MAFELAP
2016

Conference on the Mathematics
of Finite Elements and Applications

14–17 June 2016

Mini-Symposium: Numerical methods for multiphysics and coupled problems

Organisers:

Daniele Boffi and Lucia Gastaldi

Abstracts in alphabetical order

Contents

IMPROVED ARLEQUIN METHOD FOR THE HELMHOLTZ EQUATION IN 2D

J. Albella[1a], H. Ben Dhia[2], S. Imperiale[3] and J. Rodríguez[1b]

[1]Dept. of Applied Mathematics, USC,
15782 Santiago de Compostela, A Coruña (Spain).
[a]jorge.albella@usc.es, [b]jeronimo.rodriguez@usc.es

[2]Laboratoire de Mécaniques Sols Structures et Matériaux,
UMR CNRS 8579 CentraleSupelec.
hachmi.ben-dhia@centralesupelec.fr

[3]Inria Saclay-Ile de France, Team MΞDISIM, France.
sebastien.imperiale@inria.fr

The Arlequin method [1] is a flexible tool which allows to couple different models using an overlapping region. In [2] the method was used as a domain decomposition technique for the transient wave equation and in [3] some variants were presented on the 1D case to improve its flexibility on the discretization procedure. Now we apply such variants to Helmholtz equation in a 2D configuration:

$$\begin{array}{l}\text{Find the solution } u \text{ of: } \quad \rho\, k^2\, u + \operatorname{div}(\mu\, \nabla u) = 0, \text{ in } \Omega \quad \text{s.t.} \quad u = u_D, \text{ in } \partial\Omega, \\ \text{where } \rho, \mu \in L^\infty(\Omega) \text{ are strictly positive and } k \text{ denotes the wave number.}\end{array} \tag{1}$$

To present the modified Arlequin formulation of (1) we decompose the domain Ω in two subdomains Ω_1 and Ω_2 such that $\Omega_1 \cap \Omega_2 = \omega_1 \cup \omega_c \cup \omega_2 \neq \emptyset$, where those ω_1, ω_c and ω_2 are disjoint non empty sets. We also need to introduce the spaces $V = H^1(\Omega_1) \times H^1(\Omega_2)$ and $M = H^1(\omega_1) \times H^1(\omega_2)$ and the coefficients $\alpha_i, \beta_i > 0$ such that $\alpha_1 + \alpha_2 = \rho\, k^2$, $\beta_1 + \beta_2 = \mu$ and $\frac{\alpha_j}{\rho} = \frac{\beta_j}{\mu} = cte$ in ω_c. Then, considering the usual scalar product $(\cdot,\cdot)_{k,\widetilde{\Omega}}$ in $H^k(\widetilde{\Omega})$, the variational formulation for the coupled problem reads:

$$\left|\begin{array}{l}\text{Find } (u, \lambda) \in V \times M \text{ s.t. } \forall (v, l) \in V \times M \\ (\alpha_1 u_1, v_1)_{0,\Omega_1} - (\beta_1 \nabla u_1, \nabla v_1)_{0,\Omega_1} + (\alpha_2 u_2, v_2)_{0,\Omega_2} - (\beta_2 \nabla u_2, \nabla v_2)_{0,\Omega_2} + \\ (\lambda_{\omega_1}, v_1 - v_2)_{1,\omega_1} + (\lambda_{\omega_2}, v_1 - v_2)_{1,\omega_2} + (l_{\omega_1}, u_1 - u_2)_{1,\omega_1} + (l_{\omega_2}, u_1 - u_2)_{1,\omega_2} = 0.\end{array}\right. \tag{2}$$

This formulation (see [3] for the details in the 1D case) allows the use of independent meshes and offers the possibility to capture with the finest mesh the variations of the physical coefficients on $\omega_i, i \in \{1, 2\}$.

In the presentation, it will be shown that with this formulation one gets optimal convergence rate for first order finite elements but, unlike what happened in the 1D case, it provides sub-optimal results for quadratic elements. As it will be explained, this is due to the fact that the Lagrange multipliers $\lambda_{\omega_i}, i \in \{1, 2\}$ satisfy Laplace like equations set in domains with reentrant corners. The Arlequin formulation in [3] will be modified to make the method compatible with higher order elements. Numerical results will be presented to show the performance of the discretization procedure.

References

[1] H. Ben Dhia. *Problèmes mécaniques multi-échelles: la méthode Arlequin.* C.R. de l'Académie des Sciences - Series IIB, 326(12), 1998.

[2] A. Ghanem, et al. *Arlequin framework for multi-model, multi-time scale and heterogeneous time integrators for structural transient dynamics. CMAME*, 254(C):292–308, 2013.

[3] J. Albella, H. Ben Dhia, S. Imperiale and J. Rodríguez. *Improved Arlequin method for the transient wave equation.* Proceedings CEDYA-2015, 827–832, 2015.

A MATHEMATICAL MODEL FOR INDUCTION HARDENING INCLUDING NONLINEAR MAGNETIC FIELD AND CONTROLLED JOULE HEATING

Jaroslav Chovan and Marián Slodička

Department of Mathematical Analysis,
Ghent University, Galglaan 2, 9000 Ghent, Belgium,
`jaroslav.chovan@ugent.be`, `marian.slodicka@ugent.be`

We provide a derivation and an analysis of the mathematical model for induction hardening. We assume a non-linear relation between the magnetic field and the magnetic induction field.

The coupling between the electromagnetic and the thermal part is provided through the temperature-dependent electric conductivity and the joule heating term, the most crucial element, considering the mathematical analysis of the model. It functions as a source of heat in the thermal part and leads to the increase in temperature. Therefore, in order to be able to control it, we apply a truncation function.

Using the Rothe's method, we prove the existence of the global solution to the whole system. The nonlinearity in the electromagnetic part is overcome by utilizing the theory of monotone operators and the technique of Minty-Browder.

OPTIMIZED SCHWARZ METHODS FOR THE STOKES-DARCY PROBLEM

Marco Discacciati[1] and Luca Gerardo-Giorda[2]

[1]Department of Mathematical Sciences, Loughborough University, UK
m.discacciati@lboro.ac.uk

[2]BCAM – Basque Center for Applied Mathematics, Spain
lgerardo@bcamath.org

In this talk we present optimized Schwarz methods for the coupled system formed by the Stokes and the Darcy equations. Transmission conditions of Robin type are introduced and the coupled problem is reduced to a suitable interface system that can be solved using Krylov methods. A practical strategy to compute optimal Robin coefficients is considered which takes into account both the physical parameters of the problem and the size of the mesh. Numerical results show the effectiveness of our approach.

ON ω-NONLINEAR EIGENVALUE PROBLEMS WITH APPLICATIONS IN ELECTROMAGNETICS

Christian Engström

Department of Mathematics and Mathematical Statistics,
Umeå University, Sweden
christian.engstrom@math.umu.se

On a macroscopic scale dielectric materials are described by the polarization P of the material. Electromagnetic problems with time-dependent material parameters can then be expressed as a coupled problem in P and the electric field E. The corresponding eigenvalue problem is in general nonlinear in the frequency ω. Another example of a ω-nonlinear eigenvalue problem is a resonance problem with a Dirichlet-to-Neumann map on an artificial boundary. These two ω-nonlinear eigenvalue problems have similar structure, but the behaviour of eigenvalues close to a pole is completely different.

Let M_ℓ, $\ell = 1, 2, \ldots, L$ denote bounded linear operators in a Hilbert space $\mathcal{H}$ and denote by A a self-adjoint operator with compact resolvent that is bounded from below. In this talk we consider operator functions of the form

$$\mathcal{S}(\omega) = A - \omega^2 - \sum_{\ell=1}^{L} f_\ell(\omega) M_\ell, \quad \operatorname{dom} \mathcal{S}(\omega) = \operatorname{dom} A, \quad \omega \in \Omega,$$

which include operator functions that describe problems with ω-dependent material coefficients and resonance problems. We prove spectral properties of $\mathcal{S}$ and propose a new enclosure of the numerical range. Finally, we discuss convergence results for Galerkin approximations and computation of eigenvalues of matrix-valued functions.

The talk is based on joint works with Juan Carlos Araujo-Cabarcas, Luka Grubišić, Elias Jarlebring, Heinz Langer, Axel Torshage, and Christiane Tretter.

A MIXED FORMULATION FOR LARGE DEFORMATION CONTACT PROBLEM USING ISOGEOMETRIC ANALYSIS

Pablo Antolin[1], Ericka Brivadis[2a], Annalisa Buffa[2b] and Mathieu Fabre[2c]

[1]Dipartimento di Matematica "F. Casorati",
Università degli Studi di Pavia - Via Ferrata, 5 - 27100 Pavia, Italy
pablo.antolinsanchez@unipv.it

[2]IMATI CNR "E. Magenes", Via Ferrata 1, 27100 Pavia, Italy.
[a]ericka.brivadis@iusspavia.it, [b]annalisa@imati.cnr.it,
[c]mathieu@imati.cnr.it

IsoGeometric Analysis (proposed by T. Hughes and coauthors in [2]) uses B-Splines and Non-Uniform Rational B-Splines (NURBS) as basis functions to solve partial differential equations.

In this talk, we will consider rigid-deformable contact problems in large deformations. The contact constraints are treated with a mortar like approach combined with a interpolation of gap (to see [1] on a second order elliptic equations and [3] using a augmented Lagrangian method). These constraints are satisfied with a Lagrangian formulation to impose the Signorini contact conditions and an Active Set Strategy [4] ensures the complementary conditions. Some numerical results will be presented showing the good convergence properties of our algorithms.

References

[1] E. Brivadis, A. Buffa, B. Wohlmuth, and L. Wunderlich, *Isogeometric mortar methods*, Computer Methods in Applied Mechanics and Engineering, 284 (2015), pp. 292–319.

[2] T. J. R. Hughes, J. A. Cottrell and Y. Bazilevs, *Isogeometric analysis: CAD, finite elements, NURBS, exact geometry and mesh refinement*, Computer Methods in Applied Mechanics and Engineering, 194 (2005), pp. 4135–4195.

[3] L. De Lorenzis, P. Wriggers, and G. Zavarise, *A mortar formulation for 3d large deformation contact using nurbs-based isogeometric analysis and the augmented Lagrangian method*, Springer-Verlag, 49 (2012), pp. 1–20.

[4] A. E. Maliki, M. Fortin, J. Deteix, and A. Fortin, *Preconditioned iteration for saddle-point systems with bound constraints arising in contact problems*, Computer Methods in Applied Mechanics and Engineering, 254 (2013), pp. 114–125.

NUMERICAL METHODS FOR IMMERSED FSI WITH THIN-WALLED SOLIDS

Frédéric Alauzet[1], Benoit Fabrèges[2,3] Miguel A. Fernández[2,3] and Mikel Landajuela[2,3]

[1]Inria and Université Paris-Saclay, 91120 Palaiseau, France

[2]Inria Paris, 75012 Paris, France
miguel.fernandez@inria.fr

[3]Sorbonne Universités, UPMC, Lab. Jacques-Louis Lions, 75005 Paris, France

We present a Nitsche-XFEM method for fluid-structure interaction problems involving a thin-walled elastic structure (Lagrangian formalism) immersed in an incompressible fluid (Eulerian formalism). The fluid domain is discretized with an unstructured mesh not fitted to the solid mid-surface mesh. Weak and strong discontinuities across the interface are allowed for the velocity and pressure, respectively. The kinematic-dynamic interface coupling is enforced consistently using a variant of Nitsche's method involving cut elements. Robustness with respect to arbitrary interface/element intersections is guaranteed through suitable stabilization. For the temporal discretization, we introduce a semi-implicit scheme which overcomes strong coupling without compromising stability and accuracy. Numerical examples, involving static and moving interfaces, illustrate the performance of the methods.

EIGENVALUE PROBLEM FOR A NETWORK OF STRUTS MODELING AN ELASTIC ENDOVASCULAR STENT

Luka Grubišić

University of Zagreb, Faculty of Science, Department of Mathematics, Croatia
luka@math.hr

Endovascular stents are thin metallic structures which are used for treating a narrowing of blood vessels (stenosis). Stents are typically modeled as an assembly of struts and since they are a metallic structure their small deformations are sufficiently well described by 3D linearized elasticity. However, a direct numerical treatment of such model would lead to considering equations of 3D linearized elasticity in a thin domain. This is a very challenging and time consuming numerical task. As an alternative we start from a simpler analytical approximation – a reduced model – which can be obtained using a one-dimensional model of a curved elastic rod. As a result we obtain a system of ordinary differential equations on a graph. Note that our chosen model has been obtained as a limit – in an appropriate Sobolev space – of the 3D elasticity as the diameter of the strut goes to zero. Associated eigenvalue problem is discretized using a mixed finite element method. As model problems for our approach we consider four different coronary stents which are commercially available at the market and present numerical results.

This is a joint work with Josip Tambaca and Josip Ivekovic.

A NATURAL FRAMEWORK FOR ISOGEOMETRIC FLUID-STRUCTURE-INTERACTION: COUPLING BEM AND SHELL MODELS

Luca Heltai[1], Josef Kiendl[2], Antonio DeSimone[3] and Alessandro Reali[4]

[1]SISSA - International School for Advanced Studies, Trieste, Italy
luca.heltai@sissa.it

[2]Institute for Applied Mechanics,
Technische Universität Braunschweig, Germany
j.kiendl@tu-braunschweig.de

[3]SISSA - International School for Advanced Studies, Trieste, Italy
desimone@sissa.it

[4]Department of Civil Engineering and Architecture, University of Pavia, Italy
alereali@unipv.it

The interaction between thin structures and incompressible Newtonian fluids is ubiquitous both in nature and in industrial applications. We present an isogeometric formulation of such problems which exploits a boundary integral formulation of Stokes equations [1] to model the surrounding flow, and a non linear Kirchhoff-Love shell theory [2, 3] to model the elastic behaviour of the structure. We propose three different coupling strategies: a monolithic, fully implicit coupling, a staggered, elasticity driven coupling, and a novel semi-implicit coupling, where the effect of the surrounding flow is incorporated in the non-linear terms of the solid solver through its damping characteristics. The novel semi-implicit approach is then used to demonstrate the power and robustness of our method, which fits ideally in the isogeometric paradigm, by exploiting only the boundary representation (B-Rep) of the thin structure middle surface.

References

[1] Luca Heltai, Marino Arroyo, and Antonio DeSimone. Nonsingular isogeometric boundary element method for Stokes flows in 3D. *Computer Methods in Applied Mechanics and Engineering*, 268:514–539, jan 2014.

[2] J. Kiendl, K.-U. Bletzinger, J. Linhard, and R. Wüchner. Isogeometric shell analysis with Kirchhoff-Love elements. *Computer Methods in Applied Mechanics and Engineering*, 198(49-52):3902–3914, nov 2009.

[3] Josef Kiendl, Ming Chen Hsu, Michael C H Wu, and Alessandro Reali. Isogeometric Kirchhoff-Love shell formulations for general hyperelastic materials. *Computer Methods in Applied Mechanics and Engineering*, 291:280–303, 2015.

AUXILIARY SUBSPACE ERROR ESTIMATES FOR ELLIPTIC PROBLEMS

Jeffrey S. Ovall[1], Harri Hakula[2] and Michael Neilan[3]

[1]Department of Mathematics and Statistics, Portland State University, USA
jovall@pdx.edu

[2]Department of Mathematics and System Sciences, Aalto University, Finland
harri.hakula@aalto.fi

[3]Department of Mathematics, University of Pittsburgh, USA
neilan@pitt.edu

Hierarchical basis methods are implicit schemes wherein global estimates and localindicators of the discretization error in a finite element space V are obtained by solving a global residual equation in an appropriate auxiliary space W. Traditionally, the space W has been chosen such that $V \oplus W$ is a natural finite element space, e.g. if V is the degree p Lagrange space on a given mesh, then $V \oplus W$ might be the degree $p+1$ Lagrange space on the same mesh (a p-hierarchy), or the degree p Lagrange space on a uniformly-refined mesh (an h-hierarchy). We provide a very different prescription for choosing the space W that yields provably efficient and reliable error esitmates at reasonable cost. We empirically demonstrate its robustness with respect to problem parameters (e.g. discontinuous and anisotropic diffusion with high contrasts) and polynomial degree.

A FULLY-MIXED FINITE ELEMENT METHOD FOR THE NAVIER-STOKES/DARCY COUPLED PROBLEM WITH NONLINEAR VISCOSITY

Sergio Caucao[1a], Gabriel N. Gatica[1b] and Ricardo Oyarzúa[2]

[1]CI^{2}MA and Departamento de Ingeniería Matemática,
Universidad de Concepción, Chile.
[a]scaucao@ci2ma.udec.cl, [b]ggatica@ci2ma.udec.cl

[2]GIMNAP-Departamento de Matemática,
Universidad del Bío-Bío, Chile and CI^{2}MA,
Universidad de Concepción, Chile.
royarzua@ubiobio.cl

We present an augmented mixed finite element method for the coupling of fluid flow with porous media flow. The flows are governed by a class of nonlinear Navier-Stokes and the linear Darcy equations, respectively, and the transmission conditions are given by mass conservation, balance of normal forces, and the Beavers-Joseph-Saffman law. We apply dual-mixed formulations in both domains, and the nonlinearity involved in the Navier-Stokes region is handled by setting the strain and vorticity tensors as auxiliary unknowns. In turn, since the transmission conditions become essential, they are imposed weakly, which yields the introduction of the traces of the porous media pressure and the fluid velocity as the associated Lagrange multipliers. Furthermore, since the convective term in the fluid forces the velocity to live in a smaller space than usual, we augment the variational formulation with suitable Galerkin type terms arising from the constitutive and equilibrium equations of the Navier-Stokes equations, and the relation defining the strain and vorticity tensors. The resulting augmented scheme is then written equivalently as a fixed point equation, so that the well-known Schauder and Banach theorems, combined with classical results on bijective monotone operators, are applied to prove the unique solvability of the continuous and discrete systems. In particular, given an integer $k \geq 0$, piecewise polynomials of degree $\leq k$, Raviart-Thomas spaces of order k, continuous piecewise polynomials of degree $\leq k+1$, and piecewise polynomials of degree lek are employed in the fluid for approximating the strain tensor, stress, velocity, and vorticity, respectively, whereas Raviart-Thomas spaces of order k and piecewise polynomials of degree $\leq k$ for the velocity and pressure, together with continuous piecewise polynomials of degree $\leq k+1$ for the traces, constitute feasible choices in the porous medium. Finally, several numerical results illustrating the good performance of the augmented mixed finite element method and confirming the theoretical rates of convergence are reported.

A FULLY EULERIAN FINITE ELEMENT DISCRETIZATION FOR FLUID-STRUCTURE INTERACTIONS

Thomas Richter[1] and Stefan Frei[2]

[1]Department Mathematik, Universität Erlangen-Nürnberg, Germany
richter@math.fau.de

[2]Institut für Angewandte Mathematik, Universität Heidelberg, Germany
stefan.frei@iwr.uni-heidelberg.de

Problems with very large deformation, motion of the solid and even contact raise problems for the ALE formulation of fluid-structure interactions. A domain map between a reference system and the current system cannot deal with changes of topology. If a strictly monolithic system is desirable due to reasons of stability of efficiency, a Fully Eulerian formulation, where both subproblems are cast onto the current coordinate system is a promising alternative.

In this contribution we report on recent advances and applications of the Fully Eulerian Formulation for fluid-structure interactions. In particular, we focus on the question of interface accuracy, which is critical, as the Fully Eulerian method is of interface capturing type. Furthermore, we present extensions of this model to incorporate active material growth and pre-stressing.

PARTITIONED FLUID-SHELL COUPLING BASED ON A COARSE HIGHER ORDER MESH AND POSITIONAL DYNAMICS SHELL FINITE ELEMENT

Jeferson Wilian Dossa Fernandes and Rodolfo André Kuche Sanches

Department of Structural Engineering, São Carlos School of Engineering, University of São Paulo, São Calors, SP, Brazil
rodolfo.sanches@usp.br

We present a partitioned algorithm for fluid-shell interaction analysis using the finite element method (FEM) considering large structural displacements. The shell structure is modeled according to the Reissner-Mindlin kinematics, allowing thick shells modeling, and its FEM formulation is written with nodal positions and components of an unconstrained vector as degrees of freedom instead of displacements and rotations, avoiding problems related to large rotations approximations. Newmark time integrator is used for the structure and reveals to be stable and to present momentum conserving properties and enough energy conservation for most of the problems. The fluid governing equations are written in the arbitrary Lagrangian-Eulerian (ALE) description and solved by an implicit time integrator algorithm with mixed FEM approach for the incompressible cases and by one explicit characteristic based time integrator and standard finite elements for the compressible cases. The fluid-shell coupling is performed by a partitioned explicit Dirichlet-Neumann algorithm and the fluid mesh is updated by using a linear Laplacian smoothing. In order to save computing time and avoid element inversion in the Laplacian smoothing scheme, we introduce a coarse higher order auxiliary mesh which we call space mesh and use it only to capture the structural deformation and extend it to the fluid domain. Finally, the methodology is tested by numerical examples.

SCALABLE NEWTON-KRYLOV-BDDC METHODS FOR CARDIAC ELECTROMECHANICS

Simone Scacchi[1a], Piero Colli Franzone[2], Luca F. Pavarino[1b] and Stefano Zampini[3]

[1]Department of Mathematics, University of Milan, Italy
[a]simone.scacchi@unimi.it, [b]luca.pavarino@unimi.it

[2]Department of Mathematics, University of Pavia, Italy
colli@imati.cnr.it

[3]Extreme Computing Research Center,
King Abdullah University of Technology, Saudi Arabia
stefano.zampini@kaust.edu.sa

We present a Balancing Domain Decomposition by Constraints (BDDC) preconditioner for the model of cardiac mechanics. The contraction-relaxation process of the cardiac muscle, induced by the spread of the electrical excitation, is quantitatively described by a mathematical model called electro-mechanical coupling. The electric model consists of a non-linear degenerate parabolic system of two partial differential equations (PDEs), the so-called Bidomain model, which describes the spread of the electric impulse in the heart muscle. The PDE is coupled with the non-linear elasticity system, where the myocardium is considered as a nearly-incompressible transversely isotropic hyperelastic material. The discretization of the whole electro-mechanical model is performed by Q1 finite elements in space and a semi-implicit finite difference scheme in time. This approximation strategy yields at each time step the solution of a large scale linear system deriving from the discretization of the Bidomain model and a non-linear system deriving from the discretization of the finite elasticity equations. The parallel mechanical solver consists of solving the non-linear system with a Newton-Krylov-BDDC method, with different choices of coarse spaces. Three-dimensional parallel numerical tests on a Linux cluster show that the parallel solver proposed is scalable and quasi-optimal. Simulations based on the solver developed are performed to study the reliability of extracellular markers of repolarization in presence of domain deformations.

NGS-PY: A NATURAL LANGUAGE FOR HP-FEM IN MULTIPHYSICS

Joachim Schöberl

Institute for Analysis and Scientific Computing, TU Wien, Austria
joachim.schoeberl@tuwien.ac.at

Netgen/NGSolve is open source high order finite element code which provides a variety of scalar, vector-valued and tensor-valued hp-finite element spaces as needed for simulation in multiphysics. The design is object oriented, where grid-functions, bilinear- and linear forms, preconditions are C++ objects. NGSolve has a variety of built-in integrators for the classical equations and different discretisation concepts.

We present the recent redesign NGS-Py, where the C++ objects are accessible from the Python scripting language. Furthermore, equations can be provided symbolically in variational formulation, similar to the popular FEniCS system. This renders the variety of hand-written integrators obsolete. We explain in detail how element matrix calculation is now implemented, and how performance compares to the hand-written C++ code.

We show several examples demonstrating the flexibility of the interface, and the obtained performance for system assembly and solver parts.

We think this tool is in particular useful for algorithm development for multiphysics problems. The software and documentation is available from
https://gitlab.asc.tuwien.ac.at/jschoeberl/ngsolve-docu/wikis/ngspy

PARTITIONED ALGORITHMS FOR FLUID-STRUCTURE INTERACTION ARISING IN HEMODYNAMICS

Christian Vergara

MOX, Dipartimento di Matematica, Politecnico di Milano, Italy
christian.vergara@polimi.it

We propose a unified convergence analysis of the generalized Schwarz method applied to a linear elliptic problem for a general interface (flat, cylindrical or spherical) in any dimension. In particular, we provide the exact convergence set of the interface symbols related to the operators involved in the transmission conditions. We also provide a general procedure to obtain estimates of the optimized interface symbols within the constants. We apply such general results to the fluid-structure interaction problem arising in haemodynamics, obtaining partitioned algorithms based on Robin interface conditions. A proper choice of the interface parameters involved in these conditions allows us to obtain efficient algorithms which do not suffer from the high added mass effect which characterizes haemodynamic applications. The numerical results both in ideal and real geometries highlighted the suitability of our proposals.