MAFELAP 2019 abstracts for the mini-symposium High dimensional sampling and FE methods for UQ

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IMPROVED EFFICIENCY OF MULTI-INDEX FEM

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We propose a multi-index algorithm for the Monte Carlo (MC) discretization of a linear, elliptic PDE with affine-parametric input. We prove an error vs. work analysis which allows a multi-level finite-element approximation in the physical domain, and apply the multi-index analysis with isotropic, unstructured mesh refinement in the physical domain for the solution of the forward problem, for the approximation of the random field, and for the Monte-Carlo quadrature error. Our approach allows Lipschitz domains and mesh hierarchies more general than tensor grids. The improvement in complexity over multi-level MC FEM is obtained from combining spacial discretization, dimension truncation and MC sampling in a multi-index fashion. Our analysis improves cost estimates compared to multi-level algorithms for similar problems and mathematically underpins the superior practical performance of multi-index algorithms for partial differential equations with random coefficients.

MULTILEVEL QUASI-MONTE CARLO METHODS FOR RANDOM ELLIPTIC EIGENVALUE PROBLEMS

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Motivated by uncertainty quantification for the neutron diffusion criticality problem, we will study an elliptic eigenvalue problem with coefficients that depend on infinitely many stochastic parameters. The stochasticity in the coefficients causes the eigenvalues and eigenfunctions to also be stochastic, and so our goal is to compute the expectation of the minimal eigenvalue. In practice, to approximate this expectation one must: 1) truncate the stochastic dimension; 2) discretise the eigenvalue problem in space (e.g., by finite elements); and 3) apply a quadrature rule to estimate the expected value.

In this talk, we will present a multilevel Monte Carlo method for approximating the expectation of the minimal eigenvalue, which is based on a hierarchy of finite element meshes and truncation dimensions. To improve the sampling efficiency over Monte Carlo we will use a quasi-Monte Carlo rule to generate the sampling points. Quasi-Monte Carlo rules are deterministic (or quasi-random) quadrature rules that are well-suited to high-dimensional integration and can converge at a rate of $1/N$, which is faster than the rate of $1/\sqrt{N}$ for Monte Carlo. Also, to make each eigenproblem solve on a given level more efficient, we utilise two-grid scheme from to obtain the eigenvalue on the fine mesh from the coarse eigenvalue (and eigenfunction) with a single linear solve.

MULTILEVEL MONTE CARLO METHODS FOR BAYESIAN INVERSE PROBLEMS

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In this talk we will look at Bayesian static parameter estimation using multilevel Monte Carlo (MLMC) for partially observed diffusions.