

### Index tracking example question

The table below shows the stock prices and index values over a number of time periods, together with the current tracking portfolio.

Period	A	B	C	D	E	Index
0	39.7	3.1	38.2	72.9	3.6	2655.8
1	73.5	0.5	96.3	60.4	77	5039.7
2	17.3	42.3	62.2	49.7	91.5	4717.7
3	31.6	70.8	57.2	5.2	54.7	4946.7
T=4	64.4	86.9	76.1	27.1	14.3	6506.6
Current tracking portfolio	5	67	8	6	10	

For example in period 3 the stock/share price for stock A is 31.6 and the index value is 4946.7. The current tracking portfolio contains 5 units (shares) of stock A, 67 of B, etc.

Find tracking portfolios containing  $K=2, 3$  and 4 stocks.

How do these tracking portfolios compare with the current tracking portfolio in terms of their tracking performance.

Comment on anything you find noteworthy about your solution.

## Index tracking example solution

Utilising the Solver model as constructed in the lecture I get the following tracking portfolios:

	A	B	C	D	E	Error
K=2	0.00	0.00	87.98	13.42	0.00	0.047919325
K=3	0.00	34.63	53.21	0.00	0.00	0.016246539
K=4	9.11	36.75	43.08	0.00	0.00	0.014339518

The tracking error associated with the current tracking portfolio is 0.158849717, hence it is clear that (in terms of our measure of tracking performance as discussed in the lecture), the tracking portfolios above would (historically) have been much better than our current tracking portfolio.

Note here that for  $K=3$  we seem to have only two stocks in the tracking portfolio – whereas we wanted three. In fact this is implicit in the mathematics we presented. That set of mathematics (formulation of the index tracking problem as a nonlinear mixed-integer program) ensures that  $\sum_{i=1}^N z_i = K$ , where  $N=5$  and  $K=3$  here. If you look

closely at the full formulation we presented in the lecture you will see that (in the absence of any value for the minimum proportion of any stock that must be in the tracking portfolio) there is NOTHING (i.e. no constraints) to prevent  $z_i=1$  and  $x_i=0$ , i.e. nothing to prevent the  $z$  variable signifying we have a stock in the tracking portfolio being set equal to one whilst at the same time the  $x$  variable signifying how many units of the stock to buy being set equal to zero. This is precisely what has happened in the Solver solution for  $K=3$  (and indeed also for  $K=4$ ) above.

You may find that your solutions differ from those above. The essential reason for this is that (for the nonlinear mixed-integer problem we are solving) Solver is using a **heuristic** solution technique (unlike linear programming dealt with previously where we had an optimal solution technique). As such Solver cannot guarantee to find the optimal (best possible) solution.

Here the heuristic nature of the solutions given by Solver are clear. The solution presented above for  $K=4$  actually contains only three stocks. As such it is obviously a possible tracking portfolio in the event that we actually desired precisely three stocks. Now look at the tracking errors – the tracking error for the  $K=3$  portfolio found by Solver is actually larger than the tracking error for the  $K=4$  solution containing three stocks (recall we are seeking to minimise tracking error). Here therefore we actually have a clear indication that Solver cannot have possibly found the optimal solution for the  $K=3$  case since the solution given by Solver for  $K=3$  is worse than the solution shown above in the last row of the table.