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## FITTED ALE SCHEME FOR TWO-PHASE NAVIER–STOKES FLOW

Marco Agnese[a] and Robert Nürnberg[b]

Department of Mathematics, Imperial College London, UK
[a]m.agnese13@imperial.ac.uk, [b]robert.nurnberg@imperial.ac.uk

We present a novel fitted ALE scheme for two-phase Navier–Stokes flow problems that uses piecewise linear finite elements to approximate the moving interface. The meshes describing the discrete interface in general do not deteriorate in time, which means that in numerical simulations a smoothing or a remeshing of the interface mesh is not necessary.

## ESTIMATING ERRORS IN QUANTITIES OF INTEREST IN THE CASE OF HYPERELASTIC MEMBRANE DEFORMATION

Eleni Argyridou

Department of Mathematics, Brunel University, Uxbridge, UB8 3PH, UK.
Eleni.Argyridou@brunel.ac.uk

The implementation of the finite element method is described for the inflation of a thin sheet modelled as a membrane. The thin sheet is assumed to be a hyperelastic material. As well as describing how to approximately solve this problem for a sequence of increasing pressures we also outline work in progress to attempt to estimate a given quantity of interest $J(u)$ to a given accuracy where $J(.)$ denotes a functional and where $u$ denotes the exact solution. In the application $u$ denotes the displacement of the mid-surface of the membrane. With $u_h$ being our finite element approximation of $u$ and with $J(u_h)$ being our estimate of $J(u)$ we outline how to estimate $J(u) - J(u_h)$ by solving a dual problem. We consider this in the case of a quasi-static deformation when we only have space discretization errors and we also consider this in the dynamic case when we have time discretization errors as well. Results will be presented in the case of axisymmetric deformations.

## CROSS-DIFFUSION SYSTEMS FOR IMAGE PROCESSING

Adérito Araújo[1], Sílvia Barbeiro[1], Eduardo Cuesta[2] and Ángel Durán2

[1] CMUC, Department of Mathematics, University of Coimbra, Portugal
alma@mat.uc.pt, silvia@mat.uc.pt

[2]Department of Applied Mathematics, E.T.S.I. of Telecomunication, University of Valladolid, Spain
eduardo@mat.uva.es, angel@mac.uva.es

Diffusion processes are commonly used in image processing [3]. In particular, complex diffusion models have been successfully applied in medical imaging denosing [1], [2], [4]. The interpretation of a complex diffusion equation as a cross-diffusion system motivates the introduction of more general models of this type and their study in the context of image processing. In this talk we will discuss the use of nonlinear cross-diffusion systems to perform image restoration. The use of two scalar fields has the goal of distributing the features of the image and governing their relations. In this talk, special attention will be given to the well-posedness, scale-space properties and long time behaviour of the models. From a numerical point of view, a computational study of the performance of the models is carried out, suggesting their diversity and potentialities to treat image filtering problems. Examples of application will be highlighted.

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# A-POSTERIORI ERROR ESTIMATION OF DISCONTINUOUS GALERKIN METHODS FOR APPROXIMATELY-DIVERGENCE-FREE CONVECTION-DIFFUSION PROBLEMS

Samuel P. Cox[1a], Andrea Cangiani[1b] and Emmanuil H. Georgoulis[1,2]

[1]Department of Mathematics, University of Leicester, UK,
[a]spc29@leicester.ac.uk, [b]Andrea.Cangiani@le.ac.uk

[2]Physical Sciences, National Technical University of Athens, Zografou 15780, Greece.
Emmanuil.Georgoulis@le.ac.uk

Mantle convection is often modelled by a stationary Stokes system coupled to a time-dependent convection-diffusion equation for the temperature variable. Given the size of the resulting models, *a posterori* error estimators are highly desirable for the control of adaptive FE schemes in order to reduce the solution cost.

In a system containing some reaction, the reaction can typically be used to handle the convective term. However, since we have no reactive term, this option is not open to us, and so we proceed with an exponential-fitting method. Meanwhile, the numerical solution of the Stokes system may yield a convection field that is only *approximately* divergence-free. We present a derivation of an *a posteriori* error estimator for the discontinuous Galerkin discretisation of a time-dependent convection-diffusion equation with varying, nearly-divergence-free convection, based on an exponential-fitting method, along with numerical experiments to show the suitability of the error estimator.

# FINITE ELEMENT MODELING FOR COLD ROLLING OF ALUMINIUM A1200

Oluleke Oluwole[1], Aworinde A. Kehinde[2],
Emagbetere Eyere[3a] and Ahiedu I. Festus[3b]

[1] Department of Mechanical Engineering, University of Ibadan, Ibadan, Nigeria
oluwoleo2@asme.org

[2] Department of Mechanical Engineering,
Covenant University, Canaan Land, Ota, Nigeria
patientkenny@gmail.com

[3] Department of Mechanical Engineering,
Federal University of Petroleum Resources, Effurun, Delta State, Nigeria
[a] emagbetere.eyere@fupre.edu.ng, [b]ashiedu.ifeanyi@fupre.edu.ng

Sequel to losses of sheet surface integrity and dimensional accuracy observed in the cold rolling of aluminium A1200, the effect of tensional forces at mandrel, roll velocity and contact angle; on the stress distribution of the Aluminium strips and roll torque were investigated. Experiments were conducted at a four-high reversible Aluminium rolling mill. Thereafter, the Aluminium sheet was modelled for three passes of cast coil reduction from 7.0mm to 2.2mm using the Elastoplastic model with Von-Mises yield criteria and Perfectly Plastic model for hardening. The geometries were finely meshed using free quadrilateral. The roll velocities were applied as prescribed velocities and the tensional force as boundary load. The models were run on the COMSOL GUI to determine stress distributions and hence the roll force and roll torques. Simulation results compared favourably with that of experiments. Results showed that tensional forces applied at the mandrels during rolling, were higher than required as the sheet inlet thickness got smaller. The magnitudes of the roll torque were found to be strongly dependent on the amounts of draft and roll velocity for each passes. The arc length of contact was also found to be a strong leading parameter. Finite element analysis was effectively used to determine the effect of tensional forces at mandrel and roll velocities on the stress distribution in the Aluminium model during cold rolling.

## HYBRID NUMERICAL ASYMPTOTIC BOUNDARY ELEMENT METHOD FOR MULTIPLE SCATTERING PROBLEMS

Andrew Gibbs[a], Simon Chandler-Wilde, Steve Langdon and Andrea Moiola

Department of Mathematics and Statistics, University of Reading, UK,
[a]a.j.gibbs@pgr.reading.ac.uk

Standard numerical schemes for scattering problems have a computational cost that grows at least in direct proportion to the frequency of the incident wave. For many problems of scattering by single obstacles, it has been shown that a careful choice of approximation space, utilising knowledge of high frequency asymptotics, can lead to numerical schemes whose computational cost is independent of frequency. Here, we extend these ideas to multiple scattering configurations, focusing in particular on the case of two scatterers, with one much larger than the other.

## A DTN FINITE ELEMENT METHOD FOR AXISYMMETRIC ELASTICITY IN SEMI-INFINITE DOMAINS

Eduardo Godoy[1] and Mario Durán[2]

[1]INGMAT R&D Centre, Chile
eduardo.godoy@ingmat.com

[2]INGMAT R&D Centre, Chile
mario.duran@ingmat.com

In some problems arising in geophysical applications, the solid earth is mathematically modelled as an elastic semi-infinite domain. In general, to solve numerically a boundary-value problem formulated in an elastic domain, the finite element method appears to be very convenient. However, it cannot be directly applied if the involved domain is unbounded. A good alternative to overcome this drawback is to use the Dirichlet-to-Neumann (DtN) map in order to deal with the unboundedness. The DtN map provides, on an artificial boundary of regular shape, exact boundary conditions, which may be combined with a finite element discretisation of the bounded computational domain lying inside the artificial boundary. Such a procedure is known as the DtN finite element method, and it has been successfully applied to different problems formulated in infinite exterior domains, since in this case it is usually possible to compute an explicit closed-form expression for the DtN map. However, in the case of a semi-infinite elastic domain this is not, in general, possible. For this reason, the use of the DtN finite element method in geophysical applications has been rather limited.

In this work, we present a DtN finite element method for solving boundary-value problems of elasticity formulated in a locally perturbed half-space with axisymmetry about the vertical axis. The lack of a closed-form expression for the DtN map is remedied by employing an approximation procedure that combines numerical and analytical

computation techniques. Firstly, the locally perturbed half-space is truncated by means of a semi-spherical artificial boundary, dividing it into a bounded computational domain and a semi-infinite residual domain. Then, a finite element formulation of the elasticity problem is established in the computational domain, taking into account the exact boundary conditions on the artificial boundary provided by the DtN map. As it is not possible to obtain a closed-form expression for the DtN map, we approximate only those boundary integral terms occurring in the finite element formulation that involve precisely the DtN map. To do so, the boundary-value problem in the residual domain is solved by a semi-analytical technique, just for the required Dirichlet data on the artificial boundary. By applying Boussinesq potentials and separation of variables, the solution is expressed as a series with unknown coefficients, which are approximated by minimising a quadratic energy functional appropriately chosen. The minimisation yields a symmetric and positive definite linear system of equations for a finite number of coefficients, which is efficiently solved by exploiting its particular block-structure, in such a way that the coefficients of the series are in practice computed by mere forward and backward substitution. This procedure allows an approximate but very effective coupling of the DtN map with the finite element method for the semi-infinite elastic problem under study. The procedure is validated by solving a particular case where an exact solution is available, using structured triangular meshes of different sizes. The relative error between the numerical and the exact solution is calculated for each mesh size considered, corroborating the effectiveness and accuracy of the proposed procedure. Indeed, the numerical evidence shows that it achieves second-order accuracy.

## EQUIVALENT OPERATOR PRECONDITIONING FOR ELLIPTIC FINITE ELEMENT PROBLEMS

János Karátson[1] and Owe Axelsson[2]

[1]Institute of Mathematics, ELTE University Budapest, Hungary
karatson@cs.elte.hu

[2]Institute of Geonics AS CR, IT4 Inovations, Ostrava, The Czech Republic
owea@it.uu.se

A class of efficient preconditioners for discretized elliptic problems can be obtained via equivalent operator preconditioning. This means that the preconditioner is chosen as the discretization of a suitable auxiliary operator that is equivalent to the original one, see, e.g., [1, 2, 3]. Under proper conditions one can thus achieve mesh independent convergence rates. Hence, if the discretized auxiliary problems possess efficient optimal order solvers (e.g. of multigrid type) regarding the number of arithmetic operations, then the overall iteration also yields an optimal order solution, i.e. the cost $O(N)$ is proportional to the degrees of freedom.

The talk is based on the joint work of the authors, see, e.g., [4, 5, 6]. First some theoretical background is summarized, including both linear and superlinear mesh independent convergence, then various applications are shown. The results can be applied, among other things, for parallel preconditioning of transport type systems, for streamline diffusion preconditioning of convection-diffusion problems, and to achieve superlinear convergence under shifted Laplace preconditioners for Helmholtz equations.

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## A COUPLED WAVE-DIFFUSION MODEL FOR ENHANCED DRUG DELIVERY


José Augusto Ferreira[a], Daniela Jordão[b] and Luís Pinto[c]

CMUC, Department of Mathematics, University of Coimbra, Portugal
[a]ferreira@mat.uc.pt, [b]daniela.jordao@hotmail.com,
[c]luisp@mat.uc.pt


Enhanced and controlled delivery of molecules (e.g. drug, proteins, DNA) into cells with the aid of electric fields is a hot topic in molecular medicine. In this context, the role of the electric field is two-fold: one, to permeabilize the cellular membrane allowing the introduction of the molecules into the cell, a process known as electroporation; two, to advance and control the migration of the charged molecules into the cell. This last technique, known as electrophoresis, is particularly crucial when the cells have a short resealing time or when large molecules need to be loaded into cells.

The mathematical modeling of these biological and physical processes involves two main equations, a parabolic convection-diffusion equation that describes the evolution of the concentration of molecules, in and outside the cell, and Maxwell's equations for the electromagnetic waves. Ignoring the magnetic field, the Maxwell system can be reduced to a wave equation for the electric field.

In this work we study a finite difference method for the wave-diffusion coupled problem that is based on piecewise linear finite element approximations. Second order error estimates with respect to a discrete $H^1$-norm are established provided that, for each time level, the solution of the coupled problem is in $H^3(\Omega) \times H^3(\Omega)$, where $\Omega$ is the spatial domain.

# A FINITE ELEMENT FORMULATION FOR MAXWELL EIGENVALUE PROBLEM USING CONTINUOUS LAGRANGIAN INTERPOLATIONS

Önder Türk[1], Ramon Codina[2] and Daniele Boffi[3]

[1]Gebze Technical University, Gebze/Kocaeli, Turkey
onder.turk@yandex.com

[2]Universitat Politècnica de Catalunya, Barcelona, Spain
ramon.codina@upc.edu

[3]Università di Pavia, Pavia, Italy
daniele.boffi@unipv.it

In this work, we consider the stabilized finite element formulation based on the subgrid scale concept for solving the Maxwell eigenvalue problem. The application of a stabilization technique based on a projection of the residual to an eigenproblem, leads to a system resulting in a quadratic eigenvalue problem. As a consequence, eigenpairs which are not solutions of the original problem are introduced, and a considerable increase in complexity of the problem is involved. In this study, the unresolved subscales are taken to be orthogonal to the finite element space. Thus, the components leading to a quadratic structure vanish, the residual is simplified, and the implementation of term by term stabilization is allowed. Moreover, the method allows the use of continuous Lagrangian interpolations. Apart from its novelty, we show that the approach is essential to establish the original structure of the eigenproblem. We present the problem formulation, and provide some numerical results from the solution of the Maxwell eigenvalue problem on two-dimensional regions. The numerical results we have obtained from the formulation described above, demonstrate a very good agreement with the previously published results.

# NUMERICAL SOLUTION OF NONLOCAL PROBLEMS

Andrea Živčáková[a] and Václav Kučera[b]

Department of Numerical Mathematics, Faculty of Mathematical and Physics,
Charles University in Prague, Czech Republic
[a]zivcakova@karlin.mff.cuni.cz, [b]kucera@karlin.mff.cuni.cz

Classical differential equations are formulated using derivatives of various orders which are local operators, i.e. defined using only local properties of the function. The solution process is then nonlocal, where e.g. a local change of boundary conditions affects the solution in the entire domain or an open subset thereof. However there are equations, where even the problem formulation is nonlocal. A classical example are fractional differential equations. More recently, a nonlocal differential calculus was devised by Gunzburger which gives a description of various nonlocal phenomena such as nonlocal diffusion or convection-diffusion with interesting applications. Efficient solution of such problems is very challenging. Our interest in this subject originally comes from the solution of a model of flocking dynamics using the discontinuous Galerkin method.