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Abstracts in alphabetical order

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A MIXED-METHOD B-FIELD FINITE-ELEMENT FORMULATION FOR INCOMPRESSIBLE, RESISTIVE MAGNETOHYDRODYNAMICS

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Magnetohydrodynamics (MHD) models describe a wide range of plasma physics applications, from thermonuclear fusion in tokamak reactors to astrophysical models. These models are characterized by a nonlinear system of partial differential equations in which the flow of the fluid strongly couples to the evolution of electromagnetic fields. In this talk, we consider the one-fluid, viscoresistive MHD model in two dimensions. There have been numerous finite-element formulations applied to this problem, and we will briefly discuss the applications of two; a least-squares and mixed-method formulation. In the latter, we consider inf-sup stable elements for the incompressible Navier-Stokes portion of the formulation, Nedélec elements for the magnetic field, and a second Lagrange multiplier added to Faraday's law to enforce the divergence-free constraint on the magnetic field.

Regardless of the formulation, the discrete linearized systems that arise in the numerical solution of these equations are generally difficult to solve, and require effective preconditioners to be developed. Thus, the final portion of the talk, will involve a discussion of monolithic multigrid preconditioners, using an extension of a well-known relaxation scheme from the fluid dynamics literature, Vanka relaxation, to this formulation. To isolate the relaxation scheme from the rest of the multigrid method, we utilize structured grids, geometric interpolation operators, and Galerkin coarse grid operators. Numerical results are shown for the Hartmann flow problem, a standard test problem in MHD.

GEOMETRIC MULTIGRID WITH OPERATOR-DEPENDENT COARSE SPACES

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When solving nonlinear partial differential equations using the Finite Element Method, the inner linear-system solves are often the bottleneck in computation. In general, multilevel methods provide efficient solvers for these systems that are optimal and scalable to large, parallel machines. In this talk, we discuss parallel geometric multigrid methods that utilize operator-dependent coarse spaces through an AMGe-type mechanism. In particular, the coarse operator-dependent spaces can have approximation properties of the same order as the fine-grid spaces. We show results for a few linear problems in a distributed computing environment.

LEAST-SQUARES METHOD IN RELATION TO MIXED FINITE ELEMENTS FOR ELASTICITY

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The related physical equations of linear elasticity are the equilibrium equation and the constitutive equation, which expresses a relation between the stress and strain tensors. This is a first-order partial differential system such that a least squares method based on a stress-displacement formulation can be used whose corresponding finite element approximation does not preserve the symmetry of the stress [1].

In this talk, a new method is investigated by introducing the vorticity and applying the L^2 norm least squares principle to the stress-displacement-vorticity system. The question of ellipticity due to the fact that all three variables are present in one equation is discussed. Further, the supercloseness of the least squares approximation to the standard mixed finite element approximations arising from the Hellinger-Reissner principle with reduced symmetry [2], is studied. This implies that the favourable conservation properties of the dual-based mixed methods and the inherent error control of the least squares method are combined.

Additionally, a closer look will be taken at the error that appears using this formulation on domains with curved boundaries approximated by a triangulation [3]. In the higher-order case, parametric Raviart-Thomas finite elements are employed to this end.

Finally, it is shown that an optimal order of convergence is achieved and illustrated numerically on a test example.

1. Z. Cai, G. Starke. Least squares methods for linear elasticity. SIAM J. Numer. Anal. 42 (2004): 826-842

2. D. Boffi, F. Brezzi, and M. Fortin. Mixed Finite Element Methods and Applications. Springer-Verlag, Heidelberg, 2013. [Chp. 9]

3. F. Bertrand, S. Münzenmaier, and G. Starke. First-Order System Least Squares on Curved Boundaries: Lowest-Order Raviart-Thomas Elements. SIAM J. Numer. Anal. 52.2 (2014): 880-894.

A DEFLATION TECHNIQUE FOR DETECTING MULTIPLE LIQUID CRYSTAL EQUILIBRIUM STATES

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Multiple equilibrium states arise in many physical systems, including various types of liquid crystal structures. Having the ability to reliably compute such states enables more accurate physical analysis and understanding of experimental behavior. In this talk, we consider adapting and extending a deflation technique for the computation of multiple distinct solutions arising in the context of modeling equilibrium configurations of nematic and cholesteric liquid crystals. The deflation method is applied as part of an overall free-energy variational approach and is modified to fit the framework of optimization of a functional with pointwise constraints. It is shown that multigrid methods designed for the undeflated systems may be applied to efficiently solve the linear systems arising in the application of deflation. For the numerical algorithm, the deflation approach is interwoven with nested iteration, creating a dynamic and efficient method that further enables the discovery of distinct solutions. Finally, we present numerical simulations demonstrating the efficacy and accuracy of the algorithm in detecting important physical phenomena, including bifurcations and disclination behaviors.

THE NITSCHE TRICK FOR THE OBSTACLE PROBLEM – A COUNTEREXAMPLE AND CONSEQUENCES FOR OPTIMAL CONTROL

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We consider the Finite Element (FE) discretization of the obstacle problem using piecewise linear and continuous finite elements. While a priori error estimates in the energy space are standard and well known, the classical Nitsche trick for improved error estimates in $L^2(\Omega)$ seems to fail due to a lack of regularity in the dual problem. This is demonstrated by two one-dimensional counterexamples, which provide a (rigorously computable) order of convergence of $2 - 1/p$, if the obstacle is described by a function in $W^{2,p}(\Omega)$. The L^2-a priori estimate directly affects the convergence analysis for an optimal control problem governed by the obstacle problem.

DISCRETIZATION METHODS FOR ORIENTED MATERIALS

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Materials such as ferromagnets, liquid crystals, and granular media involve orientation degrees of freedom. Mathematical descriptions of such materials involve fields of nonlinear objects such as unit vectors, rotations matrices, or unitary matrices. Classical numerical methods like the finite element method cannot be applied in such situations, because linear and polynomial interpolation is not defined for such nonlinear objects. Instead, a variety of heuristic approaches is used in the literature, which are difficult to analyze rigorously. We present nonlinear generalizations of the finite element method that allow to treat problems with orientation degrees of freedom in a mathematically sound way. This allows to show solvability of the discrete problems, makes the construction of efficient solvers easier, and allows to obtain reliable bounds for the finite element approximation error. We use the technique to calculate stable configurations of chiral magnetic skyrmions, and wrinkling patterns of a thin elastic polyimide film.

HYBRID DISCONTINUOUS GALERKIN METHODS IN SOLID MECHANICS

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We introduce a novel hybrid discontinuous Galerkin method for elliptic problems with a discontinuous ansatz space in the cells and adaptively chosen constraints on the faces. This corresponds to a weakly conforming finite element space defined by primal and dual face degrees of freedom. In the solution process the interior degrees of freedom can be eliminated. We provide local criteria for the well-posedness and stability of this elimination process, and we the derive global spectral bounds for the resulting skeleton reduction. The a priori finite element error and a residual based error estimator measuring also the primal and dual consistency error are analyzed.

The face contributions of the primal and dual consistency error are used to derived a flexible strategy to increase the number of face degrees of freedom locally. The new adaptive scheme is evaluated numerically for nearly incompressible 3D linear elasticity, and the results are compared with the symmetric interior penalty discontinuous Galerkin method. Finally, we show that the method extends to nonlinear applications such as contact problems or large strain elasticity.