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NUMERICAL ANALYSIS FOR DIRICHLET CONTROL PROBLEMS

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An overview of discretization error estimates for Dirichlet control problems with L^2-regularization is presented. State and control are discretized by piecewise linear and continuous functions. Singularities of the solution at corners of the domain are taken into account by using graded finite element meshes. These singularities are different for unconstrained and control constrained problems.

OPTIMAL CONTROL PROBLEMS AND ALGEBRAIC FLUX CORRECTION SCHEMES

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Solutions of convection-diffusion-reaction equations may possess layers, i.e., narrow regions where the solution has a large gradient (in particular for convection dominated equations). Standard finite element methods lead to discrete solutions which are polluted by spurious oscillations. The main motivation for the construction of the so-called algebraic flux correction (AFC) schemes is the satisfaction of the DMP to avoid spurious oscillations in the discrete solutions. We apply an AFC scheme to an optimal control problem governed by a convection-diffusion-reaction equation. Due to the fact that the AFC schemes are nonlinear and usually non-differentiable the approaches "optimize-then-discretize" and "discretize-then-optimize" do not commute. We use the "optimize-then-discretize" approach, i.e., we discretize the state equation and besides the adjoint equation with the AFC method.

FINITE ELEMENT APPROXIMATIONS FOR AN OPTIMAL CONTROL PROBLEM RELATED TO BRUSSELATOR SYSTEM

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An optimal control problem related to the Brusselator system is considered. The Brusselator system consists of two coupled nonlinear reaction diffusion PDEs with different diffusion constants. A discontinuous (in time) Galerkin approach, combined with standard finite elements in space is considered for the discretization of the control to state mapping. Using properties of the discrete additive dynamics of the system, and a bootstrap argument, we establish stability bounds in the natural energy norm, with bounds depending polynomially upon the inverse of the diffusion constants. Special care is exercised to avoid any restriction between the temporal and spacial discretization parameters. In addition, a-priori error estimates are presented in the energy norm under a smallness assumption of on the size of the temporal discretization parameter. The error analysis also includes higher order schemes. Finally, we analyze a fully-discrete scheme for a related Robin boundary control problem. The Robin controls satisfy point-wise constraints. The state equation is discretized based on the lowest order discontinuous (in time) Galerkin scheme, while the controls are piecewise constants in space-time. Using the stability estimates under minimal regularity assumptions, we show convergence (in appropriate sense) of the associated discretized optimal control problem to the continuous one.

MULTIGOAL-ORIENTED ERROR CONTROL FOR OPTIMAL CONTROL PROBLEMS

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In this presentation, we consider an optimal control problem subject to a nonlinear PDE constraint such as the p-Laplace equation. We are interested in a posteriori error estimates for multiple quantities of interest. We combine all quantities to one and apply the dual-weighted residual (DWR) method to this combined functional. These a posteriori error estimates are then used for mesh adaptivity. In addition, the estimator allows for balancing the discretization error and the nonlinear iteration error. Several numerical examples demonstrate the excellent performance of our approach.

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OPTIMAL ERROR ESTIMATES FOR THE SEMI-DISCRETE OPTIMAL CONTROL PROBLEM OF THE WAVE EQUATION WITH TIME-DEPENDING BOUNDED VARIATION CONTROLS

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We will consider a control problem (P) for the wave equation with a standard tracking-type cost functional and a semi-norm in the space of functions with bounded variations (BV) in time. The considered controls in BV act as a forcing function on the wave equation with homogeneous Dirichlet boundary. This control problem is interesting for practical applications because the semi-norm enhances optimal controls which are piece-wise constant.

In this talk we focus on a finite element approximation of (P). In this semi-discretized version, only the state equation is discretized and the controls will not be changed. Under specific assumptions we can present optimal convergence rates for the controls, states, and cost functionals.

NUMERICAL ANALYSIS FOR THE OPTIMAL CONTROL OF SIMPLIFIED MECHANICAL DAMAGE PROCESSES

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In this talk we investigate a priori error estimates for the space-time Galerkin finite element discretization of an optimal control problem governed by a simplified damage model. The model equations are of a special structure as the state equation consists of a coupled PDE-ODE system. One difficulty for the derivation of error estimates arises from low regularity properties of solutions provided by this system. The state equation is discretized by a piecewise constant discontinuous Galerkin method in time and usual conforming linear finite elements in space. For the discretization of the control we employ the same discretization technique which turns out to be equivalent to a variational discretization approach. We provide error estimates both for the discretization of the state equation as well as for the optimal control. Numerical experiments are added to illustrate the proven rates of convergence.

MORLEY FEM FOR A DISTRIBUTED OPTIMAL CONTROL PROBLEM GOVERNED BY THE VON KÁRMÁN EQUATIONS

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Consider the distributed optimal control problem governed by the von Kármán equations that describe the deflection of very thin plates defined on a polygonal domain of $\mathbb{R}^2$ with box constraints on the control variable. The talk discusses a numerical approximation of the problem that employs the Morley nonconforming finite element method to discretize the state and adjoint variables. The control is discretized using piecewise constants. A priori error estimates are derived for the state, adjoint and control variables under minimal regularity assumptions on the exact solution. Error estimates in lower order norms for the state and adjoint variables are derived. The lower order estimates for the adjoint variable and a post-processing of control leads to an improved error estimate for the control variable. Numerical results confirm the theoretical results obtained.

This is a joint work with Sudipto Chowdhury and Devika Shylaja.

FINITE ELEMENT ERROR ESTIMATES IN L^2 FOR REGULARIZED DISCRETE APPROXIMATIONS TO THE OBSTACLE PROBLEM

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In this talk, we consider the standard obstacle problem in a convex and polygonally/ polyhedrally bounded domain. For the forcing term and the obstacle we assume for simplicity that they are arbitrarily smooth. As main result, we derive quasi-optimal error estimates in L^2 for a sequence of numerical approximations to the obstacle problem based on a regularization approach. More precisely, we obtain second order convergence in L^2 (up to logarithmic factors) with respect to the mesh size. The meshes are assumed to be quasi-uniform. The main ingredient for proving the quasi-optimal estimates is the structural and commonly used assumption that the obstacle is inactive on the boundary of the domain. No discrete maximum principle is required.

For instance, estimates of this kind may be needed when estimating the finite element error for optimal control problems subject to the obstacle problem.

FE ERROR ESTIMATES FOR SEMILINEAR PARABOLIC CONTROL PROBLEMS IN THE ABSENCE OF THE TIKHONOV TERM

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We study the FE discretization of a semilinear parabolic optimal control problem without the Tikhonov term. A priori error estimates for the FE discretization are derived based on a specific second-order sufficient optimality condition. This error estimate can be significantly improved for optimal controls with bang-bang structure. The theoretical results are illustrated by numerical experiments.

ERROR ESTIMATES FOR NORMAL DERIVATIVES AND DIRICHLET CONTROL PROBLEMS ON BOUNDARY CONCENTRATED MESHES

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The aim of this talk is to investigate the convergence behaviour of finite element approximations for the normal derivatives of the solution of the Poisson equation. In order to improve the convergence rates we use boundary concentrated meshes, i. e., meshes that are refined towards the boundary such that the diameters of elements touching the boundary are of order h^2, where h denotes the global mesh size. These meshes allow for a second-order approximation of normal derivatives in the $L^2(\Gamma)$-norm, provided that the solutions are sufficiently smooth. In the case that the computational domain is polygonal, corner singularities may reduce the regularity of solutions and thus also the convergence rates. We will observe that this is the case when an opening angle of a corner of the domain is larger than $120°$.

As an application of these results, we also investigate boundary control problems governed by elliptic equations for the case that the control is the Dirichlet datum. For these kind of problems, error estimates for the normal derivative of an adjoint state variable has to be proved. In general, solutions of these kind of problems are very irregular and hence, local mesh refinement is absolutely necessary to obtain reasonable solutions. Finally, we present numerical experiments which confirm that the predicted convergence results are sharp.

FINITE ELEMENT ERROR IN THE CONTROL OF DAMAGE PROCESSES

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Within this talk, we will discuss the finite element approximation of optimization problems subject to phase-field damage processes as constraints. To utilize standard (smooth) optimization algorithms, the irreversibility of the damage is relaxed by a penalty. We will discuss regularity of solutions to these regularized damage-models, and show that the regularity carries over to the limit in the regularization. Based on these new regularity results, we will discuss the approximability of the problem by finite elements.