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SHAPE OPTIMIZATION OF ACOUSTIC DEVICES USING CUTFEM AND LEVEL-SET GEOMETRY DESCRIPTIONS

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CutFEM is a recent framework for computations using non-body-conforming meshes [Burman, Claus, Hansbo, Larson, Massing, *Int. J. Numer. Meth. Engng.* 104, 472–501, 2015]. The framework is particularly useful in the context of shape optimization to evade the need for remeshing or deforming meshes. A less well-known added benefit with fixed meshes is that the exact expression for the shape derivative in the discrete case then is typically given by a surface integral over the design boundary. In contrast, when deforming meshes are used, the exact discrete shape derivative is more cumbersome to compute; it is given by a volume integral that explicitly involves the mesh deformation also in the interior of the domain.

In our approach to shape optimization in the CutFEM framework, the boundary is defined as the zero contour of a level-set function. The level-set function is, in turn, defined as the finite-element solution of a Poisson problem in which the nodal values of the right-hand side forcing function constitute the design variables that are actually updated by the gradient-based optimization algorithm. This conditioning of the design variables with the inverse Laplace operator enforces smooth design updates and can also be used for shape regularization by adding a Tikhonov term to the objective function.

In a recent application of this scheme, we consider the problem of designing, in full 3D, the internal geometry (the *phase plug*) of a *compression driver* aimed as the sound source for a mid-to-high-frequency acoustic horn in a public-address audio system. We rely on the FeniCS computing platform for the implementation. A particular feature of this application is that visco-thermal acoustic boundary-layer losses at solid walls are taken into account in the computations. These losses are modeled by a Wenztell boundary condition, that is, a generalized impedance (Robin) boundary condition involving a tangential diffusion operator. This condition is enforced at the boundary that cuts through the fixed background mesh in the CutFEM discretization. The presence of the tangential diffusion also affects the sensitivity analysis; since the geometry is only piecewise smooth, the jumps in the surface normal make additional contribution to the surface-based gradient expressions.

For the optimization, we chose a driver with a 2”-diameter membrane and with a phase plug consisting of 32 radial slits in circumferential symmetry, whose shape was designed using our approach. The optimization objective was to match the frequency response, in a least-squares sense, to a given idealized frequency response evaluated at 27 frequencies, 0.625–10 kHz. The target frequency response was accurately captured by the optimized design, and we concluded that the visco-thermal boundary layer losses are significant in the higher end of the spectrum and thus that the modeling of these are necessary to obtain realistic results.

SHAPE OPTIMIZATION FOR THE AUGMENTED LAGRANGIAN FORMULATION OF CONTACT PROBLEMS

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Structural optimization has become an integral part of industrial conception, with applications in more and more challenging mechanical contexts. Those contexts often lead to complex mathematical formulations involving non-linearities and/or non-differentiabilities, which causes many difficulties when considering the associated shape optimization problem.

In this talk, we get interested in finding the optimal design of an elastic body in frictional (Tresca) contact with a rigid foundation, taking into account a possible gap between the two. The proposed optimization algorithm is based on shape derivatives and a level-set representation of the shape, as introduced in [1], while the contact problem is written under its well known augmented Lagrangian formulation, see [2]. However, due to the projection operators involved in this formulation, the solution is not shape differentiable in general.

First, working with directional derivatives, we derive sufficient conditions for shape differentiability, and get an expression for the shape derivative of any generic functional. Then, some numerical results, obtained using the finite element method to solve the augmented Lagrangian formulation of the contact problem and finite differences for the advection of the level-set, are presented.

References

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FREE AND MOVING BOUNDARY PROBLEMS AND TRANSFINITE INTERPOLATIONS

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In imaging and numerical analysis a *transfinite interpolation* is a special case of an extension of a function defined on a closed subset E of the Euclidean n-dimensional space $\mathbf{R}^n$ to some larger subset of $\mathbf{R}^n$. We first generalize the construction and relax the assumptions of Dyken and Floater [*Transfinite mean value interpolation* (TMI), Computer Aided Geometric Design 26 (2009), 117–134] of an extension to an open domain Ω with compact Lipschitzian boundary and positive reach of a continuous function defined at its boundary $\partial\Omega$. Secondly, we introduce the new family of *k-Transfinite Barycentric Interpolation* (TBI) for compact H^d-rectifiable subsets E of $\mathbf{R}^n$ of arbitrary dimension $0 \leq d < n$. Modulo the specific requirements of the application at hand, it is both computationally simpler and mathematically more general than the TMI for which E is limited to the boundary of an open domain. Thirdly, dynamical versions of the TMI and the TBI are introduced to iteratively construct the rate of change of the position of the points of $\mathbf{R}^n \backslash E$ from the rate of change of the points of E as if E was a moving/deforming body in a fluid medium. This paper is motivated by pressing numerical and theoretical applications in the numerical analysis of free/moving boundary problems, *Arbitrary Lagrangian-Eulerian* (ALE) methods, and iterative schemes in shape/topological optimization and control.

MULTI-MATERIAL TOPOLOGY OPTIMIZATION BASED ON TOPOLOGICAL DERIVATIVES

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We present a topology optimization method for multiple materials which is based on the concept of topological derivatives. Here, the design, which consists of more than two materials, is represented in an implicit way by a vector-valued level set function. We show a sufficient optimality condition and an iterative algorithm which is based on this condition. Finally, we show numerical results for an academic example, where the the optimal design consists of an arbitrary, but fixed number of materials, as well as for a real world problem, the optimization of an electric motor consisting of three different materials.

NUMERICAL REALIZATION OF SHAPE OPTIMIZATION FOR UNSTEADY FLUID-STRUCTURE INTERACTION

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We consider shape optimization for unsteady fluid-structure interaction problems that couple the Navier-Stokes equations with non-linear elasticity equations. We focus on the monolithic approach in the ALE framework. It is obtained by transforming the time-dependent fluid domain to a fixed reference domain. Shape optimization by the method of mappings approach requires another transformation which maps the ALE reference domain to a nominal domain. This yields an optimal control setting and therefore can be used to drive an optimization algorithm with adjoint based gradient computation. The continuous formulation of the problem and the numerical realization are discussed. Numerical results for our implementation, which builds on FEniCS, dolfin-adjoint and IPOPT are presented.

SHAPE GRADIENTS IN FINITE ELEMENT EXTERIOR CALCULUS

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We blend two recent developments in numerical shape calculus under constraints imposed by second-order elliptic boundary value problems:

(I) The insight that the calculus of differential forms offers a powerful and unifying framework for computing shape derivatives in the sense of the velocity method [2, 1],

(II) The realization that shape gradients permit equivalent expressions based on either boundary expressions (Hadamard form) or volume integrals [4],

Hence, in the framework of exterior calculus we establish volume and boundary formulas for shape gradients. Subsequently, we generalize the duality techniques of [3] to show higher-order convergence of a Galerkin finite element approximation of the volume expressions based on discrete differential forms. This results covers the results of [3] as the case of 0-forms, and the case of shape gradients constrained by Maxwell's equations as the case of 1-forms.

References

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FIRESHAPE: A SHAPE OPTIMIZATION TOOLBOX FOR FIREDRAKE

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Fireshape is a shape optimization toolbox for the finite element library Firedrake. Fireshape can tackle optimization problems constrained by boundary value problems and offers the following features: decoupled discretization of control and state variables, automatic derivation of shape derivatives and adjoint equations, different metrics to define shape gradients, and numerous optimization algorithms. The shape optimization knowledge required to use this library is minimal.

Fireshape is available at https://github.com/fireshape/fireshape

SECOND ORDER DIRECTIONAL SHAPE DERIVATIVES OF INTEGRALS ON SUBMANIFOLDS

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We compute first and second order shape sensitivities of integrals on smooth submanifolds using a variant of shape differentiation. The result is a quadratic form in terms of one perturbation vector field that yields a second order quadratic model of the perturbed functional. We discuss the structure of this derivative, derive domain expressions and Hadamard forms in a general geometric framework, and give a detailed geometric interpretation of the arising terms.

AUTOMATED SHAPE DIFFERENTIATION IN FENICS AND FIREDRAKE

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While only relying on basic transformation rules, the calculation of shape derivatives is often a lengthy and error prone exercise. The reason for this is that even simple, linear PDEs or objectives are usually non-linear with respect to the domain under consideration.

We present a reformulation of shape derivatives in the context of finite elements as a classical Gâteaux derivative on the reference element. Based on this new approach, we show that the Unified Form Language (UFL), which underlies the popular finite element libraries Firedrake and FEniCS, can be extended to calculate first and higher order shape derivatives in an automated fashion.