MAFELAP 2016

Conference on the Mathematics of Finite Elements and Applications

14–17 June 2016

Mini-Symposium: PDE Eigenvalue problems: computational modeling and numerical analysis

Organisers:

Christian Engström, Stefano Giani, Luka Grubišić and Jeffrey Ovall

Abstracts in alphabetical order

Contents

A POSTERIORI ANALYSIS FOR MAXWELL'S EIGENVALUE PROBLEM

Daniele Boffi

Dipartimento di Matematica "F. Casorati",
University of Pavia, Italy
daniele.boffi@unipv.it

We discuss the finite element approximation of Maxwell's eigenvalue problem. A widely used tool for the analysis of this problem is a suitable mixed formulation. In this talk we show how to define an a posteriori error indicator for the mixed problem and how to implement it in the framework of the original formulation. A posteriori error analysis is performed for the proposed indicator. This is a joint work with L. Gastaldi, R. Rodríguez, and I. Šebestová.

OPTIMALITY OF ADAPTIVE FINITE ELEMENT METHODS FOR EIGENVALUE CLUSTERS

Andrea Bonito[a] and Alan Demlow

Department of Mathematics, Texas A&M University, USA
[a]bonito@math.tamu.edu

We present recent results establishing optimality of standard adaptive finite element methods of arbitrary degree for eigenfunction computations for elliptic boundary value problems. Similar previous analyses have considered only lowest-order (piecewise linear) finite element spaces or multiple eigenvalues only. In contrast to previous results, our techniques also confirm that a critical input parameter in the adaptive FEM, the marking parameter, may be chosen independent of the target cluster being approximated.

NUMERICAL APPROXIMATION OF THE SPECTRUM OF THE CURL OPERATOR IN MULTIPLY CONNECTED DOMAINS

Ana Alonso Rodríguez[1a], Jessika Camaño[2], Rodolfo Rodríguez[3], Alberto Valli[1b] and Pablo Venegas[4]

[1]Department of Mathematics, University of Trento, Italy
[a]alonso@science.unitn.it, [b]valli@science.unitn.it

[2]Departamento de Matemática y Física Aplicadas,
Universidad Católica de la Santísima Concepción and CI^{2}MA,
Universidad de Concepción, Chile
jecamano@ucsc.cl

[3]CI^{2}MA and Departamento de Ingeniería Matemática,
Universidad de Concepción, Chile
rodolfo@ing-mat.udec.cl

[4]Departamento de Matemática, Universidad del Bío Bío, Chile
pvenegas@ubb.cl

The aim of this work is to analyze the numerical approximation of the eigenvalue problem for the curl operator on a multiply connected domain. In order to obtain a well-posed eigenvalue problem, additional constraints must be imposed (see [3]). A combination between two type of constraints related to the homology of the domain have been added in order that the problem has a discrete spectrum (see [2]). A mixed variational formulation of the resulting problem and a finite element discretization are introduced and shown to be well-posed. Optimal-order spectral convergence is proved, as well as a priori error estimates, by using classical spectral approximation results (see [1]). It is described how to implement this numerical method taking care of these additional constraints. Finally the results of some numerical tests are also reported.

References

[1] I. Babuška and J. Osborn. Eigenvalue Problems. Handbook of Numerical Analysis. Vol. II, Finite Element Method (Part I) (P. G. Ciarlet, J. L. Lions, eds.). North-Holland Publ., Amsterdam, 1991, pp. 641–787.

[2] R. Hiptmair, P.M. Kotiuga, and S. Tordeux. Self–adjoint curl operators. *Ann. Mat. Pura Appl*, (4), 191 (2012), no. 3, 431–457.

[3] E. Lara, R. Rodríguez, and P. Venegas. Spectral approximation of the curl operator in multiply connected domains. *Discrete Contin. Dyn. Syst. Ser. S* (to appear).

REDUCED BASIS APPROXIMATION AND A POSTERIORI ERROR ESTIMATES FOR PARAMETRIZED ELLIPTIC EIGENVALUE PROBLEMS

Ivan Fumagalli[1a], Andrea Manzoni[2], Nicola Parolini[1b] and Marco Verani[1c]

[1]MOX - Dipartimento di Matematica, Politecnico di Milano, Italy
[a]ivan.fumagalli@polimi.it, [b]nicola.parolini@polimi.it,
[c]marco.verani@polimi.it

[2]CMCS-MATHICSE-SB, École Polytechnique Fédérale de Lausanne, Switzerland
andrea.manzoni@epfl.ch

In many applications, ranging from optics and electronics to acoustics and structural mechanics, the solution of eigenproblems plays a crucial role. Moreover, repeated solutions are required, for different physical or geometrical settings, as soon as optimal control issues or inverse problems are addressed. In this framework, the reduced basis (RB) method can represent a suitably efficient technique to contain the demanded computational effort, especially in a many-query context. Starting from the pioneering work [1], in the last fifteen years the RB method has been applied to linear and nonlinear eigenproblems, also depending on a high number of parameters [2]. Nevertheless, few results on the *a posteriori* error estimation of the reduced order solution have been published.

In [3], we develop a new RB method for the approximation of a parametrized eigenproblem for the Laplacian. This method hinges upon dual weighted residual type *a posteriori* error indicators, which give rigorous upper bounds, for any value of the parameters, of the error between the high-fidelity finite element approximation of the first eigenvalue and eigenfunction and the corresponding RB approximations. The proposed error estimators are exploited not only to certify (*online*) the RB approximation, but also to set up a greedy algorithm for the *offline* construction of the RB space. Furthermore, a computationally inexpensive approximation of the inf-sup coefficient on which the error bounds depend is provided, addressing an issue that often represents a bottleneck in the efficient application of reduced order approximations. Several numerical experiments assess the overall reliability and efficiency of the proposed RB approach, both for affine and non-affine parametrizations.

References

[1] L. Machiels, Y. Maday, I. Oliveira, A.T. Patera, and D. Rovas, *Output bounds for reduced-basis approximations of symmetric positive definite eigenvalue problems.* C. R. Acad. Sci. Paris Sér. I Math., 331(2):153-158, 2000.

[2] D.B.P. Huynh, D.J. Knezevic, and A.T. Patera, *A static condensation reduced basis element method: approximation and a posteriori error estimation.* ESAIM: Math. Model. Numer. Anal., 47:213 251, 2013.

[3] I. Fumagalli, A. Manzoni, N. Parolini, and M. Verani, *Reduced basis approximation and a posteriori error estimates for parametrized elliptic eigenvalue problems.* ESAIM: Math. Model. Numer. Anal., 2016, accepted for publication.

ADAPTIVE MIXED FINITE ELEMENTS FOR EIGENVALUES

Daniele Boffi[1a], Dietmar Gallistl[2], Francesca Gardini[1b] and Lucia Gastaldi[3]

[1]Dipartimento di Matematica "F. Casorati", University of Pavia, Italy
[a]daniele.boffi@unipv.it, [b]francesca.gardini@unipv.it

[2]Institut für Numerische Simulation, Universität Bonn, Germany
gallistl@ins.uni-bonn.de

[3]DICATAM, University of Brescia, Italy
lucia.gastaldi@unibs.it

It is shown that the h-adaptive mixed finite element method for the discretization of eigenvalue clusters of the Laplace operator produces optimal convergence rates in terms of nonlinear approximation classes. The results are valid for the typical mixed spaces of Raviart–Thomas or Brezzi–Douglas–Marini type with arbitrary fixed polynomial degree in two and three space dimensions. The talk is based on the work [1].

References

[1] D. Boffi, D. Gallistl, F. Gardini, and L. Gastaldi. Optimal convergence of adaptive FEM for eigenvalue clusters in mixed form. *ArXiv e-prints*, 1504.06418, 2015.

AN INTERIOR PENALTY METHOD WITH C^0 FINITE ELEMENTS FOR THE APPROXIMATION OF THE MAXWELL EQUATIONS IN HETEROGENEOUS MEDIA: CONVERGENCE ANALYSIS WITH MINIMAL REGULARITY

Andrea Bonito[1], Jean-Luc Guermond[1a] and Francky Luddens[2]

[1]Department of Mathematics,
Texas A&M University, College Station, TX 77843-3368, USA
[a]guermond@math.tamu.edu

[2]LMRS, CNRS-Université de Rouen Avenue de l'Université, BP.12 F76801
Saint-Étienne-du-Rouvray, France

The present paper proposes and analyzes an interior penalty technique using C^0-finite elements to solve the Maxwell equations in domains with heterogeneous properties. The convergence analysis for the boundary value problem and the eigenvalue problem is done assuming only minimal regularity in Lipschitz domains. The method is shown to converge for any polynomial degrees and to be spectrally correct.

A FRAMEWORK OF HIGH-PRECISION VERIFIED EIGENVALUE BOUNDS FOR SELF-ADJOINT DIFFERENTIAL OPERATORS

Xuefeng Liu

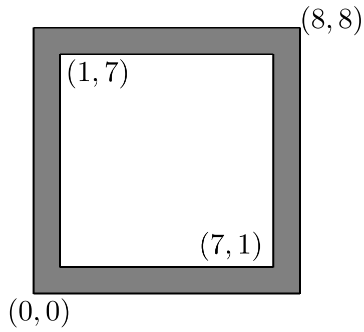
Niigata University, Japan
xfliu@math.sc.niigata-u.ac.jp

A universal framework is proposed to give high-precision explicit lower and upper bounds for the eigenvalues of self-adjoint differential operators [1]. In the case of the Laplacian operator, by applying Crouzeix–Raviart finite elements, an efficient algorithm is developed to bound the eigenvalues for the Laplacian defined in 1D, 2D and 3D spaces. For biharmonic operators, Fujino–Morley FEM is adopted to bound the eigenvalues. To obtain high-precision eigenvalue bounds, Lehmann–Goerisch's theorem along with high-order finite element methods is adopted[3, 2]. See Table 1 for a sample computation result for eigenvalue of Laplacian with homogeneous boundary condition over square-minus-square domain, where there exist singularities of eigenfunction around the reentrant corners.

By further adopting the interval arithmetic, the explicit eigenvalue bounds from numerical computations can be mathematically correct. As a computer-assited proof, the verified eigenvalue bounds have been used to investigate the solution existence of semi-linear elliptic differential equations; see, e.g., [4].

Bounds for the eigenvalues of Laplacian over square-minus-square domain [2]
(with homogeneous Dirichlet boundary condition)

λ_i	Eigenvalue bound	Relative Error
1	9.16021_{37}^{64}	2.8E-7
2	9.17008_{61}^{89}	2.9E-7
3	9.17008_{61}^{89}	2.9E-7
4	9.18056_{52}^{80}	3.0E-7
5	10.089843_{14}^{37}	2.2E-8



References

[1] Liu, X., A framework of verified eigenvalue bounds for self-adjoint differential operators, *Applied Mathematics and Computation*, 267, pp.341-355, 2015

[2] Liu, X., Okayama, T. and Oishi, S., High-precision eigenvalue bound for the Laplacian with singularities. *Computer Mathematics*, pp.311-323, Springer, 2014

[3] Liu, X. and Oishi, S., Guaranteed high-precision estimation for P_0 interpolation constants on triangular finite elements, *Japan Journal of Industrial and Applied Mathematics*, 30, pp.635-652, 2013

[4] Takayasu, A., Liu, X. and Oishi, Verified computations to semilinear elliptic boundary value problems, *Nonlinear Theory and Its Applications, IEICE*, 4(1), pp.34-61. doi:10.1588/nolta.4.34, 2013

A BAYESIAN APPROACH TO EIGENVALUE OPTIMIZATION

Sebastian Dominguez[1], Nilima Nigam[1] and Bobak Shahriari[2]

[1]Department of Mathematics, Simon Fraser University, Burnaby, Canada
nigam@math.sfu.ca

[2]Department of Computer Science, University of British Columbia, Vancouver, Canada

A celebrated conjecture by Polyá and Szegö asserts that amongst all n-sided polygons of a given area, the regular n-gon is the global optimizer of the first Dirichlet eigenvalue of the Laplacian. This conjecture has been shown to hold for triangles and quadrilaterals, but is open for pentagons.

In this talk, we present a novel framework for eigenvalue optimization combining finite element computations in a validated numerics setting, with a Bayesian optimization approach. We illustrate this approach for the specific case of the Polyá-Szegö conjecture on pentagons.

HIGH-ORDER MORTAR FINITE ELEMENT DISCRETIZATION FOR PDE EIGENVALUE PROBLEMS AND ERROR ESTIMATION

Kersten Schmidt[1a], Reinhold Schneider[1b] and Agnieszka Miedlar[2]

[1]Research Center Matheon and Department of Mathematics,
Technische Universität Berlin, Germany
[a]kersten.schmidt@math.tu-berlin.de, [b]schneidr@math.tu-berlin.de

[2]Department of Computer Science & Engineering,
University of Minnesota, USA
amiedlar@umn.edu

Mortar element methods use a decomposition of the computational domain and couple different discretization spaces in the subdomains weakly by a mortar condition. We use for example a high-order mortar element method for full-potential electronic structure calculations [1]. For this we use a spherical discretization in spherical elements around each nucleus, which is adapted to resolve the core singularity due to an unbounded potential term, is coupled to a finite element discretization in between the nuclei. We discuss the error of the mortar element method with uniform refinement as well as the reliablility of a residual error estimator. With a series of numerical experiments we illustrate the theoretical convergence results for uniform refinement also in comparison with a conforming *hp*-adaptive finite element method and a *p*-adaptive refinement strategy based on the residual error estimator.

References

[1] Huajie Chen and Reinhold Schneider. Numerical analysis of augmented plane wave methods for full-potential electronic structure calculations. *ESAIM Math. Model. Numer. Anal. (M2AN)*, 49(3):755–785, 2015.

GUARANTEED AND ROBUST A POSTERIORI BOUNDS FOR LAPLACE EIGENVALUES AND EIGENVECTORS

Benjamin Stamm[1], Eric Cancès[2], Geneviève Dusson[3], Yvon Maday[4], and Martin Vohralík[5]

[1]Center for Computational Engineering Science,
RWTH Aachen University, Aachen, Germany;
Computational Biomedicine, Institute for Advanced Simulation IAS-5
and Institute of Neuroscience and Medicine INM-9,
Forschungszentrum Jülich, Germany;
best@mathcces.rwth-aachen.de

[2]Université Paris Est, CERMICS, Ecole des Ponts and INRIA,
6 & 8 Av. Pascal, 77455 Marne-la-Vallée, France;
cances@cermics.enpc.fr

[3]Sorbonne Universités, UPMC Univ. Paris 06 and CNRS, UMR 7598,
Laboratoire Jacques-Louis Lions, F-75005 Paris, France;
Sorbonne Universités, UPMC Univ. Paris 06,
Institut du Calcul et de la Simulation, F-75005, Paris, France;
dusson@ann.jussieu.fr

[4]Sorbonne Universités, UPMC Univ. Paris 06 and CNRS, UMR 7598,
Laboratoire Jacques-Louis Lions, F-75005 Paris, France;
Institut Universitaire de France;
and Division of Applied Mathematics; Brown University,
Providence, Rhode Island, USA;
maday@ann.jussieu.fr

[5]INRIA Paris-Rocquencourt, Domaine de Voluceau - Rocquencourt,
B.P. 105, 78153 Le Chesnay, France;
martin.vohralik@inria.fr

In this talk we present a posteriori error estimates for conforming numerical approximations of the Laplace eigenvalue problem with a homogeneous Dirichlet boundary condition. In particular, upper and lower bounds for the first eigenvalue are given. These bounds are guaranteed, fully computable, and converge with the optimal speed to the exact eigenvalue. They are valid under an explicit, a posteriori, minimal resolution condition on the computational mesh and the approximate solution; we also need to assume that the approximate eigenvalue is smaller than a computable lower bound on the second smallest eigenvalue, which can be satisfied in most cases of practical interest by including the computational domain into a rectangular parallelepiped or a d-sphere. Guaranteed, fully computable, and polynomial-degree robust bounds for the energy error in the approximation of the first eigenvector are derived as well, under the same conditions. Remarkably, there appears no unknown (solution-, regularity-, or polynomial-degree-dependent) constant in our theory, and no convexity/regularity assumption on the computational domain/exact eigenvector(s) is needed.