	Choice	High	Medium	Low	Equally likely	EMV
8			A	70	20	-55	11.67	-7.5
9			B	40	15	-22.5	10.83	1.25
10			C	0	0	0	0.00	0
11								
12					Regret			
13			Choice	High	Medium	Low		
14			A	0	0	55		
15			B	30	5	22.5		
16			C	70	20	0		
17								
18		Criteria	Decision	Value				
19		Optimistic - maximax	A	70				
20		Conservative - maximin	C	0				
21		Regret - minimax	B	30				
22		Equally likely	A	11.67				
23		EMV	B	1.25				

we have the probabilities associated with the demand scenarios as 0.2 for high, 0.3 for medium and 0.5 for low (note that these probabilities must sum to one as these three scenarios are the only possibilities for the demand that might occur).

The payoffs for choice A for high, medium and low are 70, 20 and –55 respectively so the EMV for choice A is hence 70(0.2) + 20(0.3) + -55(0.5) = -7.5, as in cell H8 above. As can be seen here EMV is the probability weighted average of the numeric monetary outcomes.

The EMV for choice B is 1.25 and for choice C is 0 (cells H9 and H10 above) and so the maximum EMV is 1.25 associated with choice B.

It can be seen above that the spreadsheet shows for each of the decision criteria the decision that is 'best' and its associated value.

Discussion

Above we considered a number of different standard decision criteria on our particular example and it can be seen that, depending upon the criteria used, A, B or C could be chosen. In some senses we have achieved nothing as we knew when we first considered the problem that we could chose A, B or C. However we have articulated a number of decision criteria, each with their own logic, that enable us to systematically take the decision problem and reach a logical decision in a numeric way. The fact that there is no 'one best way' to reach a decision in problems such as the one considered above is just a fact of life. Instead the usual approach is to consider the different decisions arrived at using all criteria and then to somehow select from them a unique final decision using various ideas such as:

- voting – choose the decision that is most popular over a number of criteria; and
- personal preference – choose the decision criteria that best suits your personal preferences (e.g. a risk-taker might use the optimistic criteria, someone who is risk-adverse might use the conservative criteria) and choose the decision made by that criteria