MA3908 Goal programming question

A company manufactures three products (A, B and C) using three machines (X, Y and Z). The time required (in minutes) to produce one unit of product on these machines is as shown below:

		Product		
		A	B	C
Machine	X	1.2	2.3	2.6
	Y	3.4	4.2	3.3
	Z	3.3	2.7	4.5

For example producing one unit of product A on machine X requires 1.2 minutes. Units of A can also be produced on machines Y and Z – each unit of A produced on machine Y requires 3.4 minutes, each unit of A produced on machine Z requires 3.3 minutes.

In the forthcoming production period it is estimated that there will be (at most) 2000 minutes available on machine X, 2340 minutes available on machine Y and 2435 minutes available on machine Z. These machines are very reliable, with only 1% of available time being lost due to machine breakdown.

Technological constraints mean that for every 7 units of A produced at most 10 units of B can be produced. Production costs are £1.1 for every minute worked on machine X, £1.13 for every minute worked on machine Y and £2.5 for every minute worked on machine Z. The company would like (if possible) to meet forecast demand which for the forthcoming production period is 3900 units of A, 4460 units of B and 770 units of C. Should it not be possible to meet forecast demand the most important product is A and the second most important product is B. The company regards meeting customer demand as much more important than the production cost involved.

Formulate this problem using a weighted goal programming approach.

How does your formulation change if you use a priority approach?

MA3908 Goal programming solution

Let x(i,j) be the number of units of product i (i=A,B,C) produced on machine j (j=X,Y,Z)

Of course we could use subscripts here, but it is perfectly legitimate to use a bracket notation if we prefer.

The production time constraint is:

$1.2x(A,X) + 2.3x(B,X) + 2.6x(C,X) \leq 0.99(2000) = 1980$ m/c X
$3.4x(A,Y) + 4.2x(B,Y) + 3.3x(C,Y) \leq 0.99(2340) = 2316.6$ m/c Y
$3.3x(A,Z) + 2.7x(B,Z) + 4.5x(C,Z) \leq 0.99(2435) = 2410.65$ m/c Z

The technological constraint is:

$x(B,X) + x(B,Y) + x(B,Z) \leq (10/7)[x(A,X) + x(A,Y) + x(A,Z)]$

For the demand constraint we could define over and under variables in this goal programming formulation. However the question implies that we are concerned with meeting demand, so falling short (under) the key factor. Hence here we only need under variables.

Let A^-, B^- and C^- be the under variables for products A, B and C respectively (where all of these variables are ≥ 0)

The demand constraints are then

$x(A,X) + x(A,Y) + x(A,Z) \geq 3900 - A^-$
$x(B,X) + x(B,Y) + x(B,Z) \geq 4460 - B^-$
$x(C,X) + x(C,Y) + x(C,Z) \geq 770 - C^-$

(note need ≥ here to preserve the flexibility to produce more than the demand).

The total cost is:

$F = 1.1[1.2x(A,X) + 2.3x(B,X) + 2.6x(C,X)] + 1.13[3.4x(A,Y) + 4.2x(B,Y) + 3.3x(C,Y)] + 2.5[3.3x(A,Z) + 2.7x(B,Z) + 4.5x(C,Z)]$

Here we need to set up a table relating for the under variables a weight associated with percentage deviation from the goal. Here we have three goals relating to the demand for each of the three products.

Variable	Current goal	Weight for one per cent deviation from this goal	One per cent of goal
A^-	3900	10	39
B^-	4460	5	44.6
C^-	770	2	7.7

Students can put any weights in here, but to satisfy the wording of the question the weight for A > the weight for B > the weight for C.

The objective here is then

Minimise $10(A^-/39) + 5(B^-/44.6) + 2(C^-/7.7)$

If we have a priority approach the first program to solve is:

Minimise A^-
subject to all the constraints seen above.

Let the solution to this be A* (say)

The second program to solve is then
Minimise B^-
subject to all the constraints seen above and $A^- = A^*$

Let the solution to this be B* (say)

The third program to solve is then
Minimise C^-
subject to all the constraints seen above and $A^- = A^*$ and $B^- = B^*$

Let the solution to this be C* (say)

The final program to solve is then
Minimise the total cost F
subject to all the constraints seen above and $A^- = A^*$ and $B^- = B^*$ and $C^- = C^*$