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HOW TO SOLVE TRANSMISSION PROBLEMS WITH SIGN-CHANGING COEFFICIENTS: PART II. WHEN HYPER-SINGULAR BEHAVIORS APPEAR.

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In electromagnetic theory, the effective response of specifically designed materials can be modeled by strictly negative coefficients: these are the so-called negative materials. Transmission problems with discontinuous, sign-changing coefficients then occur in the presence of negative materials surrounded by classical materials. For general geometries, establishing Fredholmness of these transmission problems is well-understood thanks to the T-coercivity approach [3].
Let σ be a parameter that is strictly positive in some part of the computational domain (characterizing a classical material), and strictly negative elsewhere (characterizing the negative material). We focus on the scalar source problem in two dimensions: find u such that $\mathrm{div}(\sigma\nabla u)+\omega^2 u = f$ plus boundary condition, where f is some data and ω is the pulsation. Denoting by σ^+ the strictly positive value, and by σ^- the strictly negative value, one can prove that there exists a critical interval, such that the scalar source problem is well-posed in the Fredholm sense if, and only, if, the ratio σ^-/σ^+ lies outside the critical interval. The bounds of this critical interval depends on the shape of the interface separating the two materials.
When the ratio σ^-/σ^+ lies inside the critical interval, the problem is ill-posed due to hyper-singular solutions appearing at the corners of the interface. One can recover a well-posed formulation by taking into account those singularities in an extended framework.
We present two approaches to approximate the solution in the extended framework using finite element method. The first approach consists in rescaling the mesh according to the singularities' oscillations, and using Perfectly Matched Layers at the corners [2]. The second approach consists in explicitly computing the singularities, and subtract them from the solution to use standard meshes (Singular Complement Method, [1]).

The case where the ratio σ^-/σ^+ lies outside the critical interval is addressed in the talk given by P. Ciarlet "How to solve problems with sign-changing coefficients: part I. Classical theory".

References

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HOW TO SOLVE PROBLEMS WITH SIGN-CHANGING COEFFICIENTS: PART I. CLASSICAL THEORY

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In electromagnetic theory, the effective response of specifically designed materials can be modeled by strictly negative coefficients: these are the so-called negative materials. Transmission problems with discontinuous, sign-changing coefficients then occur in the presence of negative materials surrounded by classical materials. For general geometries, establishing Fredholmness of these transmission problems is well-understood thanks to the T-coercivity approach [2].
Let σ be a parameter that is strictly positive in some part of the computational domain (characterizing a classical material), and strictly negative elsewhere (characterizing the negative material). We focus on the scalar source problem: find u such that $\mathrm{div}(\sigma\nabla u) + \omega^2 u = f$ plus boundary condition, where f is some data and ω is the pulsation. Denoting by σ^+ the strictly positive value, and by σ^- the strictly negative value, one can prove that there exists a critical interval, such that the scalar source problem is well-posed in the Fredholm sense if, and only, if, the ratio σ^-/σ^+ lies outside the critical interval. The bounds of this critical interval depends on the shape of the interface separating the two materials [2, 1].
When the ratio σ^-/σ^+ lies outside the critical interval, one can apply the T-coercivity approach at the discrete level to solve the problems numerically. We propose a treatment which allows to design meshing rules for an arbitrary polygonal interface and then recover standard error estimates. This treatment relies on the use of simple geometrical transforms to define the meshes.

The case where the ratio σ^-/σ^+ lies inside the critical interval is addressed in the talk given by C. Carvalho "How to solve problems with sign-changing coefficients: part II. When hyper-singular behaviors appear".

References

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NUMERICAL METHODS FOR MAXWELL'S EQUATIONS WITH RANDOM POLARIZATION

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Electromagnetic wave propagation in complex dispersive media is governed by the time dependent Maxwell's equations coupled to equations that describe the evolution of the induced macroscopic polarization, so-called Auxiliary Differential Equations (ADE). We consider "polydispersive" materials represented by distributions of dielectric parameters in a polarization model. The work focuses on the novel computational framework for such problems introduced in [1], involving Polynomial Chaos Expansions as a method to improve the modeling accuracy of the ADE model and allow for easy simulation using standard numerical methods. We discuss generalizations of the approach and stability and dispersion analyzes of the resulting fully discrete schemes.

References

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DEVELOPMENT AND ANALYSIS OF FINITE ELEMENT AND FOURTH-ORDER FINITE DIFFERENCE METHODS FOR AN EQUIVALENT BERENGER'S PML MODEL

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In this paper, we continue our study of the equivalent Berenger's PML model formulated by Bécache and Joly in 2002. Here we will focus on developing and analyzing both finite element and high-order finite difference methods for solving the model. Numerical stability similar to the continuous model for both methods are established. Numerical examples implementing both methods are presented.

HYBRID SPECTRAL AND TIME DOMAIN NONLINEAR MAXWELL SOLVER

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In this talk, we report our recent progress on the development of a hybrid spectral and time domain method for solving Maxwell equations in complex media. Our goal is to be able to accurately model the electromagnetic wave propagation in general nonlinear media over a distance longer than hundreds or thousands of wavelengths. We apply the unidirectional pulse propagation equation (UPPE) to propagates the optical wave in spectral domain. The UPPE is derived from the Maxwell equations by assuming that the backward-scattered field can be neglected. In the region where we want to consider the backward-scattered field, we apply the finite difference time domain (FDTD) method. The FDTD method is a full vector Maxwell solver in time domain. To test the performance of our method, we simulate the optical pulse propagation in complex media with Lorentz dispersion and Kerr nonlinearity.