Markowitz extra constraints example question

The table below shows stock prices for five stocks over a number of time periods.

Period	A	B	C	D	E
0	39.7	3.1	38.2	72.9	3.6
1	73.5	0.5	96.3	60.4	77
2	17.3	42.3	62.2	49.7	91.5
3	31.6	70.8	57.2	5.2	54.7
T=4	64.4	86.9	76.1	27.1	14.3

For example in period 3 the stock/share price for stock A is 31.6.

Plot the efficient frontier for these stocks incorporating the following conditions:

- No more than 60% of the total investment can be made in any stock
- No more than 40% of the total investment can be made in stocks A and B (combined)
- No more than 75% of the total investment can be made in stocks C, D and E (combined)

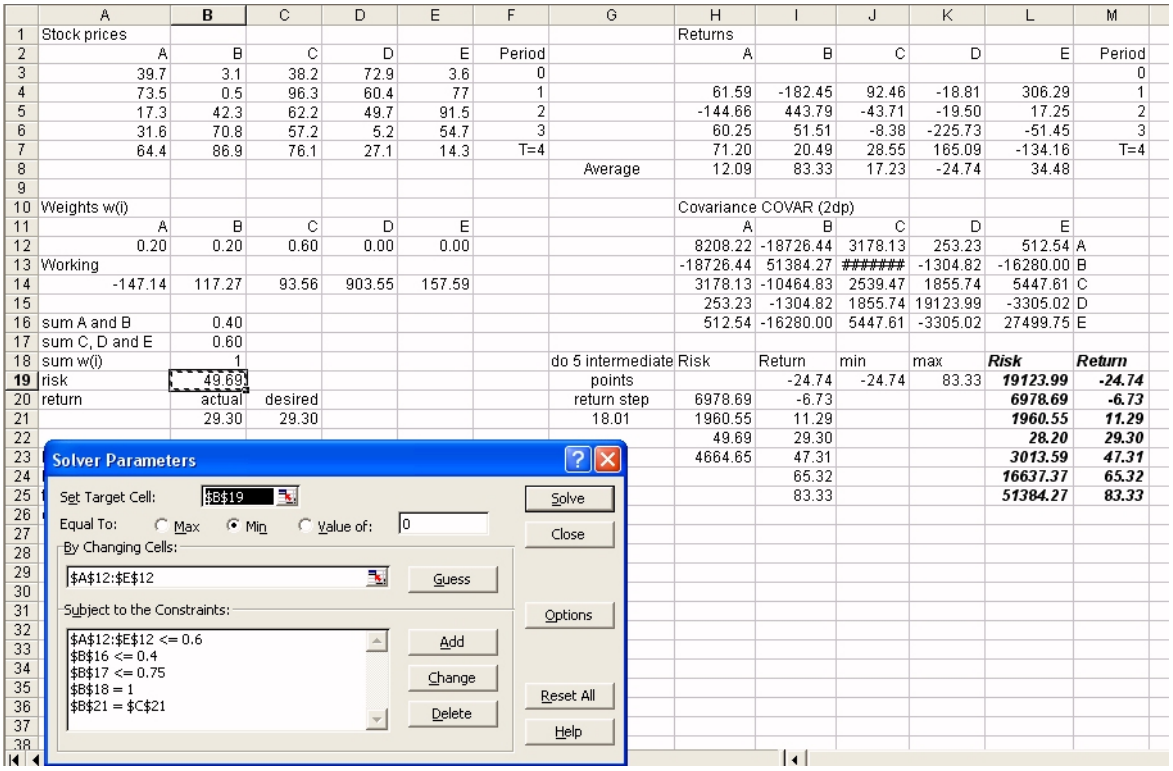
Compare this efficient frontier with the efficient frontier you get if these conditions are not imposed.

Markowitz extra constraints example solution

We have the standard Markowitz model (as in the lecture notes) but now the additional restrictions lead to the following constraints:

- No more than 60% of the total investment can be made in any stock
  $w_i \leq 0.6 \qquad i=A,B,\ldots,E$
- No more than 40% of the total investment can be made in stocks A and B (combined)
  $w_A + w_B \leq 0.4$
- No more than 75% of the total investment can be made in stocks C, D and E (combined)
  $w_C + w_D + w_E \leq 0.75$

A suitable spreadsheet is shown below



	A	B	C	D	E	F	G	H	I	J	K	L	M
1	Stock prices							Returns					
2	A	B	C	D	E	Period		A	B	C	D	E	Period
3	39.7	3.1	38.2	72.9	3.6	0							0
4	73.5	0.5	96.3	60.4	77	1		61.59	-182.45	92.46	-18.81	306.29	1
5	17.3	42.3	62.2	49.7	91.5	2		-144.66	443.79	-43.71	-19.50	17.25	2
6	31.6	70.8	57.2	5.2	54.7	3		60.25	51.51	-8.38	-225.73	-51.45	3
7	64.4	86.9	76.1	27.1	14.3	T=4		71.20	20.49	28.55	165.09	-134.16	T=4
8							Average	12.09	83.33	17.23	-24.74	34.48	
9													
10	Weights w(i)							Covariance COVAR (2dp)					
11	A	B	C	D	E			A	B	C	D	E	
12	0.20	0.20	0.60	0.00	0.00			8208.22	-18726.44	3178.13	253.23	512.54	A
13	Working							-18726.44	51384.27	#######	-1304.82	-16280.00	B
14	-147.14	117.27	93.56	903.55	157.59			3178.13	-10464.83	2539.47	1855.74	5447.61	C
15								253.23	-1304.82	1855.74	19123.99	-3305.02	D
16	sum A and B	0.40						512.54	-16280.00	5447.61	-3305.02	27499.75	E
17	sum C, D and E	0.60											
18	sum w(i)	1					do 5 intermediate	Risk	Return	min	max	***Risk***	***Return***
19	risk	49.69					points		-24.74	-24.74	83.33	***19123.99***	***-24.74***
20	return	actual	desired				return step	6978.69	-6.73			***6978.69***	***-6.73***
21		29.30	29.30				18.01	1960.55	11.29			***1960.55***	***11.29***
22								49.69	29.30			***28.20***	***29.30***
23								4664.65	47.31			***3013.59***	***47.31***
24									65.32			***16637.37***	***65.32***
25									83.33			***51384.27***	***83.33***
26													
27													
28													
29													
30													
31													
32													
33													
34													
35													
36													
37													
38													

Solver Parameters

Set Target Cell: $B$19

Equal To: Max Min Value of: 0

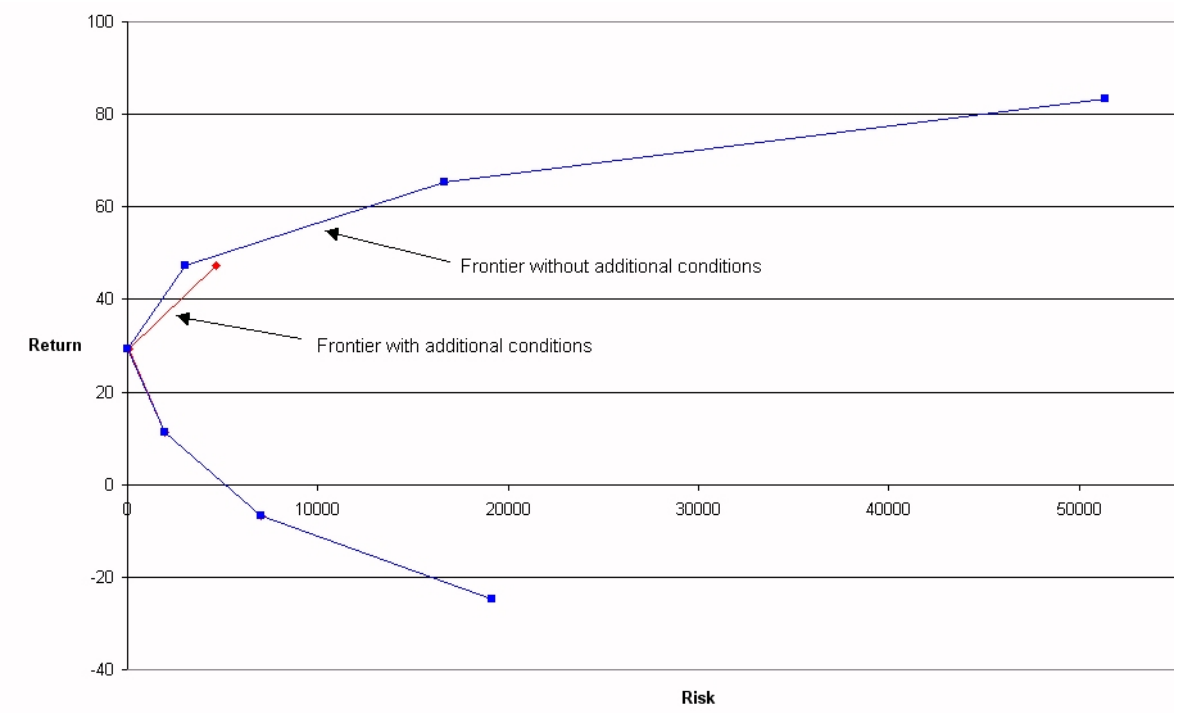
By Changing Cells: $A$12:$E$12

Subject to the Constraints:
$A$12:$E$12 <= 0.6
$B$16 <= 0.4
$B$17 <= 0.75
$B$18 = 1
$B$21 = $C$21

Solve | Close | Guess | Options | Add | Change | Delete | Reset All | Help

The efficient frontier (more strictly the entire frontier, including both efficient and inefficient parts) is shown below. Here the frontier that you get with these conditions not imposed is also shown. Note that at any given return level the point you get on the frontier must have an equal (or higher) risk as a result of imposing these additional conditions.

Note how some return levels (e.g. the higher levels) are not achievable given these additional conditions.


100
80
60
40
20
0
-20
-40
Return
0
10000
20000
30000
40000
50000
Risk
Frontier without additional conditions
Frontier with additional conditions