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Abstracts in alphabetical order

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ADVECTION-DIFFUSION EQUATIONS WITH RANDOM COEFFICIENTS ON MOVING HYPERSURFACES

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Sometimes the partial differential equations with random coefficients can be better formulated on moving domains, especially in biological applications. We will introduce and analyse the advection-diffusion equations with random coefficients on moving hypersurfaces. Since we will consider evolving domains, for the definition of the Bochner type solution space we will use the approach that transforms the equation onto a fixed reference domain. Under suitable regularity assumptions, using Banach-Nečas-Babuška theorem, we will prove existence and uniqueness of the weak solution and also we will give some regularity results about the solution. For discretization in space, we will apply the evolving surface finite element method to the weak form of the equation for which we approximate the hypersurface by an evolving interpolated polyhedral surface. Numerical approximation of uncertainty is performed by the Monte-Carlo-Method. We plan to illustrate our theoretical findings by numerical computations.

ADAPTIVE STOCHASTIC GALERKIN FEM WITH HIERARCHICAL TENSOR REPRESENTATIONS

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Parametric PDEs have gained a lot of attention in recent years, especially in the context of uncertainty quantification (UQ) where the parameters are random variables. Since in practice often many parameters of the considered problems cannot be determined precisely or are stochastic by nature, modelling and simulation with uncertain data is particularly relevant with engineering applications.

Spectral methods for PDEs with random data are based on the functional representation of the solution manifold in some polynomial chaos basis, including all dependencies on the stochastic parameters of the model. While the implementation of these numerical methods can be more involved than popular sampling techniques such as Monte Carlo and its more advanced variants, they potentially lead to optimal convergence rates with respect to the regularity of the considered problem, i.e. higher regularity can be fully exploited. As another advantage, they allow for the computation of a posteriori error indicators or estimators based on a hierarchical discretisation or on the residual. In case of a Galerkin method, the latter even leads to reliable a posteriori error estimation similar to what has become standard in deterministic FEM. When using equilibration error estimators, the error bound of the mean energy error is even guaranteed.

While sampling techniques solely rely on the evaluation of single realisations, the full discretisation of the stochastic problem in a Galerkin approach usually results in very high-dimensional algebraic problems which easily become unfeasible for numerical computations because of the dense coupling structure of the stochastic differential operator. Recently, an adaptive SGFEM based on a residual a posteriori error estimator was presented and the convergence of the adaptive algorithm was shown [Eigel, Gittelson, Schwab, Zander 2014]. This approach leads to a drastic reduction of the complexity of the problem due to the iterative discovery of the sparsity of the solution and a subsequent quasi optimal discretisation. To allow for larger and more general problems, in [Eigel, Pfeffer, Schneider 2015] we exploit the tensor structure of the parametric problem by representing operator and solution iterates in the modern tensor train (TT) format. The (successive) compression carried out with such a (linearised) hierarchical representation can be seen as a generalisation of some other model reduction techniques, e.g. reduced basis methods. The suggested approach facilitates the efficient computation of different error indicators related to the computational mesh, the active polynomial chaos index set, and the TT rank. Most notably, the curse of dimension is circumvented despite the use of a full stochastic tensor space.

EFFICIENT ERROR ESTIMATION AND FAST SOLVERS FOR STOCHASTIC GALERKIN FINITE ELEMENT APPROXIMATION

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We discuss two issues related to the efficient implementation of stochastic Galerkin finite element methods (SGFEMs) for elliptic PDEs with random coefficients: a posteriori error estimation and fast iterative solvers.

An a posterior error estimator was recently proposed in [A. Bespalov, C.E. Powell, D. Silvester, Energy norm a posteriori error estimation for parametric operator equations, SIAM Journal Sci. Comp. 36(2), A339–A363, 2014]. A strengthened Cauchy Schwarz (or CBS) constant associated with a deterministic problem related to the mean diffusion coefficient determines both the efficiency of the error estimate, and the estimate of the error reduction that would be achieved by enriching the SGFEM approximation space. We present new analysis of CBS constants for use in developing adaptive SGFEM algorithms.

A novel reduced-basis solver for the associated discrete linear systems of equations was also recently introduced in [C.E. Powell, V. Simoncini, D. Silvester, An efficient reduced basis solver for stochastic Galerkin matrix equations, submitted (2015)]. When we re-cast the linear systems as matrix equations, the solution matrix often has low rank and can be well approximated in a low-dimensional space. We describe a novel strategy for adaptively building such a space, leading to an algorithm with lower memory requirements than standard Krylov solvers.

AN OPTIMAL SOLVER FOR LINEAR SYSTEMS ARISING FROM STOCHASTIC FEM APPROXIMATION OF DIFFUSION EQUATIONS WITH RANDOM COEFFICIENTS

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This paper discusses the design and implementation of efficient solution algorithms for symmetric linear systems associated with stochastic Galerkin approximation of elliptic PDE problems with correlated random data. The novel feature of our preconditioned MINRES solver is the incorporation of error control in the natural "energy" norm in combination with a reliable and efficient a posteriori estimator for the PDE approximation error. This leads to a robust and optimally efficient inbuilt stopping criterion: the iteration is terminated as soon as the algebraic error is insignificant compared to the approximation error.

ANALYSIS OF THE ENSEMBLE KALMAN FILTER FOR INVERSE PROBLEMS

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The ideas from the Ensemble Kalman Filter introduced by Evensen in 1994 can be adapted to inverse problems by introducing artifical dynamics. In this talk, we will discuss an analysis of the EnKF based on the continuous time scaling limits, which allows to derive estimates on the long-time behavior of the EnKF and, hence, provides insights into the convergence properties of the algorithm. In particular, we are interested in the properties of the EnKF for a fixed ensemble size, in order to better understand current practice, and to suggest future directions for development of the algorithm. Results from various numerical experiments supporting the theoretical findings will be presented.

ADAPTIVE ALGORITHMS DRIVEN BY A POSTERIORI ESTIMATES OF ERROR REDUCTION FOR PDES WITH RANDOM DATA

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An efficient adaptive algorithm for computing stochastic Galerkin finite element approximations of elliptic PDE problems with random data will be outlined in this talk. The underlying differential operator will be assumed to have affine dependence on a large, possibly infinite, number of random parameters. Stochastic Galerkin approximations are sought in a tensor-product space comprising a standard h-finite element space associated with the physical domain, together with a set of multivariate polynomials characterising a p-finite-dimensional manifold of the (stochastic) parameter space.

Our adaptive strategy is based on computing distinct error estimators associated with the two sources of discretisation error. These estimators, at the same time, will be shown to provide effective estimates of the error reduction for enhanced approximations. Our algorithm adaptively 'builds' a polynomial space over a low-dimensional manifold of the infinite-dimensional parameter space by reducing the energy of the combined discretisation error in an optimal manner. Convergence of the adaptive algorithm will be demonstrated numerically.

GAUSSIAN PROCESS REGRESSION IN BAYESIAN INVERSE PROBLEMS

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A major challenge in the application of sampling methods to large scale inverse problems, is the high computational cost associated with solving the forward model for a given set of input parameters. To overcome this difficulty, we consider using a surrogate model that approximates the solution of the forward model at a much lower computational cost. We focus in particular on Gaussian process emulators, and analyse the error in the posterior distribution resulting from this approximation.

MULTILEVEL MONTE CARLO ANALYSIS FOR OPTIMAL CONTROL OF ELLIPTIC PDES WITH RANDOM COEFFICIENTS

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This work is motivated by the need to study the impact of data uncertainties and material imperfections on the solution to optimal control problems constrained by partial differential equations. We consider a pathwise optimal control problem constrained by a diffusion equation with random coefficient together with box constraints for the control. For each realization of the diffusion coefficient we solve an optimal control problem using the variational discretization [M. Hinze, Comput. Optim. Appl., 30 (2005), pp. 45-61]. Our framework allows for lognormal coefficients whose realizations are not uniformly bounded away from zero and infinity. We establish finite element error bounds for the pathwise optimal controls. This analysis is nontrivial due to the limited spatial regularity and the lack of uniform ellipticity and boundedness of the diffusion operator. We apply the error bounds to prove convergence of a multilevel Monte Carlo estimator for the expected value of the pathwise optimal controls. In addition we analyze the computational complexity of the multilevel estimator. We perform numerical experiments in 2D space to confirm the convergence result and the complexity bound.