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Question 1

Shorting is where we sell a stock that we do not own in the hope of achieving a profit. To do this we would find someone who already owns some shares in the stock. We would borrow them from them and sell them now. The idea being that when we have to return them to whomever we borrowed them from the stock will have dropped in price and we can buy them in the open market for less than we sold them for. The risk is that when we buy in the market the price has risen and so we will have to pay more than we received when we sold them, meaning we make a loss

[5 marks]

For the index tracking model, with proportion constraints, and a transaction cost constraint let:

N be the total number of distinct stocks (companies) in which we can invest

K be the desired number of distinct stocks in the tracking portfolio (TP)

ε_i be the minimum proportion, and

δ_i be the maximum proportion, of the TP that must be held in stock i if any of stock i is held

X_i be the number of units of stock i in the current TP

V_{it} be the value of one unit of stock i at time t

T be such that we have observed historical values for stocks and the index over the time period 0,1,2,...,T. The time T represents a decision point, a time at which we may switch from our current TP [X_i] to a new TP.

I_t be the value of the index at time t

C be the total value of the current TP [X_i] at time T plus any cash change in the portfolio (either new cash available for investment or cash being withdrawn at time T), i.e.

$$C = \sum_{i=1}^{N} V_{iT}X_i + \text{cash change}$$

γ be the limit on the proportion of C that can be consumed by transaction cost

Decision variables are:

x_i the number of units of stock i that we choose to hold in the new tracking portfolio

z_i = 1 if any of stock i is held in the new tracking portfolio

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= 0 otherwise

Without significant loss of generality we allow $[x_i]$ to take fractional values.

The constraints associated with the index tracking problem are:

$\sum_{i=1}^{N} z_i = K$	choose exactly K stocks
$\varepsilon_i z_i \le V_{iT} x_i / C \le \delta_i z_i \quad i=1,...,N$	relate amount held to the zero-one variable
$C_{trans} = \sum_{i=1}^{N}$ transaction cost($X_i \rightarrow x_i$ at time T)	define transaction cost
$C_{trans} \le \gamma C$	limit transaction cost
$\sum_{i=1}^{N} V_{iT} x_i = C - C_{trans}$	everything invested, balance constraint
$x_i \ge 0 \quad i=1,...,N$	
$z_i \in [0,1] \quad i=1,...,N$	

Letting $R_t = \log_e(I_t/I_{t-1})$ and $r_t = \log_e[[\sum_{i=1}^{N} V_{it}x_i]/[\sum_{i=1}^{N} V_{it-1}x_i]]$, be the (continuous time) return on the index and tracking portfolio respectively our objective is:

minimise $\sum_{t=1}^{T} (r_t - R_t)^2/T$

i.e. minimise average squared error

[15 marks]

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Question 2

The payoff calculations are shown below

		Scenario			
Payoff	Choice	S1	S2	S3	Equally likely
	A	-17	59	19	20.33
	B	27	63	-16	24.67
	C	-12	-5	32	5.00

	Regret		
Choice	S1	S2	S3
A	44	4	13
B	0	0	48
C	39	68	0

Criteria	Decision	Value
Optimistic - maximax	B	63
Conservative - maximin	C	-12
Regret - minimax	A	44
Equally likely	B	24.67

Hence on a majority vote basis we should choose B

[13 marks]

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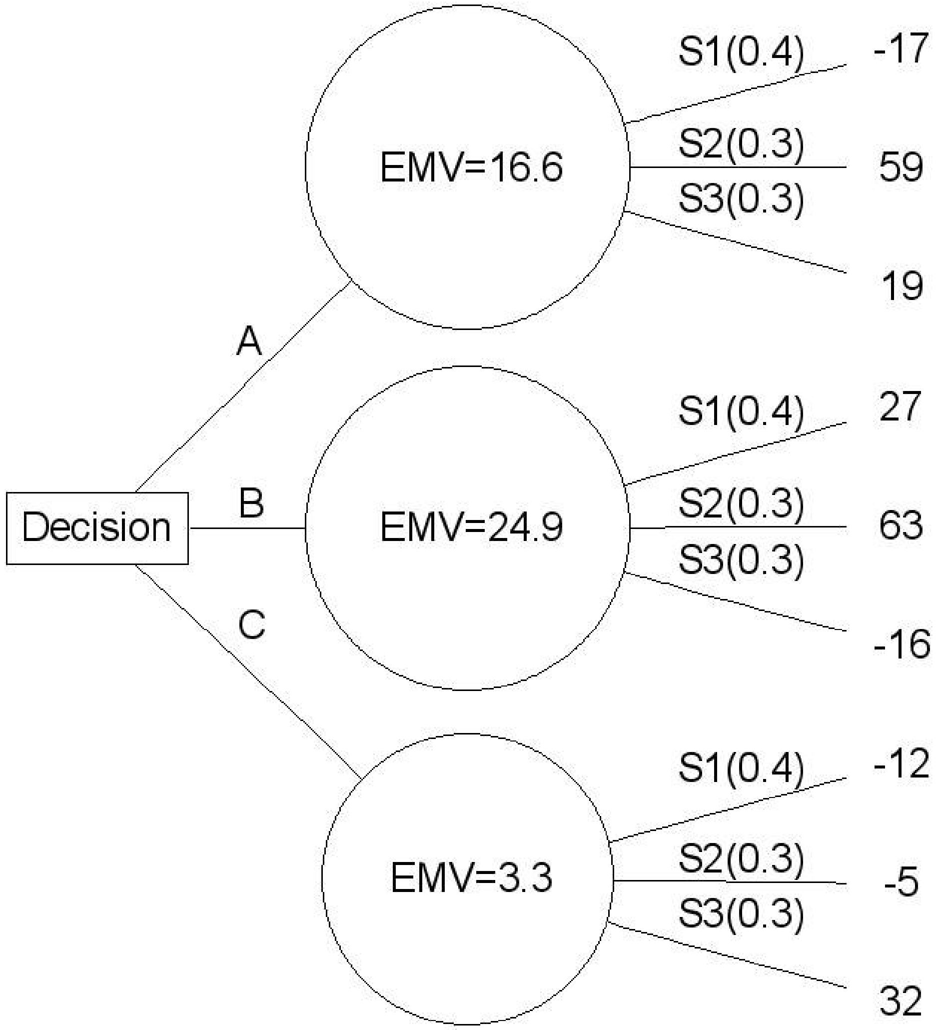
With the probability information it is possible to analyse the situation using the decision tree below



Hence the best solution here is to choose B with an EMV of 24.9

Note that this choice of B does not perturb the choice we made under the payoff criteria we considered above (which was also B).

[7 marks]

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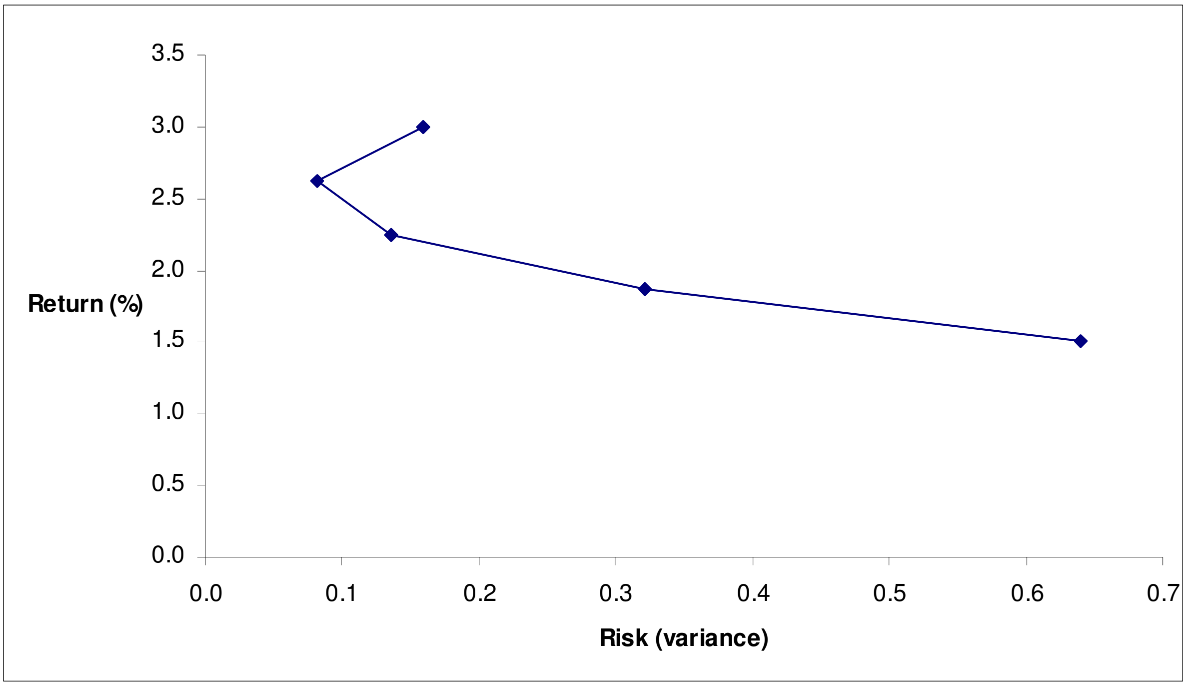
Question 3

For the pair AC we have:

Stock A weight	Stock C weight	Return	Risk
0	1	1.50	0.64
0.25	0.75	1.88	0.32
0.5	0.5	2.25	0.14
0.75	0.25	2.63	0.08
1	0	3.00	0.16



The efficient part of this frontier is just the two upper portfolios

[6 marks]

For the pair AB we have:

Stock A weight	Stock B weight	Return	Risk
0	1	2.00	1.00
0.25	0.75	2.25	0.57

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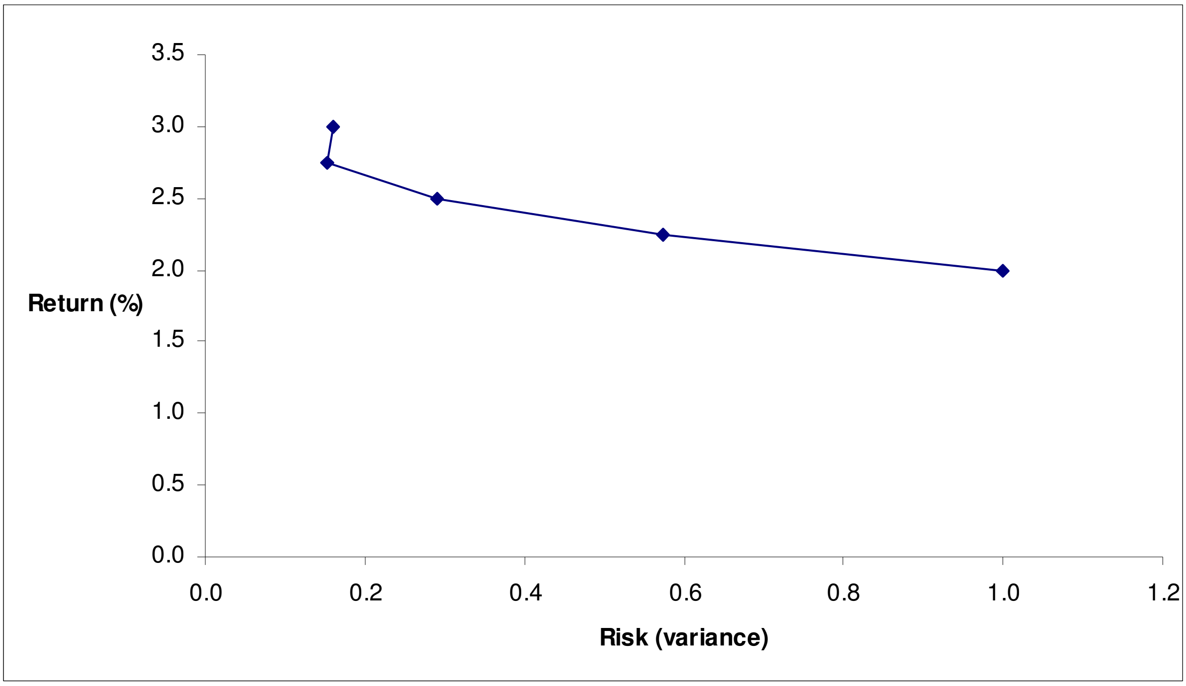
0.5	0.5	2.50	0.29
0.75	0.25	2.75	0.15
1	0	3.00	0.16



For the pair BC we have:

Stock B weight	Stock C weight	Return	Risk
0	1	1.50	0.64
0.25	0.75	1.63	0.27
0.5	0.5	1.75	0.21
0.75	0.25	1.88	0.45
1	0	2.00	1.00

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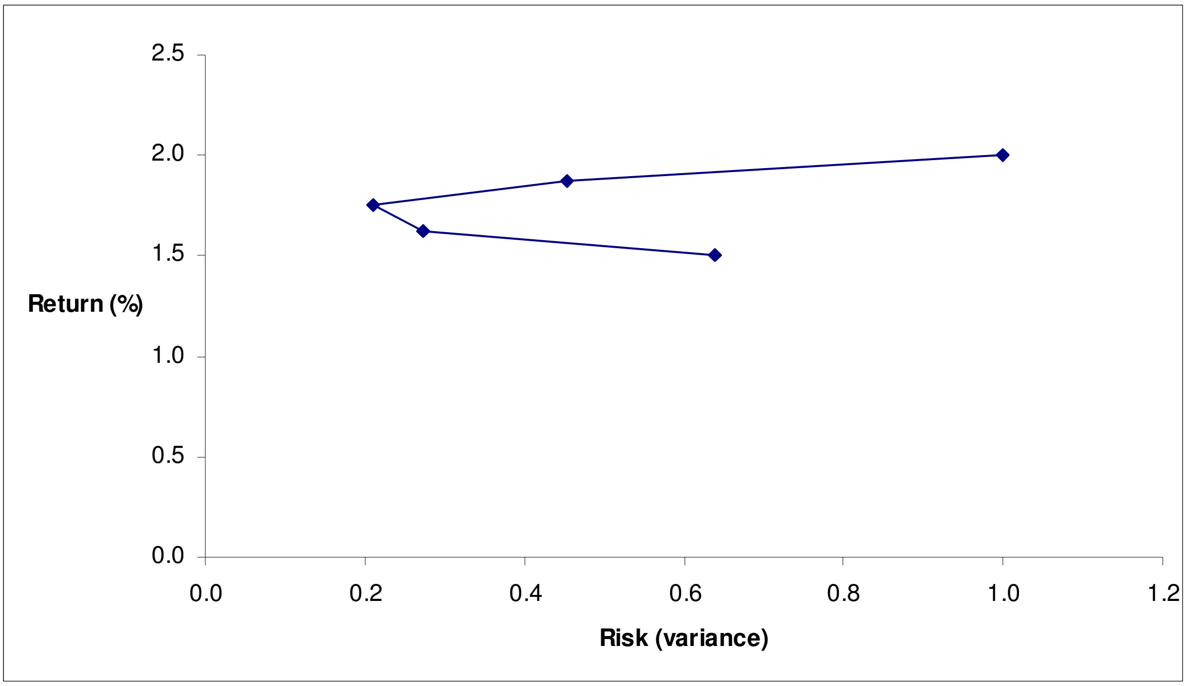
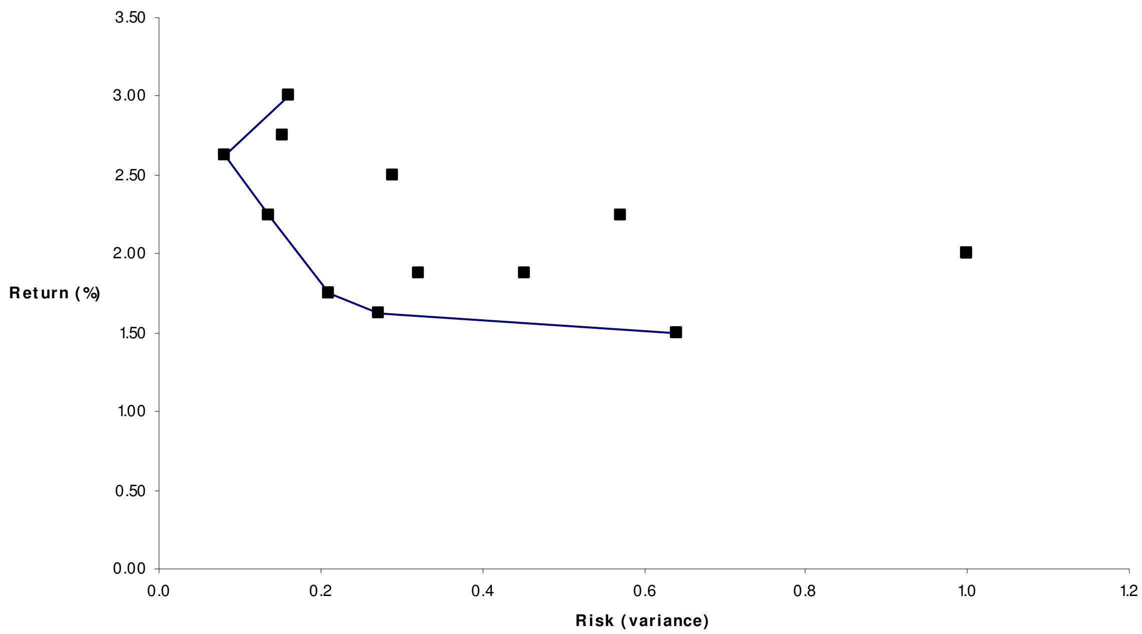


Plotting all these points on the same graph and connecting up the frontier we have:



The inefficient part of this frontier is just the four lower portfolios

[14 marks]

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Question 4

We compare the two numerically, as below:

Period	Index value	Portfolio value	Index return (%)	Portfolio return (%)	Difference
1	60.4	45329			
2	62.0	43631	2.615	-3.818	-6.432
3	68.5	42773	9.970	-1.986	-11.956
4	69.7	42687	1.737	-0.201	-1.938
		average	4.774	-2.002	
				Average squared error	62.693

Here we see the index has increased, but the portfolio has decreased, with the average squared error being high. Hence we could conclude that the portfolio is not tracking the index at all well (albeit we have only limited data).

[7 marks]

Let x_A, x_B, …., x_F be the zero-one variables taking the value one if the company takes the investment opportunity, zero otherwise.

The constraints are:

Budget:

$160x_A + 310x_B + 55x_C + 95x_D + 115x_E + 210x_F \leq 500$

Opportunities D and F are mutually exclusive, so if the company chooses opportunity D they cannot choose opportunity F (and vice-versa)

$x_D + x_F \leq 1$

If the company opportunity B they must also choose opportunity D

$x_D \geq x_B$

The objective is a maximisation objective:

maximise $5x_A + 9x_B + 1x_C + 2x_D + 8x_E + 6x_F$

[13 marks]