MAFELAP 2016

Conference on the Mathematics of Finite Elements and Applications

14–17 June 2016

Mini-Symposium: Space-time discretization methods

Organisers:

Martin Gander, Ulrich Langer and Olaf Steinbach

Abstracts in alphabetical order

Contents

HIGH-ORDER DISCONTINUOUS GALERKIN METHODS IN TIME FOR THE WAVE EQUATION

Fatima Al-Shanfari

Department of Mathematics, Brunel University, UK
fatima.al-shanfari@brunel.ac.uk

In this paper, we analyse the high-order in time discontinuous Galerkin finite element method (DGFEM) for second-order in time evolution problems. We use a generalization of C Johnson (CMAME, 1993), with high orders in time, non-homogeneous boundary data; leading to an abstract Hilbert space variational formulation. Based on our abstract Hilbert space variational formulation we re-write the second order in time problem as a first-order system in time and we apply the discretization approach in time for the variational formulation of abstract parabolic problems introduced by D Schötzau (PhD Thesis, 1999).

We prove a priori error estimates and unconditional stability estimates within our abstract framework for finite polynomial degrees in time. Finally we apply our abstract framework to the acoustic wave equation.

EXTENSION OF LINEAR TIME-PARALLEL ALGORITHMS TO NON-LINEAR PROBLEMS

Martin J. Gander

Section of Mathematics, University of Geneva, Switzerland
martin.gander@unige.ch

Once an evolution problem has been disretized in space-time, it is of interest due to its size to solve it on a large scale parallel computer. Several recent time parallel methods have been developed only for linear problems, and they use linearity in essential ways, for example the ParaExp algorithm, or the parallelization method based on diagonalization of the time stepping matrix. I will use the latter to explain how one can use such an essentially linear method also in the context of a non-linear evolution problem. I will first explain the method for a scalar model problem, and then give a formulation for a non-linear partial differential equation based on tensorization. I will also illustrate the methods with numerical experiments.

TIME-DOMAIN BOUNDARY ELEMENT METHODS FOR INTERFACE PROBLEMS

Heiko Gimperlein[1,2], Ceyhun Özdemir[3], David Stark[1] and Ernst P. Stephan[3]

[1]Maxwell Institute for Mathematical Sciences and Department of Mathematics, Heriot-Watt University, Edinburgh EH14 4AS, UK,
h.gimperlein@hw.ac.uk

[2]Institute for Mathematics, University of Paderborn, Warburger Str. 100, 33098 Paderborn, Germany

[3]Institute for Applied Mathematics, Leibniz University Hannover, Welfengarten 1, 30167 Hannover, Germany

We consider well-posedness, convergence and a posteriori error estimates for fluid-structure interaction and contact problems in time-domain.
In the case of an elastic body immersed in a fluid, a Galerkin time-domain boundary element method (TDBEM) for the wave equation in the exterior is coupled to a finite element method for the Lamé equation. Based on ideas from the time–independent coupling formulation and its a posteriori error analysis, we give a priori and a posteriori error estimates, which demonstrate the convergence and give rise to adaptive mesh refinement procedures.
We then discuss a first error analysis for dynamic Signiorini problems with flat contact area, a variational inequality involving the Dirichlet-Neumann operator for the wave equation. Here refined information about the Dirichlet-Neumann operator allows to prove well-posedness as well as a priori and a posteriori error estimates for the TDBEM solutions.
The talk concludes with a survey of recent computational work on TDBEM in our group.

SPACE-TIME AND REDUCED BASIS METHODS

Silke Glas[a] and Karsten Urban[b]

Institute for Numerical Mathematics, University of Ulm, Germany
[a]silke.glas@uni-ulm.de, [b]karsten.urban@uni-ulm.de

Parametrized parabolic problems often occur in industrial or financial applications, e.g. as pricing of options on the stock market. If we want to calibrate an option pricing model, we need several evaluations for different parameters. Fine discretizations, that are needed for these problems, resolve in large scale problems and thus in long computational times. To reduce the size of those problems, we use the Reduced Basis Method (RBM) [2, 1]. The ambition of the RBM is to efficiently reduce discretized parametrized partial differential equations given in a variational form. Using space-time formulations, we do not use a time-stepping scheme, but take the time as an additional variable in the variational formulation of the problem.

Well-posedness for the space-time variational approach has been shown for a wide range of problems. For the general case of a parabolic variational equation, see [3].

Combining the RBM with the space-time formulation, we derive a possibly noncoercive Petrov–Galerkin problem, where improved error estimators for parabolic equations could be achieved [4].

In this talk we consider a comparison between space-time methods and the often used time-stepping scheme for the RBM. We conclude with an overview where the space-time methods has been successfully applied to RBM.

References

[1] Jan S. Hesthaven, Gianluigi Rozza, and Benjamin Stamm. *Certified reduced basis methods for parametrized partial differential equations*. Springer Briefs in Mathematics. Springer, Cham; BCAM Basque Center for Applied Mathematics, Bilbao, 2016. BCAM SpringerBriefs.

[2] Alfio Quarteroni, Andrea Manzoni, and Federico Negri. *Reduced basis methods for partial differential equations*, volume 92 of *Unitext*. Springer, Cham, 2016. An introduction, La Matematica per il 3+2.

[3] Christoph Schwab and Rob Stevenson. Space-time adaptive wavelet methods for parabolic evolution problems. *Math. Comp.*, 78(267):1293–1318, 2009.

[4] Karsten Urban and Anthony T. Patera. An improved error bound for reduced basis approximation of linear parabolic problems. *Math. Comp.*, 83(288):1599–1615, 2014.

MULTILEVEL APPROACHES IN SPACE AND TIME

Rolf Krause[1a], Pietro Benedusi[1b], Xiaozhou Li[1c], Daniel Hupp[2d] and Peter Arbenz[2e]

[1]Institute of Computational Science, USI, Lugano, Switzerland,
[b]pietro.benedusi@usi.ch, [a]rolf.krause@usi.ch, [c]xiaozhou.li@usi.ch

[2]Computer Science Department, ETH Zurich, Switzerland,
[d]huppd@inf.ethz.ch, [e]arbenz@inf.ethz.ch

Time parallel algorithms are more and more a promising strategy to extend the scalability of PDEs solvers. In fact the sequential time integration limits the parallelism of a solver to the spatial variables.

In this context, firstly we present a space-time multilevel algorithm for the nonlinear systems arising from the discretization of Navier-Stokes (N-S) equations with finite differences. In particular we study the incompressible, unsteady N-S equations with periodic boundary condition in time.

Time periodic flows, that we find, for example, in biomechanics or engineering, can be conveniently discretized in space-time, where adding parallelism in the time direction is natural.

To achieve fast convergence, we used a multigrid algorithm with parallel box smoothing, the properties of which are studied using local Fourier analysis. We used numerical experiments to analyze the scalability and the convergence of the solver, focusing on the case of a pulsatile flow in three dimensions.

We also present some recent results for an iterative time integrator based on Discontinuous Galerkin (DG) and the Spectral Deferred Correction method (SDC). The DG approach can improve stability, convergence and flexibility of SDC, preserving its structure. This algorithm may find application as a smoother in time-parallel multilevel solvers, as the popular PFASST [Emmett, M. and Minion, M., Toward an Efficient Parallel in Time Method for Partial Differential Equations, Comm. in App. Math. and Comp. Science, 2012, v. 7, pp. 105–132].

A TREFFTZ POLYNOMIAL SPACE-TIME DISCONTINUOUS GALERKIN METHOD FOR THE SECOND ORDER WAVE EQUATION

Lehel Banjai[1a], Emmanuil Geogoulis[2] and Oluwaseun Lijoka[1b]

[1]Department of Mathematics,
Heriot Watt University EH14 4AS, Edinburgh, UK.
[a]L.Banjai@hw.ac.uk, [b]ofl1@hw.ac.uk

[2]Department of Mathematics,
University of Leicester, University Road Leicester, LE1 7RH UK.
Emmanuil.Georgoulis@le.ac.uk

A new space-time discontinuous Galerkin (dG) method utilising special Trefftz polynomial basis functions is proposed and fully analysed for the scalar wave equation in second order formulation. The dG method considered is motivated by the class of interior penalty dG methods, as well as by the classical work of Hulbert and Hughes [4]. The choice of the penalty terms included in the bilinear form is essential for both the theoretical analysis and for the practical behaviour of the method for the case of lowest order basis functions. A best approximation result is proven for this new space-time dG method with Trefftz-type basis functions. Rates of convergence are proved in any dimension and verified numerically in spatial dimensions $d = 1$ and $d = 2$. Numerical experiments highlight the effectiveness of the Trefftz method in problems with energy at high frequencies.

References

[1] A. Maciąg, and J.Wauer, Solution of the two-dimensional wave equation by using wave polynomials, *Journal of Engineering Mathematics 339–350, 2005.*

[2] Petersen, Steffen and Farhat, Charbel and Tezaur, Radek, A space-time discontinuous Galerkin method for the solution of the wave equation in the time domain, *International Journal for Numerical Methods in Engineerin, 275–295, 2009.*

[3] A. Moiola, R. Hiptmair, and I. Perugia, Plane wave approximation of homogeneous Helmholtz solutions, *Journal of Applied Mathematics and Physics. Journal de Mathématiques et de Physique Appliquées, 809–837, 2011.*

[4] G. M. Hulbert, and T. J. R. Hughes, Space-time finite element methods for second-order hyperbolic equations, *Computer Methods in Applied Mechanics and Engineering, 327–348, 1990.*

[5] P. Monk and G. R. Richter, A discontinuous Galerkin method for linear symmetric hyperbolic systems in inhomogeneous media, *Journal of Scientific Computing, 443–477, 2005.*,

HIGH-ORDER MARCHING-ON-IN-TIME (MOT) FOR 2D TIME DOMAIN BOUNDARY ELEMENT METHODS (TD-BEM)

Matthias Maischak

Department of Mathematics, Brunel University London, UK
matthias.maischak@brunel.ac.uk

In this talk we study the transient scattering of acoustic waves by an obstacle in an infinite two dimensional domain, where the scattered wave is represented in terms of time domain boundary layer potentials. The problem of finding the unknown solution of the scattering problem is thus reduced to finding the unknown density of the time domain boundary layer operators on the obstacle's boundary, subject to the boundary data of the known incident wave. Using a Galerkin approach, the unknown density is approximated by a piecewise polynomial function, the coefficients of which can be found by solving a linear system. The entries of the system matrix of this linear system involve, for the case of the two dimensional scattering problem under consideration, integrals over four dimensional space-time manifolds. An accurate computation of these integrals is crucial for the stability of this method.

Using piecewise polynomials of arbitrary order, the two temporal integrals can be evaluated analytically, leading to kernel functions for the spatial integrals with complicated domains of piecewise support.

These spatial kernel functions can be generalised into a class of admissible kernel functions which, as we prove, belong to countably normed spaces [1].

Therefore, a quadrature scheme for the approximation of the two dimensional spatial integrals with admissible kernel functions converges exponentially [3]. Similar results for the three dimensional case can be found in [2, 4].

This talk concentrates on an efficient scheme to evaluate the integrals with high order polynomials and stability results for the Galerkin scheme We also show numerical experiments underlining the theoretical results, cf. [1].

References

[1] M. Gläfke. Adaptive Methods for Time Domain Boundary Integral Equations. PhD Thesis, Brunel University, 2013. 160

[2] E. Ostermann. Numerical Methods for Space-Time Variational Formulations of Retarded Potential Boundary Integral Equations. PhD Thesis, Institut für Angewandte Mathematik, Leibniz Universität Hannover, 2010.

[3] C. Schwab. Variable order composite quadrature of singular and nearly singular integrals. Computing 53, 2 (1994), 173–194.

[4] E. P. Stephan, M. Maischak, E. Ostermann. Transient boundary element method and numerical evaluation of retarded potentials. In Computational Science – ICCS 2008, Vol. 5102 of Lecture Notes in Computational Science , Springer, 2008, 321–330.

PARALLEL TIME-DOMAIN BOUNDARY ELEMENT METHOD FOR 3-DIMENSIONAL WAVE EQUATION

Michal Merta[a], Alexander Veit, Jan Zapletal and Dalibor Lukas

Centre of Excellence IT4Innovations, VŠB-TU Ostrava, Czech Republic.
[a]michal.merta@vsb.cz

We present a boundary element method for 3-dimensional sound-hard scattering. It relies on an indirect formulation for the retarded double-layer potential introduced by Bamberger and Ha Duong in 1986 and on smooth time ansatz functions recently proposed by Sauter and Veit. The latter allows for an efficient use of Gauss quadrature within the assembly of the resulting boundary element system matrix. The assembling process is implemeted in parallel and we numerically document its scalability. Further, a heuristical preconditioner, which accelerates flexible GMRES iterations, is presented. The efficiency of our approach is documented for a problem on a sphere with known analytical solution as well as for a scattering from a real-world geometry.

SPACE–TIME TREFFTZ DISCONTINUOUS GALERKIN METHODS FOR WAVE PROBLEMS

Andrea Moiola[1] and Ilaria Perugia[2]

[1]Department of Mathematics and Statistics,
University of Reading, UK
a.moiola@reading.ac.uk

[2]Faculty of Mathematics, University of Vienna, Austria
and
Department of Mathematics, University of Pavia, Italy
ilaria.perugia@univie.ac.at

We present a space–time discontinuous Galerkin method for linear wave propagation problems. The special feature of the scheme is that it is a *Trefftz method*, namely that trial and test functions are solution of the partial differential equation to be discretised in each element of the (space–time) mesh. The method considered, described in [2] and [4], is a modification of the schemes of [3] and [5].

The DG scheme is defined for unstructured meshes whose internal faces need not be aligned to the space–time axes. The Trefftz approach can be used to improve and ease the implementation of explicit schemes based on "tent-pitched" meshes, cf. [1] and [5].

We show that the scheme is well-posed, quasi-optimal and dissipative, and prove a priori error bounds for general Trefftz discrete spaces. A concrete discretisation can be obtained using piecewise polynomials that satisfy the wave equation elementwise, for which we show high orders of convergence.

References

[1] J. Gopalakrishnan, J. Schöberl, and C. Wintersteiger, *Mapped tent pitching schemes for hyperbolic systems*, arXiv:1604.01081v1, (2016).

[2] F. Kretzschmar, A. Moiola, I. Perugia, and S. M. Schnepp, *A priori error analysis of space-time Trefftz discontinuous Galerkin methods for wave problems.* IMA J. Numer. Anal., (2015).

[3] F. Kretzschmar, S. M. Schnepp, I. Tsukerman, and T. Weiland, *Discontinuous Galerkin methods with Trefftz approximations.* J. Comput. Appl. Math. 270 (2014), 211–222.

[4] A. Moiola, *Trefftz discontinuous Galerkin methods on unstructured meshes for the wave equation*, arXiv preprint, arXiv:1505.00120, (2015).

[5] P. Monk and G. R. Richter, *A discontinuous Galerkin method for linear symmetric hyperbolic systems in inhomogeneous media.* J. Sci. Comput., 22/23 (2005), 443–477.

SPACE-TIME CFOSLS METHODS WITH AMGE UPSCALING

Martin Neumüller[1], Panayot S. Vassilevski[2] and Umberto E. Villa[3]

[1]Institute of Computational Mathematics,
Johannes Kepler University Linz, Austria
neumueller@numa.uni-linz.ac.at

[2]Center for Applied Scientific Computing,
Lawrence Livermore National Laboratory, U.S.A.
panayot@llnl.gov

[3]Institute for Computational Engineering and Sciences (ICES),
The University of Texas at Austin, U.S.A.
uvilla@ices.utexas.edu

This work considers the combined space-time discretization of time-dependent partial differential equations by using first order least square methods. We also impose an explicit constraint representing space-time mass conservation. To alleviate the restrictive memory demand of the method, we use dimension reduction via accurate element agglomeration AMG coarsening, referred to as AMGe upscaling. Numerical experiments demonstrating the accuracy of the studied AMGe upscaling method are provided.

TIME DOMAIN BOUNDARY ELEMENT FORMULATION WITH VARIABLE TIME STEP SIZE

Martin Schanz

Institute of Applied Mechanics, Graz University of Technology, Austria,
m.schanz@tugraz.at

The numerical solution of wave propagation problems requires discretizations in space and time. Latest since the great success of Discontinuous Galerkin methods it is accepted that adaptive space-time methods are preferable against time stepping techniques. In the context of Boundary Element Methods (BEM) space-time methods are used from the beginning on [Mansur(1983)]. Using a constant time step size results in a lower triangular Toeplitz system for the discretized retarded potentials. Hence, the complexity in time is linear. Also the convolution quadrature method (CQM) in its initial form requires a constant time step size [Lubich(1988)], which results as well in a linear complexity in time.

A variable time step size for BEM has been proposed by [Sauter and Veit(2013)] using a global shape function in time and by [Lopez-Fernandez and Sauter(2013)] with a generalized convolution quadrature method. The latter approach shares all benefits of the original CQM but allows a variable time step size. The complexity in time is $\mathscr{O}(N \log N)$. This approach is used in this presentation to formulate a BE formulation for acoustics and elastodynamics. Numerical studies will show the behaviour of this formulation with respect to temporal discretization. The formulation will be based on a collocation approach in space.

References

[Lopez-Fernandez and Sauter(2013)] M. Lopez-Fernandez and S. Sauter. Generalized convolution quadrature with variable time stepping. *IMA J. of Numer. Anal.*, 33 (4):1156–1175, 2013.

[Lubich(1988)] C. Lubich. Convolution quadrature and discretized operational calculus. I. *Numer. Math.*, 52(2):129–145, 1988.

[Mansur(1983)] W. J. Mansur. *A Time-Stepping Technique to Solve Wave Propagation Problems Using the Boundary Element Method*. Phd thesis, University of Southampton, 1983.

[Sauter and Veit(2013)] S. Sauter and A. Veit. A Galerkin method for retarded boundary integral equations with smooth and compactly supported temporal basis functions. *Numer. Math.*, 123(1):145–176, 2013.

MAPPED TENT PITCHING METHOD FOR HYPERBOLIC CONSERVATION LAWS

Jay Gopalakrishnan[1], Joachim Schöberl[2a] and Christoph Wintersteiger[2b]

[1]Portland State University, USA
gjay@pdx.edu

[2]Institute for Analysis and Scientific Computing, TU Wien, Austria
[a]joachim.schoeberl@tuwien.ac.at, [b]christoph.wintersteiger@tuwien.ac.at

Tent pitching algorithms construct space-time meshes by vertically erecting canopies over vertex patches. The main advantage is the ability to advance in time by different amounts at different spacial locations. These tent pitched meshes are usually combined with a space-time discretization, which leads to a rather large local problem on each tent. This talk considers a novel discretization technique, that exploits the structure of tent pitched meshes to reduce the local problem size. The reduction is obtained by transforming the tents to a reference domain with a space-time tensor product structure, which then allows to discretize space and time independently. These Mapped Tent Pitching (MTP) schemes can be applied to both, linear and non-linear systems. For linear systems a fully implicit MTP scheme is presented in [1] and this talk will focus on non-linear systems (see [1, 2]). Numerical results for the Euler equations in 2+1 dimensions and the linear wave equation in 3+1 dimensions will be shown.

References

[1] J. Gopalakrishnan, J. Schöberl and C. Wintersteiger. Mapped Tent Pitching Schemes for Hyperbolic Systems. arXiv:1604.01081

[2] C. Wintersteiger. Mapped Tent Pitching Method for Hyperbolic Conservation Laws. Master's thesis, TU Wien, 2015.

AN ENERGY APPROACH TO TIME-DOMAIN BOUNDARY INTEGRAL EQUATIONS FOR THE WAVE EQUATION

Marco Zank

Institute of Computational Mathematics, Graz University of Technology, Austria
zank@math.tugraz.at

For the discretisation of the wave equation by boundary element methods the starting point is the so-called Kirchhoff's formula, which is a representation formula by means of boundary potentials. In this talk different approaches to derive weak formulations of related boundary integral equations are considered. First, weak formulations based on the Laplace transform and second, time-space energetic formulations are introduced. In both cases coercivity is shown in appropriate Sobolev spaces.

Finally, some numerical examples are presented and discussed.