

# MAFELAP 2019 abstracts for the mini-symposium FE for moving boundary problems: current approaches and applications

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# FINITE ELEMENT ANALYSIS OF A COUPLED SYSTEM OF VARIATIONAL INEQUALITIES MODELING BIOFILM GROWTH AT THE PORESCALE

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We consider a nonlinear system of diffusion-reaction PDEs for the evolution of biofilm and nutrient. Biofilm is a collection of microbial cells embedded in a protective extracellular polymer substance (EPS). While the model for biomass-nutrient dynamics is well-known, we consider the process at a micro-scale at which one recognizes a free boundary between the biofilm and the surrounding fluid through which nutrient and individual cells migrate. The model is motivated by the available micro-CT imaging data of the porescale invaded by microbial growth.

The coupled PDE system involves thus a free boundary modeled by a variational inequality. We describe the model and its Finite Element discretizations and present convergence proof and experiments both in the Galerking and mixed finite element setting.

# PHASE FIELD MODELLING FOR DISSOLUTION AND PRECIPITATION

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We consider a model for precipitation and dissolution in a porous medium. The ions transported by a fluid through the pores can precipitate at the pore walls. The mineral formed in this way can dissolve, increasing the amount of dissolved ions in the fluid. These processes lead to changes in the flow domain, which are not known a-priori but depend on the concentration of the solute.

One possible approach is to consider the fluid and mineral phases as different phases, separated by an interface that moves in time depending on the model unknowns. Here we discuss an alternative approach, based on a phase field variable having a smooth, diffuse transition of non-zero width from the fluid into the mineral phase. The evolution of the phase field variable is determined through the Allen-Cahn equation.

We first show that as the width of the diffuse transition zone approaches zero, the sharp-interface formulation is recovered. Then, considering a periodically perforated domain mimicking a porous medium, we employ homogenization techniques to derive an upscaled model, valid at the Darcy scale. This involves solving so-called cell problems, providing the effective diffusion and permeability matrices that are depending on the phase field variable.

Finally, we extend this approach to non-periodic media, and propose an adaptive upscaling procedure. Coupled with a linearisation scheme, this becomes an efficient numerical homogenisation scheme for simulating such multi-scale processes involving freely moving interfaces at the micro-scale.

## References

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## EFFICIENT SIMULATION OF LINEAR AND NONLINEAR POROELASTICITY

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Poroelasticity, i.e. fully coupled porous media flow and mechanics has many societal important applications including geothermal energy, enhanced oil recovery or  $CO_2$  storage. A typical mathematical model for poroelasticity is the quasi-static, linear Biot model, see e.g. [3]. Nevertheless, the linearity assumption is not valid in many practical situations and extension of the model should be considered.

In this work we present efficient numerical schemes for the linear and nonlinear Biot models [1]. The Bulk modulus (Lame coefficient) and the fluid compressibility are nonlinear functions satisfying certain assumptions. We use the  $L$ -scheme, see e.g. [4, 5] or the Newton method for linearization, either monolithically or combined with a fixed stress type splitting [2, 3, 6]. Additionally, the optimisation of the stabilisation parameter in the fixed-stress scheme will be discussed [7].

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# NUMERICAL SOLUTION OF TWO DIMENSIONAL TUMOUR GROWTH WITH MOVING BOUNDARY

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The two-phase tumour growth problem in two dimensions is a coupled system of a hyperbolic, elliptic and parabolic partial differential equations that respectively model volume fraction, velocity and pressure of tumour cells, and nutrient concentration within the tumour tissue. We present a numerical technique that overcomes the challenges associated with the time-dependent boundary of a growing tumour. The hyperbolic equation is extended to a fixed domain, which encompasses all time-dependent domains up-to a fixed time, without applying any domain transformation. This extension correctly embeds the dynamics of the moving boundary. A finite volume - finite element method is used to solve the system where the tumour boundary is recovered by locating the discontinuity in the volume fraction of the tumour cells. The new method overcomes the re-meshing issues associated with numerically solving similar moving boundary problems. The ability of the current technique to predict the evolution of irregularly shaped tumours, and thus relaxing the assumption of radially symmetric growth in previous works, is a novelty and an advantage.