Index tracking – building on the basic Excel model

Previously we considered the mathematics for, and constructed an Excel sheet for, basic index tracking. In that basic model we assumed:

- we have zero transaction cost, i.e. buying and selling stocks costs us nothing
- we have no limits on the proportion of the tracking portfolio in each stock (i.e. ε_i=0 and δ_i=1)

Here we consider a number of uses of, and extensions to, this basic Excel model for index tracking.

K tradeoff
The first use we might make of the basic model is to investigate how the (optimised) objective function might change as K, the number of stocks in the index tracking portfolio, changes. Logically as K increases would you expect the optimised objective function value to decrease or not?

As K is our choice all we can really do is to see how the (optimised) objective function changes as K changes – in other word we have a tradeoff curve to construct.

Downside risk
In our basic model the objective was

$$\text{minimise} \sum_{t=1}^{T} (r_t - R_t)^2/T$$

so we desired to have r_t and R_t precisely the same in each and every time period t. In other words if, for example:

- r_t was below R_t by 1%

this was equally as bad

- r_t being above R_t by 1%

But in the real world are these two situations equivalent? If you were running an index tracking fund which of these two situations would you prefer?

This leads us on to what is known as ***downside risk*** (when r_t is below R_t). Here a simple mathematical way of capturing downside risk is:

$$\text{minimise} \sum_{t=1}^{T} \max\,[0, R_t - r_t]/T$$

When downside risk is calculated in this form then we are seeking to avoid following below index return, but are indifferent as to how much we exceed index return.

On a technical point here although we have divided by T in both of the expressions seen above (for average squared error and downside risk) this is not actually necessary. Because T is fixed dividing by it makes no difference to the optimal portfolio – it just effects (scales) the objective function value.

Proportions

We can add back into our basic model the proportion constraints that we previously neglected by assuming $\varepsilon_i=0$ and $\delta_i=1$, these constraints are:

$$\varepsilon_i z_i \leq V_{iT} x_i / C \leq \delta_i z_i \qquad i=1,\ldots,N$$

Sector constraints

An extension to constraining the proportion in any particular asset/stock is to deal with **sector constraints**. Typically this assumes that the assets can be classified as belonging to one of a number of sectors (e.g. energy, banking, telecommunications, etc) and then constraining the total investment in any sector.

Other constraints

Because we have a general mathematical approach it is easy to build in other constraints – for example if we have two assets (stocks) such that we can invest in one or the other but not both.

Transaction cost

We can add back into our basic model transaction cost that we previously neglected by assuming buying and selling stocks costs us nothing. If we do this then we include:

$$C_{trans} = \sum_{i=1}^{N} \text{transaction cost}(X_i \rightarrow x_i \text{ at time T})$$

$$C_{trans} \leq \gamma C$$

$$\sum_{i=1}^{N} V_{iT} x_i = C - C_{trans}$$

This last constraint replaces the constraint

$$\sum_{i=1}^{N} V_{iT} x_i = C$$

that we have before

For example we might assume that:

- transaction cost is 15 basis points, so we pay (15/100) of one percent = 0.0015 as a fraction of any value traded. So if we trade £10,000 of stock it costs us 10000(0.0015) = £15.
- γ is 0.001 so we are willing to sacrifice one tenth of one percent of the value of our portfolio in transaction cost in order to achieve (we hope, in the future) better tracking.