

Mathematical Sciences

BACHELOR DEGREE EXAMINATIONS

MAY 2011

MA3908

**APPLIED RISK AND OPTIMISATION IN FINANCIAL
PLANNING**

Time allowed: **THREE hours plus five minutes** reading time.

Graph paper will be provided

This paper consists of **FOUR** questions.

All questions carry equal marks.

Full marks may be obtained for satisfactory solutions to **THREE** questions.

If more than **THREE** questions are attempted the marks for the best **THREE** solutions will be counted.

An indication of the marks allocated to each sub-section of a question is in the right hand margin.

Question 1

Part a

What is meant by shorting? Explain how you might short a stock. What are the risks involved?

[5 marks]

Part b

Present a mathematical model for index tracking involving proportion constraints and a constraint upon transaction cost. Clearly explain your notation and the meaning of each constraint.

[15 marks]

Question 2

The payoff table below shows for a company the profit expected (£m) from three possible choices (A, B and C) with respect to a possible contract bid depending upon three scenarios for the forthcoming year (S1 to S3 respectively).

	S1	S2	S3
A	-17	59	19
B	27	63	-16
C	-12	-5	32

Here, for example, if the company makes choice A (and it can only choose one of A, B or C) and the scenario for the forthcoming year turns out to be S3 then it makes a profit of £19m. Using standard decision criteria what would you suggest the company does?

[13 marks]

Given the additional information that the probabilities for scenarios S1, S2 and S3 are estimated to be 0.4, 0.3 and 0.3 respectively it has been suggested that this situation could be analysed using a decision tree. Comment on this suggestion and, if appropriate, perform a decision tree analysis for this problem.

[7 marks]

Question 3

We have three stocks A, B and C that have the following (annual) returns and standard deviations in returns.

Stock	Return (%)	Standard deviation
A	3.0	0.4
B	2.0	1.0
C	1.5	0.8

The correlations between these three stocks have been estimated from data to be:

	A	B	C
A	1	0	-0.4
B	0	1	-0.5
C	-0.4	-0.5	1

So, for example, the correlation between stocks B and C is -0.5.

Plot the frontier (the curve representing the tradeoff between risk and return) that consists of just stocks A and C. Which portfolios on this frontier are efficient?

[6 marks]

Plot the frontier assuming that you are allowed to choose just two stocks to have in an investment portfolio. Which portfolios on this frontier are inefficient (dominated)?

[14 marks]

Note that throughout this question when plotting a frontier for any two stocks you should plot the frontier that results when the percentage of the investment allocated to any stock is 100%, 75%, 50%, 25% and 0%.

Question 4

Part a

The data below shows the value of an index, and of a tracking portfolio, over four time periods. How well do you think this tracking portfolio tracks the index?

Period	Index value	Portfolio value (£)
1	60.4	45329
2	62.0	43631
3	68.5	42773
4	69.7	42687

[7 marks]

Part b

A company has six possible investment opportunities, labelled A to F inclusive. The cost of each opportunity, and the company rating of the attractiveness of each opportunity, is:

Opportunity	Cost (£'000)	Rating
A	160	5
B	310	9
C	55	1
D	95	2
E	115	8
F	210	6

Here, for example, opportunity D would require investment of £95,000 and is rated 2. The higher the rating the more attractive the opportunity is. The company has some constraints relating to their choice of opportunity:

- opportunities D and F are mutually exclusive, so if the company choose opportunity D they cannot choose opportunity F (and vice-versa)
- if they choose opportunity B they must also choose opportunity D

The total budget that the company has to spend on opportunities is £0.5million.

Formulate the problem of choosing the opportunities in which to invest so as to optimise the total rating of the opportunities taken as an integer program with linear constraints.

[13 marks]