MA3908 Linear programming solution - questions

For the LP problems formulated previously (and for reference these are given below) use Solver in Excel to find the optimal solution for each LP.

Question

A company manufactures two products (A and B) and the profit per unit sold is £3 and £5 respectively. Each product has to be assembled on a particular machine, each unit of product A taking 12 minutes of assembly time and each unit of product B 25 minutes of assembly time. The company estimates that the machine used for assembly has an effective working week of only 30 hours (due to maintenance/breakdown).

Technological constraints mean that, for every five units of product A produced, at least two units of product B must be produced.

Formulate the problem of how much of each product to produce as a linear program.

Answer

Let

x_A = number of units of A produced

x_B = number of units of B produced

then the constraints are:

$12x_A + 25x_B \leq 30(60)$ (assembly time)

$x_B \geq 2(x_A/5)$ (technological)

i.e. $x_B - 0.4x_A \geq 0$

i.e. $5x_B >= 2x_A$

where $x_A, x_B \geq 0$ and the objective is maximise $3x_A + 5x_B$

Question

A food is manufactured by refining raw oils and blending them together. The raw oils come in two categories:

Vegetable oil:

- VEG1
- VEG2

Non-vegetable oil:

- OIL1
- OIL2
- OIL3

The prices for buying each oil are given below (in £/tonne)

VEG1	VEG2	OIL1	OIL2	OIL3
115	128	132	109	114

The final product sells at £180 per tonne. Vegetable oils and non-vegetable oils require different production lines for refining. It is not possible to refine more than 210 tonnes of vegetable oils and more than 260 tonnes of non-vegetable oils. There is no loss of weight in the refining process and the cost of refining may be ignored.

There is a technical restriction relating to the hardness of the final product. In the units in which hardness is measured this must lie between 3.5 and 6.2. It is assumed that hardness blends linearly and that the hardness of the raw oils is:

VEG1	VEG2	OIL1	OIL2	OIL3
8.8	6.2	1.9	4.3	5.1

It is required to determine what to buy and how to blend the raw oils so that the company maximises its profit. Formulate the above problem as a linear program.

Answer

Variables

We need to decide how much of each oil to use so let x_i be the number of tonnes of oil of type i used (i=1,...,5) where i=1 corresponds to VEG1, i=2 corresponds to VEG2, i=3 corresponds to OIL1, i=4 corresponds to OIL2 and i=5 corresponds to OIL3 and where $x_i \geq 0$ i=1,...,5

Constraints

- cannot refine more than a certain amount of oil

  $x_1 + x_2 \leq 210$

  $x_3 + x_4 + x_5 \leq 260$

- hardness of the final product must lie between 3.5 and 6.2

  $(8.8x_1 + 6.2x_2 + 1.9x_3 + 4.3x_4 + 5.1x_5)/(x_1 + x_2 + x_3 + x_4 + x_5) \geq 3.5$

  $(8.8x_1 + 6.2x_2 + 1.9x_3 + 4.3x_4 + 5.1x_5)/(x_1 + x_2 + x_3 + x_4 + x_5) \leq 6.2$

As written above the hardness constraints are non-linear but can be transformed to the two linear constraints:

$(8.8x_1 + 6.2x_2 + 1.9x_3 + 4.3x_4 + 5.1x_5) \geq 3.5(x_1 + x_2 + x_3 + x_4 + x_5)$

$(8.8x_1 + 6.2x_2 + 1.9x_3 + 4.3x_4 + 5.1x_5) \leq 6.2(x_1 + x_2 + x_3 + x_4 + x_5)$

i.e.

$3.5(x_1+x_2+x_3+x_4+x_5) \leq (8.8x_1+6.2x_2+1.9x_3+4.3x_4+5.1x_5) \leq 6.2(x_1+x_2+x_3+x_4+x_5)$

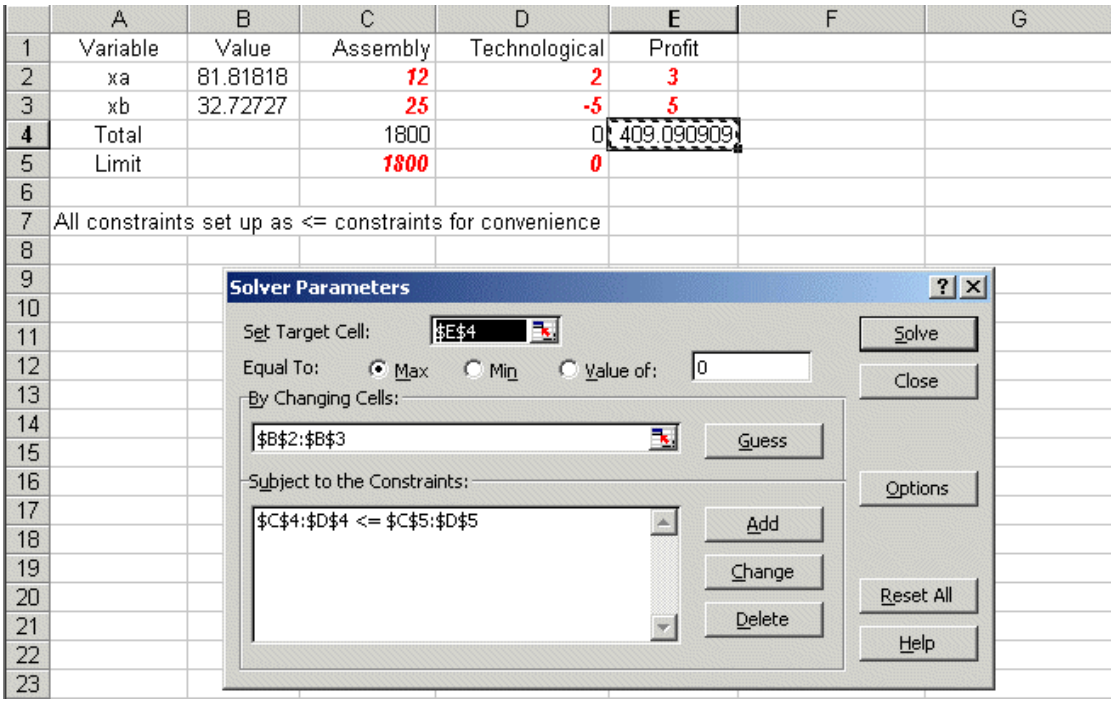
Objective

The objective is to maximise total profit, i.e.

maximise $180(x_1 + x_2 + x_3 + x_4 + x_5) - 115x_1 - 128x_2 - 132x_3 - 109x_4 - 114x_5$

MA3908 Linear programming solution - answers

An Excel sheet that shows the answer for the two questions is given below.



	A	B	C	D	E	F	G
1	Variable	Value	Assembly	Technological	Profit		
2	xa	81.81818	12	2	3		
3	xb	32.72727	25	-5	5		
4	Total		1800	0	409.090909		
5	Limit		1800	0			
6							
7	All constraints set up as <= constraints for convenience						
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	A	B	C	D	E	F	G
1	Variable	Value	Constraint 1	Constraint 2	Constraint 3	Constraint 4	Profit
2	VEG1 - x1	190	*1*	*0*	*-5.3*	*2.6*	*65*
3	VEG2 - x2	20	*1*	*0*	*-2.7*	*0*	*52*
4	OIL1 - x3	0	*0*	*1*	*1.6*	*-4.3*	*48*
5	OIL2 -x4	260	*0*	*1*	*-0.8*	*-1.9*	*71*
6	OIL3 -x5	0	*0*	*1*	*-1.6*	*-1.1*	*66*
7	Total		210	260	-1269	0.00	31850
8	Limit		*210*	*260*	*0*	*0*	
9							
10	All constraints set up as <= constraints for convenience						
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