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A WEAKLY SYMMETRIC FINITE VOLUME METHOD FOR ELASTICITY WITH APPLICATION TO FRACTURES IN POROUS MEDIA

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Simulations of fluid flow through deformable porous media are of increasing importance in subsurface applications. While the flow equations are usually discretized by a finite volume method, it is common practice to apply finite elements to the elasticity equation. This situation has the disadvantage that finite volume and finite element methods inherently use different data structures, and are best adapted to different grid types.

Recently, a finite volume method for elasticity, termed multi-point stress approximations (MPSA) has been proposed, and extended to poro-elastic systems. The schemes have been proven convergent both for for elastic and poro-elastic problems. The proof highlights the role of local coercivity conditions, which are functions of the local geometry, material parameters and discretization scheme. These conditions can be verified for many classes of grids, but for simplex grids the situation is less clear, and existing MPSA-type may fail unless strict conditions are placed on the grid geometry.

Here we offer a resolution to these issues by the introduction of a new MPSA-method. The key tool is to enforce symmetry of the stress tensor weakly, motivated by similar approaches in mixed finite elements for elasticity. This removes the issues with local coercivity, and also significantly reduces the computational cost of discretization. The resulting method is stable also on simplex grids, and we verify its convergence for heterogeneous and nearly incompressible media. Furthermore, we discuss the extension of the weakly symmetric MPSA method to fractured media, and show applications of the method for hydraulic stimulation of geothermal systems.

ITERATIVE METHODS FOR COUPLED FLOW AND GEOMECHANICS PROBLEMS IN POROUS MEDIA

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Coupling of geomechanics and flow in a poroelastic porous medium has several energy and environmental applications including subsidence events and ground water remediation. The geomechanical effects account for the influence of deformations in the porous media caused due to the fluid pore pressure whereas the changes in the pore structure due to mechanical stresses affect the flow field. Single phase quasi-static Biot model is typically used to model these coupled flow and deformation processes. The model consists of quasi-static elliptic linear elastic equation coupled to a parabolic flow equation.
We report here some of the developments in suitable iterative schemes for such models and their extensions. Our work has two components: 1. Developing suitable iterative schemes for the extensions of the Biot model to include more physics such as fractures and non-linearities, 2. Developing multirate schemes by exploiting the different time scales of mechanics and flow solve by taking coarser time step for mechanics and smaller time steps for flow. The iterative multirate schemes combine the advantages of both implicit and explicit approaches. They are efficient, allow larger time steps, are robust, and the decoupling allows us to solve the linear systems efficiently. We analyse these iterative and explicit multirate schemes and rigorously analyse the convergence and stability properties of these schemes. The flow equation is discretised using the mixed method whereas the mechanics equation is solved using conformal Galerkin. Our approach can deal with a wide variety of discretizations.

PHASE-FIELD FRACTURE PROPAGATION: VALIDATIONS AND APPLICATIONS

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This work presents phase field modeling of fluid-filled fracture propagation in a poroelastic medium. Here lower-dimensional fracture surface is approximated by using the phase field function. The two-field displacement phase-field system solves fully-coupled constrained minimization problem due to the crack irreversibility. This constrained optimization problem is handled by using active set strategy. The pressure is obtained by using a diffraction equation where the phase-field variable serves as an indicator function that distinguishes between the fracture and the reservoir. Then the above system is coupled via a fixed-stress iteration. In addition, we couple with transport system for proppant filled fracture by using a power-law fluid system.

The numerical discretization in space is based on Galerkin finite elements for displacements and phase-field, and an Enriched Galerkin method is applied for the pressure equation in order to obtain local mass conservation. The concentration is solved with cell-centered finite elements. Nonlinear equations are treated with Newton's method. Predictor-corrector dynamic mesh refinement allows to capture more accurate interface of the fractures with reasonable number for degrees of freedom.

EFFICIENT SOLVERS FOR SUBSURFACE FLOW PROBLEMS

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Many problems in porous media science and geophysics comprise interactions of processes, and are typically formulated as a system of coupled PDEs. In most cases these systems are transient and often also non-linear. Developing efficient solvers is a delicate task, since one needs to must combine suitable schemes for (i) time integration, (ii) linearization, and (iii) (geometric and/or algebraic) multilevel solvers, finally being employed in a (iv) parallel computing environment. In this presentation, we take an application oriented approach, and focus on the problem classes of poroelasticity problems and density-driven-flow. For these two examples, we outline a common solution strategy, and provide numerical results.

NON-STATIONARY ADVECTION-DIFFUSION PROBLEMS IN NETWORKS OF FRACTURES WITH AN OPTIMIZATION APPROACH

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Time-dependent advection-diffusion problems in large discrete fracture networks (DFN) are addressed via an optimization-based approach. In DFN models, underground fractures are modelled individually and are represented by planar polygons intersecting each other in the three dimensional space and forming an intricate network resembling the fracture-system in the underground. Fracture geometrical data and hydraulic properties are only known through probability distribution functions, tuned through sampling and testing on specific geological sites. DFN models are particularly well suited for the simulation of transport phenomena in which the directionality of the flow is of paramount importance but some difficulties are be addressed to perform effective simulations. We have: geometrical complexities, as the generation of a mesh suitable for finite elements and conforming to interfaces (i.e. fracture intersections) on intricate networks of fractures often results infeasible or leads to poor quality elements; the multiscale nature of the problem, due to the simultaneous presence of large geological entities (as, e.g., faults) and very small fractures; domain size with networks for practical applications counting up to millions of fractures; and uncertainty in input data. A solution to the above mentioned issues is proposed, based on a PDE constrained optimization method [2, 4, 3]. The method allows for an independent mesh generation on each fracture of the network, resorting to the minimization of a cost functional to enforce conditions at the interfaces on the non-conforming meshes. In such a way the mesh can be adjusted locally to fit with the scale of each fracture. The method can also be readily implemented in parallel computers, thus effectively handling problem dimensions. Thanks to the robustness of the method stochastic analyses considering randomness in DFN data are performed, also exploiting modern uncertainty quantification techniques [1].

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AN ADAPTIVE MIXED FINITE ELEMENT METHOD FOR DARCY FLOW IN FRACTURED POROUS MEDIA

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In this work, we propose an adaptive mixed finite element method for simulating the single-phase Darcy flow in fractured porous media. The reduced model that we use for simulation is a discrete fracture model coupling Darcy flows in the matrix and the fractures, and the fractures are modeled by lower-dimensional fractures. The Raviart-Thomas mixed finite element methods are utilized for the solution of the coupled Darcy flows in the matrix and the fractures. In order to improve the efficiency of the simulation, we use adaptive mixed finite element method based on the residual-based *a posteriori* error estimators. Several examples of Darcy flow in the fractured porous media are provided to demonstrate the robustness of the algorithm.

NUMERICAL METHODS FOR P-LAPLACE TYPE PROBLEMS

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In this presentation, we consider numerical methods for solving power-law diffusion problems, e.g. p-Laplace type problems. For the space discretization we use continuous Galerkin finite element methods (FE) with high order polynomial spaces. For the solution of the resulting nonlinear system we employ different Newton methods, such as residual-based and error-oriented globalization techniques. In addition, we also transform the original problem into a saddle point problem using an augmented Lagrangian (ALG) decomposition technique.

Assuming sufficient regularity for the solution, we derive high order interpolation and error estimates in relevant quasi-norms. We mainly focus on a systematic comparison of first and second order finite element approximations in order to confirm our theoretical findings. Our second goal is a very detailed comparison of two different Newton methods: a residual-based procedure and an error-oriented procedure. Lastly, we discuss the solution of the produced ALG saddle point problem. We discretize it using a FE methodology and then we present two iterative methods for solving the resulting nonlinear algebraic system. The first iterative method is the classical ALG1 iterative method, which is usually used in the literature. It can be interpreted as a variant of the Uzawa algorithm, where the Lagrange multiplier is separately updated. The second proposed iterative method can be characterized as a monolithic approach where all the unknown variable are simultaneously computed in one step.
All, proposed methods are compared with respect to computational cost and to the convergence rates in several examples.
This talk is based on a joint work with Thomas Wick, [1]. We gratefully acknowledge the financial support of this research work by the Austrian Science Fund (FWF) under the grant NFN S117-03.

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A GENERALIZED MULTISCALE FINITE ELEMENT METHOD FOR PROBLEMS IN FRACTURED MEDIA

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Complex processes in fractured media lead to multiscale problems because of a hierarchy of fracture sizes. To represent the microscale interaction between the fractures and the matrix, various coarse-grid models have been developed. These include dual-continua like approaches, coarse-scale continuum model, upscaling methods, Multiscale Finite Volume, and so on. In this talk, I will describe an approach, which is based on Generalized Multiscale Finite Element Method. The main idea of the approach is to extract important local information from local snapshot spaces via local spectral problems. This computational approach leads to extracting important flow patterns in fractured media and results to accurate predictions when using a few basis functions. In my talk, I will describe the algorithm. I will show numerical results for two applications. One is for a model problem describing the transport of shale gas and the other is wave propagation.

PHASE-FIELD FRACTURE PROPAGATION: MODELING AND NUMERICAL METHODS

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Currently, fracture propagation is a major topic in applied mathematics and engineering. It seems to turn out that one of the most promising methods is based on a variational setting and more specifically on a thermodynamically consistent phase-field model. Here a smoothed indicator function determines the crack location and is characterized through a model regularization parameter. In addition, modeling assumes that the fracture can never heal, which is imposed through a temporal constraint, leading to a variational inequality system. The basic fracture model problem is augmented with several hints and discussions of serious challenges in developing numerical methods for fracture propagation. Key aspects are robust and efficient algorithms for imposing the previously mentioned crack irreversibility constraint, treatment of the indefinite Jacobian matrix, computational analysis of the interplay of model and discretization parameters, goal-functional evaluations, coupling to other multiphyics problems such as pressurized fractures, fluid-filled fractures, proppant-filled fractures in porous media, fluid-structure interaction, and aspects of high performance computing for tackling practical field problems.

A LAGRANGE MULTIPLIER METHOD FOR A BIOT-STOKES MODEL OF FLOW IN FRACTURED POROELASTIC MEDIA

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We study a mathematical model and its finite element approximation for solving the coupled problem arising in the interaction between fluid in a poroelastic material and fluid in a fracture. The fluid flow in the fracture is governed by the Stokes equations, while the poroelastic material is modeled using the Biot system. The continuity of normal velocity on the interface is imposed via a Lagrange multiplier. A stability and error analysis is performed for the semidiscrete continuous-in-time formulation. We present a series of numerical experiments to illustrate the convergence of the method and its applicability to modeling physical phenomena, as well as the sensitivity of the model with respect to its parameters.

This is joint work with Ilona Ambartsumyan, Eldar Khattatov, and Paolo Zunino from University of Pittsburgh.