MAFELAP 2016

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Rolf Krause, Pietro Benedusi, Xiaozhou Li, Daniel Hupp and Peter Arbenz

A Trefftz polynomial space-time discontinuous Galerkin method for the second order wave equation
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Stable FEMs with applications
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Finite Element – Eulerian Lagrangian Localized Adjoint Method for an oil recovery model: breakdown, and correction
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Stable splitting of polyharmonic operators
Dietmar Gallistl

A fictitious domain approach with a distributed Lagrange multiplier for fluid-structure interactions
Daniele Boffi and Lucia Gastaldi

Robust residual-based a posteriori Arnold-Winther mixed finite element analysis in elasticity
Joscha Gedicke and Carsten Carstensen

Convergent semi-Lagrangian methods for the Monge-Ampère equation on unstructured grids
Max Jensen and Xiaobing Feng

Underpenalized discontinuous Galerkin methods for radiation transport
Guido Kanschat

Quasi-optimal Variational Multiscale stabilization of convection-diffusion equations
Guanglian Li, Daniel Peterseim and Mira Schedensack
A posteriori error estimates for the finite element approximations of the von Kármán equations
Neela Nataraj, Gouranga Malik and Carsten Carstensen

Fully stable and fully consistent nonconforming Galerkin methods
Andreas Veeser and Pietro Zanotti

Uncertainty quantification using stochastic PDEs and finite elements
Advection-diffusion equations with random coefficients on moving hypersurfaces
Ana Djurdjevac, Charles M. Elliott, Ralf Kornhuber and Thomas Ranner

Adaptive stochastic Galerkin FEM with hierarchical tensor representations
Martin Eigel, Max Pfeffer and Reinhold Schneider

Efficient error estimation and fast solvers for stochastic Galerkin finite element approximation
Catherine E. Powell, Adam Crowder, David Silvester and Valeria Simoncini

An optimal solver for linear systems arising from stochastic FEM approximation of diffusion equations with random coefficients
David Silvester and Pranjal

Analysis of the Ensemble Kalman Filter for Inverse Problems
Claudia Schillings and Andrew Stuart

Adaptive algorithms driven by a posteriori estimates of error reduction for PDEs with random data
David Silvester, Alex Bespalov and Catherine E. Powell

Gaussian process regression in Bayesian inverse problems
Aretha Teckentrup and Andrew Stuart

Multilevel Monte Carlo Analysis for Optimal Control of Elliptic PDEs with Random Coefficients
Ahmad Ahmad Ali, Elisabeth Ullmann and Michael Hinze
In this talk we deal with the integral version of the Dirichlet homogeneous fractional Laplace equation. For this problem, weighted and fractional Sobolev a priori estimates are provided in terms of the Hölder regularity of the data. By relying on these results, optimal order of convergence for the standard linear finite element method is proved for adapted meshes designed to handle the singular behavior of solutions near the boundary. Some numerical examples are given showing results in agreement with the theoretical predictions.
Magnetohydrodynamics (MHD) models describe a wide range of plasma physics applications, from thermonuclear fusion in tokamak reactors to astrophysical models. These models are characterized by a nonlinear system of partial differential equations in which the flow of the fluid strongly couples to the evolution of electromagnetic fields. In this talk, we consider the one-fluid, viscoresistive MHD model in two dimensions. There have been numerous finite-element formulations applied to this problem, and we will briefly discuss the applications of two: a least-squares and mixed-method formulation. In the latter, we consider inf-sup stable elements for the incompressible Navier-Stokes portion of the formulation, Nedélec elements for the magnetic field, and a second Lagrange multiplier added to Faraday’s law to enforce the divergence-free constraint on the magnetic field.

Regardless of the formulation, the discrete linearized systems that arise in the numerical solution of these equations are generally difficult to solve, and require effective preconditioners to be developed. Thus, the final portion of the talk, will involve a discussion of monolithic multigrid preconditioners, using an extension of a well-known relaxation scheme from the fluid dynamics literature, Vanka relaxation, to this formulation. To isolate the relaxation scheme from the rest of the multigrid method, we utilize structured grids, geometric interpolation operators, and Galerkin coarse grid operators. Numerical results are shown for the Hartmann flow problem, a standard test problem in MHD.
Fitted ALE Scheme for Two-Phase Navier–Stokes Flow

Marco Agnese\textsuperscript{a} and Robert N"urnberg\textsuperscript{b}

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We present a novel fitted ALE scheme for two-phase Navier–Stokes flow problems that uses piecewise linear finite elements to approximate the moving interface. The meshes describing the discrete interface in general do not deteriorate in time, which means that in numerical simulations a smoothing or a remeshing of the interface mesh is not necessary.

An Isogeometric Approach to Symmetric Galerkin Boundary Element Method

Alessandra Aimi\textsuperscript{1a}, Mauro Diligenti\textsuperscript{1b}, Maria Lucia Sampoli\textsuperscript{2} and Alessandra Sestini\textsuperscript{3}

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The Isogeometric Analysis (IgA) approach, introduced by Hughes and collaborators \cite{Hughes}, establishes a strict relation between the geometry of the problem domain and the approximate solution representation, giving surprising computational advantages. It has also brought a renewed interest for Boundary Element Methods (BEMs), since one has to discretize only the boundary of the problem domain and this can be done in an accurate way by powerful geometric modeling techniques.

Among BEMs, the Symmetric Galerkin version (SGBEM) \cite{SGBEM} is recognized as particularly suitable for mixed boundary value problems and for coupling with FEM. In this context, we have recently introduced the IgA concept into SGBEM, using classical B-splines \cite{B-splines} to represent both the boundary and the approximate solution. In this talk we will discuss about an extension including NURBS and generalized B-splines \cite{NURBS, Generalized B-splines}. The computational advantages over standard and curvilinear SGBEMs, where the numerical solution is given by means of Lagrangian basis functions, will be underlined by several numerical results.
High order finite element methods have been analysed extensively for a wide variety of applications and are known to be capable of producing exponential rates of convergence, even for challenging problems with singularities, sharp boundary layers and high frequency oscillations. High order polynomial approximations are commonplace in many areas of scientific computing including computer graphics, computer aided-geometric design, and spectral methods for PDEs. It is commonplace to see the spectral method used with approximation orders in the 100s or even 1000s. Yet, despite theory giving the nod to the use of very high order finite element methods, the range of polynomial degree used in practical finite element computations is rarely larger than eighth order! Few commercial codes allow the use of high order finite elements. The rather modest polynomial degrees seen in high order finite element analysis are due to efficiency considerations rather than any theoretical barriers. Bernstein-Bezier polynomials have a number of interesting properties that have led to their being the industry standard for visualisation and CAGD. We explore the use of Bernstein polynomials as a basis for finite element approximation.
In this paper, we analyse the high-order in time discontinuous Galerkin finite element method (DGFEM) for second-order in time evolution problems. We use a generalization of C Johnson (CMAME, 1993), with high orders in time, non-homogeneous boundary data; leading to an abstract Hilbert space variational formulation. Based on our abstract Hilbert space variational formulation we re-write the second order in time problem as a first-order system in time and we apply the discretization approach in time for the variational formulation of abstract parabolic problems introduced by D Schötzau (PhD Thesis, 1999).

We prove a priori error estimates and unconditional stability estimates within our abstract framework for finite polynomial degrees in time. Finally we apply our abstract framework to the acoustic wave equation.
The Arlequin method [1] is a flexible tool which allows to couple different models using an overlapping region. In [2] the method was used as a domain decomposition technique for the transient wave equation and in [3] some variants were presented on the 1D case to improve its flexibility on the discretization procedure. Now we apply such variants to Helmholtz equation in a 2D configuration:

Find the solution $u$ of:

$$\rho k^2 u + \text{div}(\mu \nabla u) = 0, \quad \text{in } \Omega \quad \text{s.t.} \quad u = u_D, \quad \text{in } \partial \Omega,$$

where $\rho, \mu \in L^\infty(\Omega)$ are strictly positive and $k$ denotes the wave number. (1)

To present the modified Arlequin formulation of (1) we decompose the domain $\Omega$ in two subdomains $\Omega_1$ and $\Omega_2$ such that $\Omega_1 \cap \Omega_2 = \omega_1 \cup \omega_c \cup \omega_2 \neq \emptyset$, where those $\omega_1$, $\omega_c$ and $\omega_2$ are disjoint non empty sets. We also need to introduce the spaces $V = H^1(\Omega_1) \times H^1(\Omega_2)$ and $M = H^1(\omega_1) \times H^1(\omega_2)$ and the coefficients $\alpha_1, \beta_1 > 0$ such that $\alpha_1 + \alpha_2 = \rho k^2$, $\beta_1 + \beta_2 = \mu$ and $\frac{\alpha_1}{\rho} = \frac{\beta_1}{\mu} = \text{cte}$ in $\omega_c$. Then, considering the usual scalar product $(\cdot, \cdot)_{k,\tilde{\Omega}}$ in $H^k(\tilde{\Omega})$, the variational formulation for the coupled problem reads:

$$\text{Find } (u, \lambda) \in V \times M \text{ s.t. } \forall (v, l) \in V \times M$$

$$\begin{align*}
(\alpha_1 u_1, v_1)_{\omega_1} &- (\beta_1 \nabla u_1, \nabla v_1)_{\omega_1} + (\alpha_2 u_2, v_2)_{\omega_2} - (\beta_2 \nabla u_2, \nabla v_2)_{\omega_2} + \\
(\lambda_{\omega_1}, v_1 - v_2)_{\omega_1} + (\lambda_{\omega_2}, v_1 - v_2)_{\omega_2} + (l_{\omega_1}, u_1 - u_2)_{\omega_1} + (l_{\omega_2}, u_1 - u_2)_{\omega_2} &= 0.
\end{align*}$$

This formulation (see [3] for the details in the 1D case) allows the use of independent meshes and offers the possibility to capture with the finest mesh the variations of the physical coefficients on $\omega_i, i \in \{1, 2\}$.

In the presentation, it will be shown that with this formulation one gets optimal convergence rate for first order finite elements but, unlike what happened in the 1D case, it provides sub-optimal results for quadratic elements. As it will be explained, this is due to the fact that the Lagrange multipliers $\lambda_{\omega_i}, i \in \{1, 2\}$ satisfy Laplace like equations set in domains with reentrant corners. The Arlequin formulation in [3] will be modified to make the method compatible with higher order elements. Numerical results will be presented to show the performance of the discretization procedure.
We present a recently developed rate-dependent cohesive-zone model which simulates crack growth along rubber interfaces. Postulating the existence of a rate-independent rupture energy, associated with the rupture of bonds, a damage variable is introduced, which is assumed to evolve as a rate-independent function of part of the elastic energy. The overall rate-dependent response is retrieved by introducing additional internal variables associated with viscous dissipation. The approach was validated against test results for a DCB made of two steel arms bonded along a rubber interface, with prescribed cross-head opening speeds ranging 5 logarithmic decades. Using a Mittag-Leffner relaxation function for the undamaged interface resulted in the first cohesive-zone model based on fractional viscoelasticity, which provides excellent correlation of experimental and numerical results across the entire range of tested speeds [3, 4].

We also discuss the accuracy and the computational cost of the numerical time integration of the fractional differential equations, which we determine via the Grünwald-Letnikov expression of the fractional derivative [2, 5].

Finally, we revisit a recently proposed thermodynamical derivation of our model [1], discussing alternative choices for the damage evolution law and how they can be physically justified for different polymeric materials.

References


A POSTERIORI ERROR ANALYSIS FOR A VISCOUS FLOW–TRANSPORT PROBLEM

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In this talk we present an a posteriori error analysis for an augmented mixed–primal finite element approximation of a stationary viscous flow and transport problem. The governing system corresponds to a scalar, nonlinear convection-diffusion equation coupled with a Stokes problem with variable viscosity, and it serves as a prototype model for sedimentation-consolidation processes and other phenomena where the transport of species concentration within a viscous fluid is of interest. The solvability of the continuous mixed–primal formulation along with a priori error estimates for a finite element scheme using Raviart-Thomas spaces of order k for the stress approximation, and continuous piecewise polynomials of degree ≤ k + 1 for both velocity and concentration, have been recently established in [M. Alvarez et al., ESAIM: Math. Model. Numer. Anal. 49 (5) (2015) 1399–1427]. Here we derive two efficient and reliable residual-based a posteriori error estimators for that scheme: For the first estimator, and under suitable assumptions on the domain, we apply a Helmholtz decomposition and exploit local approximation properties of the Clément interpolant and Raviart-Thomas operator to show its reliability. On the other hand, its efficiency follows from inverse inequalities and the localization arguments based on triangle-bubble and edge-bubble functions. Secondly, an alternative error estimator is proposed, whose reliability can be proved without resorting to Helmholtz decompositions. Our theoretical results are then illustrated via some numerical examples, highlighting also the performance of the scheme and properties of the proposed error indicators.

References

We present multigrid algorithms for the efficient solution of the linear system of equations arising from high-order discontinuous Galerkin discretizations of second-order elliptic problems on polygonal/polyhedral meshes. We prove that, under suitable assumptions on the agglomerated coarse grid, the two-level version of the method converges uniformly with respect to the granularity of the grid and the polynomial approximation degree $p$, provided that the number of smoothing steps, which depends on $p$, is chosen sufficiently large. An analogous result is then obtained for the multigrid algorithms, under an additional assumption on the maximum number of levels. Numerical experiments confirm the effectiveness of the proposed schemes. Moreover, we also demonstrate numerically that the proposed solvers are convergent in practice, even when some of the theoretical assumptions are not fully satisfied.
HIGH-ORDER DISCONTINUOUS GALERKIN APPROXIMATIONS TO SECOND-ORDER ORDINARY DIFFERENTIAL EQUATIONS WITH APPLICATIONS TO ELASTODYNAMICS

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In this talk we propose and analyze a high-order discontinuous Galerkin finite element method for the time integration of linear second-order ordinary differential equations. These equations typically arise after space semi-discretization of second order hyperbolic-type differential problems, e.g., the elastodynamics equation. After introducing the new method, we analyze its well-posedness and present a-priori error estimates in a suitable (mesh-dependent) norm. Numerical results are also presented to verify the theoretical estimates.

FINITE ELEMENTS FOR A CLASS OF PHASE TRANSITION PROBLEMS WITH NOISE

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We consider a class of nonlinear Stochastic PDEs with additive noise, stemming from various phase separation models for binary alloys. These equations may depend on a small parameter \( \varepsilon > 0 \) which describes the inner interfacial regions width between the two phases. We construct continuous and discontinuous Galerkin schemes for the numerical approximation of solutions, and prove optimal order of accuracy.
In this talk we discuss convergence results for finite element discretized Dirichlet control problems in polygonal domains. We investigate unconstrained as well as control constrained problems. In both cases we discretize the state and the control by piecewise linear and continuous functions. The error estimates, which we obtain, mainly depend on the size of the interior angles but also on the presence of control constraints and the structure of the underlying mesh. For instance, considering non-convex domains, the convergence rates of the discrete optimal controls in the unconstrained case can even be worse than in the control constrained case.
A MULTISCALE HYBRID-MIXED METHOD FOR THE STOKES AND BRINKMAN EQUATIONS

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In this work a multiscale hybrid-mixed method (MHM) ([1]), applied to the Stokes and the Brinkman equations on heterogeneous media, is introduced and analyzed. Given a coarse partition of the domain and using a hybrid formulation ([2]), the MHM method consists of independent Stokes (or Brinkman) local problems brought together by a face-based weak formulation on the skeleton of the partition. The multiple scales of the media are incorporated in the basis functions which are driven by the local problems with prescribed Neumann boundary conditions. Once available (exactly or approximatively), the multiscale basis functions are used to compute the degrees of freedom from a face-based global variational problem defined on the skeleton of the partition. The numerical solution shares the important properties of the continuum as the local equilibrium with respect to external forces and local mass conservation. Several numerical tests assess the accuracy and the conservative properties of MHM method on academic and highly heterogeneous cases.

References


NEW MIXED FINITE ELEMENTS ON QUADRILATERALS OF MINIMAL DIMENSION

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We develop two families of mixed finite elements on quadrilateral meshes for approximating \((u, p)\) solving a second order elliptic equation in mixed form. Standard Raviart-Thomas (RT) and Brezzi-Douglas-Marini (BDM) elements are defined on rectangles and extended to quadrilaterals using the Piola transform, which are well-known to lose optimal approximation of \(\nabla \cdot u\). Arnold-Boffi-Falk (ABF) spaces rectify the problem by increasing the dimension of RT, so that approximation is maintained after Piola mapping. Our two families of finite elements are uniformly inf-sup stable, achieve optimal rates of convergence, and have minimal dimension. The elements for \(u\) are constructed from vector polynomials defined directly on the quadrilaterals, rather than being transformed from a reference rectangle by the Piola mapping, and then supplemented by two (one for the lowest order) basis functions that are Piola mapped. One family has full \(H(\text{div})\)-approximation (\(u\), \(p\), and \(\nabla \cdot u\) are approximated to the same order like RT) and the other has reduced \(H(\text{div})\)-approximation (\(p\) and \(\nabla \cdot u\) are approximated to one less power like BDM). The two families are identical except for inclusion of a minimal set of vector and scalar polynomials needed for higher order approximation of \(\nabla \cdot u\) and \(p\), and thereby we clarify and unify the treatment of finite element approximation between these two classes. The key result is a Helmholtz-like decomposition of vector polynomials, which explains precisely how a divergence is approximated locally. We develop an implementable local basis and present numerical results confirming the theory.
The implementation of the finite element method is described for the inflation of a thin sheet modelled as a membrane. The thin sheet is assumed to be a hyperelastic material. As well as describing how to approximately solve this problem for a sequence of increasing pressures we also outline work in progress to attempt to estimate a given quantity of interest $J(u)$ to a given accuracy where $J(\cdot)$ denotes a functional and where $u$ denotes the exact solution. In the application $u$ denotes the displacement of the mid-surface of the membrane. With $u_h$ being our finite element approximation of $u$ and with $J(u_h)$ being our estimate of $J(u)$ we outline how to estimate $J(u) - J(u_h)$ by solving a dual problem. We consider this in the case of a quasi-static deformation when we only have space discretization errors and we also consider this in the dynamic case when we have time discretization errors as well. Results will be presented in the case of axisymmetric deformations.
ELLIPTIC PROBLEMS IN A NON-LIPSCHITZ DOMAIN

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In this work we review and analyze the approximation, by standard piecewise linear finite elements, of some elliptic problems in the plane domain \( \Omega = \{(x;y) : 0 < x < 1; 0 < y < x^\alpha\} \); which gives, for \( \alpha > 1 \), the simplest model of an external cusp. The focus of interest resides in the fact that, since the domain is curved and non- Lipschitz, the problems under consideration had not been covered by the standard literature which only had dealt with polygonal or smooth domains.

First, since many of the results on Sobolev spaces, which are fundamental in the usual error analysis, do not apply to cusp domains \[5\], we had to develop trace and extension theorems in weighted Sobolev spaces, with the weight being a power of the distance to the cuspidal. These estimates allowed us to prove, for the Poisson problem, that the optimal order, with respect to the number of nodes, could be recovered by using appropriate graded meshes \[3\ 4\ 1\].

Then, we studied the Laplacian eigenvalue problem, in which the classical spectral theory could not be applied directly, and in consequence, this eigenvalue problem had to be reformulated in a proper setting in order to obtain quasi optimal order of convergence for the eigenpairs \[2\].

At present, we are studying a Steklov eigenvalue problem and the particular difficulties that arise in this problem.

References


The puzzling and important phenomenon of wave localization arises in many physical and mathematical contexts, with applications range from the quantum mechanics of electrical conduction through the design of optical devices to the construction of noise abatement systems, to name but a few. Although studied by physicists and mathematicians for the better part of a century, localization of eigenmodes is still not fully understood nor controlled. In this talk we will describe recent major strides which have been made towards a comprehensive theory. In particular, it is now possible to predict and control the spectrum—both the eigenfunctions and the eigenvalues—of a large class of elliptic PDE, such as Schrödinger operators with random potentials. The talk will feature numerous high fidelity large scale finite element computations which have played a crucial role in guiding our understanding, validating theoretical results, and highlighting mysteries as yet unexplained.
VIRTUAL MODELING AND ADDITIVE MANUFACTURING (3D PRINTING) FOR ADVANCED MATERIALS (3D@UNIPV): A NEW RESEARCH ARENA

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Additive manufacturing (also known as 3D printing) is becoming more and more a prominent technology, which however still requires deep investigations in terms of materials, virtual modeling, applications, as well as effective economic impact evaluation. In particular, 3D printing cuts across many different areas, involving several research subjects and allowing the development of new high-impact applications.

Aware of all these aspects, after a quick overview of 3D printing in general, as well as of the new University of Pavia strategic project entitled Virtual modeling and additive manufacturing (3D printing) for advanced materials (3D@UniPV), the talk will discuss some specific area of active research, ranging from plastic sintering to metal 3D printing, from the production of high performing materials to new civil engineering structural applications.

LOCAL BOUNDED COCHAIN PROJECTIONS ON CUBICAL MESHES

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We review the construction of the two main families of finite element differential forms on cubical meshes and construct local bounded cochain projections for the spaces. We provide an outline of the construction of the projections due to Falk and Winther. A crucial step of the construction is an inclusion property for the analogue on cubical meshes of the space of Whitney forms into spaces of finite element differential forms. Preliminary report on a joint work with Ragnar Winther.
We prove a convergence result for a natural discretization of the Dirichlet problem of the elliptic Monge-Ampère equation using finite dimensional spaces of piecewise polynomial $C^1$ functions. Discretizations of the type considered in this paper have been previously analyzed in the case the equation has a smooth solution and numerous numerical evidence of convergence were given in the case of non smooth solutions. Our convergence result is valid for non smooth solutions, is given in the setting of Aleksandrov solutions, and consists in discretizing the equation in a subdomain with the boundary data used as an approximation of the solution in the remaining part of the domain. Our result gives a theoretical validation for the use of a non monotone finite element method for the Monge-Ampère equation.

We investigate new PDE discretization approaches for solving variational formulations with different types of trial and test spaces. The general mixed formulation we consider assumes a stability LBB condition and a data compatibility condition at the continuous level. For our proposed discretization method a discrete inf − sup condition is automatically satisfied by natural choices of test spaces (first) and corresponding trial spaces (second). For the proposed iterative method, nodal bases for the trial space are not required, and a cascadic multilevel algorithm can be adopted to speed up the approximation process. The level change criterion is based on matching the order of the the iteration error with the the order of the expected discretization error. Applications of the new approach include discretization of second order PDEs with oscillatory or rough coefficients and first order systems of PDEs, such as $\text{div} - \text{curl}$ systems and time-hamonic Maxwell equations.
MONOTONICITY PRESERVING NONLINEAR STABILIZATION
FOR HYPERBOLIC SCALAR PROBLEMS

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Hyperbolic problems can satisfy maximum principles or positivity preservation. In this work, we focus on the development of fully implicit finite element methods with nonlinear stabilization based on artificial diffusion techniques that keep at the discrete level these interesting properties.

The nonlinear viscosity is equal to a nonlinear shock detector times a linear viscosity. The idea is to activate the full linear viscosity on discontinuities/shocks, and switch it off in smooth regions. The shock detector must be such that it takes value 1 on local extrema (to satisfy discrete maximum principles), and switch off for linear functions, i.e., linearity-preserving (for accuracy purposes). In this sense, we work with shock detectors in the line of [1]. Another ingredient is the expression of the Laplacian term. Following [3], we consider a graph-Laplacian term. Finally, the linear viscosity is edge-based and follows the ideas in flux-corrected transport methods [2]: we consider the minimum amount of viscosity needed to prove monotonicity properties.

The resulting scheme satisfy discrete maximum principles and positivity, and is local variation diminishing. Further, following the ideas in [3] we can prove Lipschitz continuity. However, it is not enough to end up with a useful numerical method, since the nonlinear convergence of the resulting algorithms is extremely complicated. In this sense, we have developed a smooth version of the framework, in which we can prove that the resulting nonlinear stabilization term is $C^2$-continuous. The resulting schemes can still keep the monotonicity properties, but are much cheaper (due to a much better nonlinear convergence). We have considered the Newton method with line search and Anderson acceleration techniques. For a large smoothing parameter, the method is cheaper and more dissipative, whereas for a zero smoothing parameter, we recover the original scheme.

References


In this talk we will present our results concerning the stability of the space-time discontinuous Galerkin method (STDGM) for the solution of nonstationary, linear convection-diffusion-reaction problem in time-dependent domains. In the first part of the talk we formulate the continuous problem using the arbitrary Lagrangian-Eulerian (ALE) method, which replaces the classical partial time derivative with the so-called ALE-derivative and an additional convective term. After that we discretize our problem using the space-time discontinuous Galerkin method. In the formulation of the numerical scheme we use the nonsymmetric, symmetric and incomplete versions of the space discretization of the diffusion term and interior and boundary penalization. Finally in the third part of the talk we present our results concerning the unconditional stability of the method.

The results were obtained in cooperation with M. Feistauer.

The contribution will be presented in the mini-symposium “Finite Element methods for PDEs in time-dependent domains”.

AN $HP$-ADAPTIVE $C^0$-INTERIOR PENALTY METHOD FOR THE OBSTACLE PROBLEM OF CLAMPED KIRCHHOFF PLATES

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In this talk we consider an $hp$-adaptive $C^0$-interior penalty method for the bilaplace obstacle problem. The a posteriori error estimate consists of two stages. In the first part the error contributions associated with the obstacle condition are split off, and in the second part, a residual based a posteriori error estimate for the simpler biharmonic equation is generalized to higher order $h$- and $p$-versions. Essential for the a posteriori error estimate is the computation of a discrete Lagrange multiplier, representing the residual of the variational inequality, either by solving a mixed formulation directly, or by post-processing it after solving a discrete variational inequality. The choice of the finite element sets and whether the discrete inf-sup condition holds uniformly or at all are not of importance for the a posteriori error estimate. Numerical experiments demonstrate the behavior of the a posteriori error estimate and the superior convergence rate of the $hp$-adaptive scheme compared with uniform and $h$-adaptive schemes.
CROSS-DIFFUSION SYSTEMS FOR IMAGE PROCESSING

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Diffusion processes are commonly used in image processing [3]. In particular, complex diffusion models have been successfully applied in medical imaging denosing [1], [2], [4]. The interpretation of a complex diffusion equation as a cross-diffusion system motivates the introduction of more general models of this type and their study in the context of image processing. In this talk we will discuss the use of nonlinear cross-diffusion systems to perform image restoration. The use of two scalar fields has the goal of distributing the features of the image and governing their relations. In this talk, special attention will be given to the well-posedness, scale-space properties and long time behaviour of the models. From a numerical point of view, a computational study of the performance of the models is carried out, suggesting their diversity and potentialities to treat image filtering problems. Examples of application will be highlighted.

References


Acknowledgments

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STABLE AND STABILISED FINITE ELEMENT METHODS ON ANISOTROPIC MESHES

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In this talk I will review some recent results on the stability of finite element methods for the Stokes problem on anisotropic quadrilateral meshes. In the first part of the talk I will talk about the stability of the pair $Q_{k+1} \times P_{k-1}$ for $k \geq 1$. This pair is inf-sup stable, but its stability depends on the aspect ratio of the triangulation. Then, a strategy that consists on identifying the pressure modes responsible for this behavior has been followed. This spurious modes are then removed (to create then a new uniformly inf-sup stable pair), or penalised weakly in the formulation (thus creating a stabilised method). In the second part of the talk I will move on to show the stability properties of the Taylor-Hood scheme. In that part minimal assumptions on the mesh will be made in order to be able to show stability independent of the aspect ratio. This talk gathers contributions in collaboration with M. Ainsworth (Brown, US), and A. Wachtel (Strathclyde, UK).

NONLINEAR EDGE DIFFUSION METHODS: LINK TO AFC SCHEMES AND ADAPTIVITY

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In this talk I will review some recent developments on a maximum principle preserving discretisation of the convection-diffusion-reaction equation. To guarantee the discrete maximum principle, the method adds nonlinear diffusion to the formulation. The particularity of this extra diffusion is that it is based on the edges of the triangulation. We prove existence of solutions, as well as convergence to the exact solution, and preservation of the discrete maximum principle. The role of the linearity preservation is discussed, and also a link to Algebraic-Flux Correction schemes is presented. Finally, an adaptive procedure, based on an a posteriori error estimator, is proposed and tested. This talk will include contributions made in collaboration with A. Allendes (UTFSM, Valparaíso, Chile), E. Burman (UCL, UK), F. Karakatsani (Chester, UK), and R. Rankin (UTFSM, Valparaíso, Chile).
ADAPTIVE APPROXIMATION OF THE MONGE-KANTOROVICH PROBLEM

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Optimal transportation problems define high-dimensional linear programs. An efficient approach to their numerical solution is based on reformulations as nonlinear partial differential equations. If transportation cost is proportional to distance this leads to the Monge–Kantorovich problem which is a constrained minimization problem on Lipschitz continuous functions. We discuss the iterative solution via splitting methods and devise an adaptive mesh refinement strategy based on an a posteriori error estimate for the primal-dual gap.

SPACE-TIME FINITE ELEMENT APPROXIMATION OF FLOW IN DEFORMABLE POROUS MEDIA

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The modelling of coupled mechanical deformation and flow in porous media has become of increasing importance in several branches of natural sciences and technology including environmental, mechanical, petroleum and reservoir engineering, biomechanics and medicine. The numerical simulation of coupled mechanical deformation and flow is complex due to the structure of the model equations and continues to remain a challenging task. Recently, iterative coupling techniques have attracted researchers’ interest and schemes were proposed\cite{1,5}. The appreciable advantage of these approaches is that by coupling the model components iteratively already highly developed simulation techniques for each component of the overall system can be used fully.

In this contribution we consider the quasi-static Biot system of poroelasticity,

\begin{align}
-\nabla \cdot (\sigma_0 + C : \epsilon(u) - b(p - p_0)I) &= \rho_b g, \quad (1) \\
\partial_t \left( \frac{1}{M} p + \nabla \cdot (bu) \right) + \nabla \cdot q &= f, \quad q = -\frac{K}{\eta} (\nabla p - \rho_f g), \quad (2)
\end{align}

We present a higher order space-time finite element approximation of the system\cite{1},\cite{2} that is based on an iterative coupling of properly defined subproblems of mechanical deformation and fluid flow; cf.\cite{1}. For the discretization in time a discontinuous Galerkin method is used. Mixed finite element methods are applied for the spatial
discretization of the flow subproblem. Error estimates for the discretization and efficient solution techniques for the arising algebraic systems of equations are addressed; cf. [2, 3, 4]. The stability and performance properties of the techniques are illustrated by applications of practical interest.

References


DIVERSION FREE VIRTUAL ELEMENTS FOR THE STOKES PROBLEM

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We present a Virtual Element Method for the Stokes problem that, with respect to a more standard Virtual approximations of the same problem, holds two different advantages: (1) the discrete solution is exactly divergence free (2) the discrete problem is equivalent to a reduced version with much less degrees of freedom. This two advantages sum up with the traditional Virtual Element benefits of polygonal meshes and potentially high order polynomial degree. In the talk we present the construction of the method, theoretical convergence results and related numerical tests.
The advent of Advanced/Additive Manufacturing and the Materials Genome Initiative has placed significant emphasis on accelerating the qualification of new materials for use in real applications. Within these workflows lies both the engineering scale qualification through building and testing components at scale and full-scale modeling with integrated continuum computer codes and the materials scale qualification through revolutionary methods to non-destructively measure microstructure (3DXRD) and physics specific experiments coupled with meso-scale mechanics simulations of the same physics specific experiment using the same microstructure. This Integrated Computational Materials Engineering (ICME) process is one of the use cases that drives the Exascale Materials Co-design Center (ExMatEx). The goal of the Co-design Center is very analogous to the acceleration of new materials deployment within the MGI, rather co-design accelerates the deploying of laboratory concepts for future computer components to enable a productive exascale computer system. Our science strategy applies adaptive physics refinement, whereby a coarse-scale simulation dynamically spawns fine-scale simulations as needed. This direct coupling between the continuum integrated code (continuum plasticity) and direct numerical simulation of the meso-scale phenomena (crystal plasticity) involves a coarse-scale simulation, dynamically spawned fine-scale simulation tasks, a database for storing the results of fine-scale tasks, and an adaptive sampling layer which queries the database, interpolates results, and decides when to spawn new fine-scale tasks. Here we review the ExMatEx project, and its use cases.

This was joint work with Timothy Germann (LANL) and was performed under the auspices of the U.S. Department of Energy by Lawrence Livermore National Laboratory under contract DEAC52-07NA27344 and supported by Office of Science, Office of Advanced Scientific Computing Research.
When solving nonlinear partial differential equations using the Finite Element Method, the inner linear-system solves are often the bottleneck in computation. In general, multilevel methods provide efficient solvers for these systems that are optimal and scalable to large, parallel machines. In this talk, we discuss parallel geometric multigrid methods that utilize operator-dependent coarse spaces through an AMGe-type mechanism. In particular, the coarse operator-dependent spaces can have approximation properties of the same order as the fine-grid spaces. We show results for a few linear problems in a distributed computing environment.
A NITSCH-E-TYPE METHOD FOR HELMHOLTZ EQUATION WITH AN EMBEDDED, ACOUSTICALLY PERMEABLE INTERFACE

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We consider the Helmholtz equation of acoustic wave propagation in the situation where a permeable interface is embedded in the computational domain. The presence of the interface is represented by a complex-valued impedance function $Z$ that relates the jump in the solution over the interface to the flux through the interface. Thus, the flux is assumed to be continuous over the interface whereas the solution may contain jump discontinuities. Such an interface condition constitutes, for instance, a macro model of a perforated plate through which sound is leaking. The real part of $Z$, assumed to be nonnegative, represents losses in the interface, whereas the imaginary part, which can be of either sign, corresponds to reactive effects. For low-loss interfaces with negative imaginary part of $Z$, so-called surface waves can appear in a layer around the interface.

The straight-forward, standard finite-element discretization of this problem leads to a variational form in which the impedance function appears in the denominator of a surface integral along the interface, which means that partly or fully vanishing impedance functions cannot be handled without this term blowing up. We propose another formulation, based on a variant of Nitsche’s method, which seamlessly handles a complex-valued impedance function $Z$ that is allowed to vanish. The method can be seen as an interpolation between the standard method and a classic Nitsche method that weakly enforces continuity over the interface.

We show stability of the method, in terms of a discrete Gårding inequality, for a quite general class of surface impedance functions, provided that possible surface waves are sufficiently resolved by the mesh. Moreover, we prove an a priori error estimate under the assumption that the absolute value of the impedance is bounded away from zero almost everywhere. Numerical experiments illustrate the performance of the method for a number of test cases in 2D and 3D with different interface conditions, with and without surface waves.
The related physical equations of linear elasticity are the equilibrium equation and the constitutive equation, which expresses a relation between the stress and strain tensors. This is a first-order partial differential system such that a least squares method based on a stress-displacement formulation can be used whose corresponding finite element approximation does not preserve the symmetry of the stress [1].

In this talk, a new method is investigated by introducing the vorticity and applying the $L^2$ norm least squares principle to the stress-displacement-vorticity system. The question of ellipticity due to the fact that all three variables are present in one equation is discussed. Further, the supercloseness of the least squares approximation to the standard mixed finite element approximations arising from the Hellinger-Reissner principle with reduced symmetry [2], is studied. This implies that the favourable conservation properties of the dual-based mixed methods and the inherent error control of the least squares method are combined.

Additionally, a closer look will be taken at the error that appears using this formulation on domains with curved boundaries approximated by a triangulation [3]. In the higher-order case, parametric Raviart-Thomas finite elements are employed to this end.

Finally, it is shown that an optimal order of convergence is achieved and illustrated numerically on a test example.


We discuss the finite element approximation of Maxwell’s eigenvalue problem. A widely used tool for the analysis of this problem is a suitable mixed formulation. In this talk we show how to define an a posteriori error indicator for the mixed problem and how to implement it in the framework of the original formulation. A posteriori error analysis is performed for the proposed indicator. This is a joint work with L. Gastaldi, R. Rodríguez, and I. Šebestová.
HIGH ORDER EDGE ELEMENTS AND DOMAIN DECOMPOSITION PRECONDITIONING FOR THE TIME-HARMONIC MAXWELL’S EQUATIONS

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Edge elements are finite elements particularly suited for the approximation of the electric field, and high order elements provide, at a fixed number of unknowns, a more accurate solution. The high order generators presented in [1] have a rather simple expression since they are defined only in terms of barycentric coordinates, and a convenient set of degrees of freedom can be chosen to facilitate their implementation in the finite elements framework (see [2] for practical details).

However, the matrices of the linear systems resulting from this high order discretization are ill conditioned, so that preconditioning becomes necessary when using iterative solvers. Indeed, direct solvers are more robust, but for the considered large scale simulations they can’t be used since they require a high memory cost. As preconditioners we choose domain decomposition preconditioners, which are naturally suited for parallel computing and make it possible to deal with smaller subproblems. We present numerical results for the simulation of Maxwell’s equations in high frequency regime and for dissipative and heterogeneous media (the tests were performed on the Curie supercomputer of GENCI-CEA).

References


ADAPTIVE FINITE ELEMENT METHODS FOR THE LAPLACE-BELTRAMI OPERATOR

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Elliptic partial differential equations on surfaces are ubiquitous from geometry and relativity theory to applications in phase transitions, materials science, and image processing. They are typically governed by the Laplace-Beltrami operator, but more general operators arise as well. We present and analyze a new adaptive finite element method (AFEM) for the Laplace-Beltrami problem. In this context, the efficiency of the algorithm is challenged by two sources of errors: the geometric error due to the approximation of the surface and the error corresponding to the finite element resolution of the partial differential equation on the approximate surface. The nonlinear interplay between these two errors plays a critical role the AFEM needs to accommodate for.

In a first step, we consider smooth surfaces or surfaces with possible discontinuity lines exactly captured by the numerical method. However, in several contexts such as when the discontinuity lines are not piecewise polynomial or when the position of the discontinuities are part of the unknown, this aforementioned assumption cannot hold. In a second step, we describe and analyze a new algorithm able to cope with this additional difficulty.

OPTIMALITY OF ADAPTIVE FINITE ELEMENT METHODS FOR EIGENVALUE CLUSTERS

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We present recent results establishing optimality of standard adaptive finite element methods of arbitrary degree for eigenfunction computations for elliptic boundary value problems. Similar previous analyses have considered only lowest-order (piecewise linear) finite element spaces or multiple eigenvalues only. In contrast to previous results, our techniques also confirm that a critical input parameter in the adaptive FEM, the marking parameter, may be chosen independent of the target cluster being approximated.
LARGE DEFORMATIONS OF BILAYER PLATES

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The bending of bilayer plates is a mechanism which allows for large deformations via small externally induced lattice mismatches of the underlying materials. Its mathematical modeling consists of a geometric nonlinear fourth order problem with a nonlinear pointwise isometry constraint and where the lattice mismatches act as a spontaneous curvature. A gradient flow is proposed to decrease the system energy and is coupled with finite element approximations of the plate deformations based on Kirchhoff quadrilaterals. In this talk, we focus on the convergence of the iterative algorithm towards stationary configurations and the \(\Gamma\)-convergence of their finite element approximations. We also explore the performances of the numerical algorithm as well as the reduced model capabilities via several insightful numerical experiments involving large (geometrically nonlinear) deformations.

ROBUST DISCRETIZATION OF FLOW IN FRACTURED POROUS MEDIA

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Fractures are ubiquitous in natural rocks, and in many cases have a leading order impact on the structure of fluid flow. Since fractures frequently have high aspect ratios, it is appealing to consider them as lower-dimensional features.

We present a modelling approach based on mixed finite element methods and the mortar method which fully couples the physics in domains with different dimensions. In particular, we apply the approach to Darcy flow in fractured media and show how abrupt fracture tips as well as fracture intersections are naturally handled. The proposed discretization is applicable to both two and three spatial dimensions and is capable of handling conductive as well as blocking fractures. Furthermore, the method respects mass conservation and handles non-matching grids. We establish both theoretically and through numerical examples that our method is convergent in all relevant physical limits.
THE VIRTUAL ELEMENT METHOD FOR DARCY FLOWS IN COMPLEX GEOMETRIES

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The simulation of flows in fractured media is a very challenging issue in applications and is often tackled by considering stochastically generated Discrete Fracture Networks (DFN) as models for the medium. These are sets of planar polygons, representing fractures, intersecting each other in the three dimensional space in such a way that the physical properties of the medium are well represented. The polygon sizes span many orders of magnitude and the stochastic nature of intersections between polygons causes infeasibility when generating meshes that require some kind of conformity. The Virtual Element Method (VEM) was recently developed to enable the use of arbitrarily shaped polygons to discretize the spatial domain. This flexibility can be exploited to handle the issue of discretizing DFNs using elements which are conforming to intersections, thus enabling the application of domain decomposition techniques to compute the distribution of hydraulic head. From this distribution, it is possible to obtain the Darcy velocity, that can then be used as input for the simulation of the transport of a passive scalar, e.g. the density of a pollutant. This requires to solve an additional advection-diffusion problem that is naturally advection dominated, which is known to lead to instabilities when discretised by the standard Galerkin approach. In order to tackle this issue, in we show that a Streamline Upwind Petrov-Galerkin stabilization can be derived for the VEM, in a consistent way such that the rate of convergence of the method is preserved. This approach can be applied to DFN simulations to obtain the steady state solution of the transport problem in the DFN.

References


A high-order Discontinuous Galerkin Method with Lagrange Multipliers (DGLM) is presented for the solution of the unsteady advection-diffusion equation in the high Péclet number regime. In this regime, this equation models transport problems for which the standard Finite Element Method (FEM) is typically inadequate at practical mesh resolutions, as it produces non-physical oscillations in the numerical solution.

Like a Discontinuous Enrichment Method (DEM), the DGLM method described in this presentation overcomes the issue of spurious oscillations near boundary or internal layers by attempting to resolve them using appropriate shape functions. Specifically, these are chosen as polynomials that are additively enriched with free-space solutions of the governing differential equation. Also like a DEM, the DGLM method presented herein enforces a weak continuity of the solution across inter-element boundaries using Lagrange multipliers. It operates directly on the second-order form of the advection-diffusion equation and does not require any stabilization.

DGLM approximates the solutions of both homogeneous and non-homogeneous instances of the unsteady advection-diffusion equation using carefully constructed combinations of discontinuous polynomials enriched by free-space solutions of the advection-diffusion-reaction equation. Time-integration is performed using an implicit family of schemes based on the Backward Differential Formula whose numerical stability for the resulting differential-algebraic equations is rigorously proven. A theoretical analysis of the well-posedness of the proposed overall DGLM method and optimal performance results are also presented.
Given $s \in (0, 1)$, the fractional Laplacian of order $s$ of a smooth function $u$ is defined by

$$(-\Delta)^s u(x) = C(n, s) \text{ p.v.} \int_{\mathbb{R}^n} \frac{u(x) - u(y)}{|x - y|^{n+2s}} \, dy,$$

where $C(n, s)$ is a normalization constant. In this talk we address the equation

$$\begin{cases}
(-\Delta)^s u = \lambda u & \text{in } \Omega, \\
u = 0 & \text{in } \Omega^c,
\end{cases}$$

where $\Omega \subset \mathbb{R}^n$ is a bounded set. Even if $\Omega$ is an interval, it is very challenging to obtain closed analytical expressions for the eigenvalues and eigenfunctions of the fractional Laplacian. This motivates the utilization of discrete approximations of this problem; we study a conforming, piecewise linear finite element method. The main advantage of such an approximation is that it provides upper bounds for the eigenvalues, regardless of the regularity of the domain $\Omega$.

Unlike the classical Laplacian, eigenfunctions of the fractional Laplacian in $\Omega$ are not smooth up to the boundary; in particular, the first eigenfunction behaves as $d(x, \partial \Omega)^s$, and therefore it should not be expected to be more regular than $H^{s+1/2-\epsilon}(\Omega)$ for $\epsilon > 0$. We study the order of convergence for eigenvalues and eigenfunctions, both in the energy and the $L^2$-norm, and perform numerical experiments that illustrate the optimality of our theoretical findings. The eigenvalue estimates we provide are in good agreement with previous work by other authors.
ON THE DESIGN OF DUAL-COMPATIBLE T-MESH REFINEMENT ALGORITHMS

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Amongst the several types of adaptive spline spaces proposed in recent years, also in connection with related application in isogeometric analysis, the spaces of T-splines have some interesting and unique features. The functions spanning the space, the T-splines, are a natural generalization of tensor-product B-splines depending on the local topology of the T-mesh. If the T-mesh is dual-compatible (or, equivalently, analysis-suitable), the T-splines are linear independent, and therefore they form a basis (see, for instance, [L. Beirão da Veiga, A. Buffa, G. Sangalli and R. Vazquez, Analysis-suitable T-splines of arbitrary degree: definition and properties, Math. Mod. Meth. Appl. Sci. 23 (2013), pp. 1979-2003]). As a consequence, a refinement algorithm that preserves the dual-compatible structure of the T-mesh guarantees that the corresponding T-splines form a basis. In this talk we will discuss possible alternatives to existing T-mesh refinement algorithms (see [M.A. Scott, X. Li, T.W. Sederberg and T.J.R. Hughes, Local refinement of analysis-suitable T-splines, Comput. Methods Appl. Mech. Engrg. 213 (2012) pp. 206-222] and [P. Morgenstern and D. Peterseim, Analysis-suitable adaptive T-mesh refinement with linear complexity, Comput. Aided Geom. D. 34 (2015), pp. 50-66]). Our arguments are based on studying the influence of refinements on the local preservation of the dual-compatible structure, and allow us to study the complexity of the algorithm, a fundamental ingredient for the analysis of adaptive isogeometric methods.

A-POSTERIORI ERROR ESTIMATES FOR PRESSURE-PROJECTION SCHEMES

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We give a short introduction and the historical development of pressure-correction methods for time discretization of the incompressible Stokes equations and discuss advantages and disadvantages of the different schemes. Further we present a-posteriori error estimates for the two-step backward differential formula method (BDF2) for the pressure-correction scheme in rotational form.
RECENT VARIANTS OF MIXED VEM SPACES

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We recall the definitions and the basic properties of the original $H(\text{div})$ and $H(\text{curl})$-conforming Virtual Element spaces. Then we propose a new, simpler, presentation, and the Serendipity variants of them. Some applications (to Darcy flows or to magnetostatic problems) are also briefly discussed.

A HAMILTONIAN FINITE ELEMENT METHOD FOR NONLINEAR POTENTIAL FLOW FREE SURFACE WAVES

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An important mathematical model to describe nonlinear water waves is provided by the potential flow equation in combination with nonlinear free surface boundary conditions. This model assumes that the flow field is inviscid and irrotational and is suitable for large amplitude, non-breaking water waves away from boundary layers, where the Navier-Stokes equations are required to account for viscous effects. Computing large amplitude potential flow water waves is, however, non-trivial since the mesh deformation, necessary to accommodate for the free surface motion, and the nonlinearities can easily result in numerical instabilities. Many numerical discretisations include therefore additional stabilisation terms, but this results in unphysical wave damping that seriously reduces the numerical accuracy.

In this presentation we will present an alternative approach. The nonlinear free surface potential flow equations, when written in terms of the free surface potential and wave height, have a Hamiltonian structure. Preserving this Hamiltonian structure in the finite element discretisation results in an energy preserving numerical discretisation with superior (long time) accuracy and no artificial wave damping.


$$\mathcal{L}(\phi, \eta) = -\int_0^T \int_{\Omega_t} g \cdot x + \partial_t \phi + \frac{1}{2} |\nabla \phi|^2 \, d\Omega \, dt,$$

where $g$ denotes gravity, $x$ is the coordinate vector, $\phi$ the potential velocity. The free surface height $\eta$ appears only implicitly as part of the boundary of the domain $\Omega_t$. The Lagrangian functional can be rewritten as a Hamiltonian, where the restriction of $\phi$ to the free surface takes the role of generalised momentum and $\eta$ takes the role of generalised position.
The Lagrangian functional is used to obtain a system of ordinary differential equations for the nodal values in the finite element discretisation. After introducing a suitable transformation and a lengthy computation we can rewrite these ordinary differential equations as a Hamiltonian system. This Hamiltonian structure can be proven even for time-dependent, unstructured, moving and deforming meshes, including a wave maker and general bottom surface. Combined with a symplectic time integrator this results in a numerical discretisation with extraordinary stability properties, no artificial wave damping, and very good long time accuracy. We will demonstrate the preservation of the discrete energy and the accuracy of the finite element discretisation, including simulations of strongly interacting waves resulting in a large, highly nonlinear splash.

MULTISCALE METHODS IN POROELASTICITY

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Many problems in geoscience and more generally porous media are inherently multiscale. To bridge the scales and create computational methods, sub-grid or local problems are solved in order to build accurate and efficient spaces in which to compute on the coarse grids. This has been achieved successfully in many different frameworks such as homogenization, variational multiscale, and multiscale finite element methods. In this talk, we will primarily focus on homogenization based methods of the multiscale finite element method and its generalization using local model reduction techniques and partition of unity methods. Many local problems are computed in one coarse grid block. To reduce the dimension and capture relevant information a model reduction step is performed that may include a local eigenvector computation or proper orthogonal decomposition. Then, a partition of unity truncates these enriched basis to a generalized finite element space.

Many of these techniques were first developed to understand flow in porous media. For this talk, we will addition to flow through porous media also discuss the mechanical interactions via poroelasticity. We will introduce the complexities of modeling and simulating deformable porous media by first discussing the homogenization of Stokes-Elasticity systems and fluid-structure interaction. When pore-scale deformation occurs, the resulting homogenized models can be nonlinear in the coefficients. This motivates the need for computational multiscale methods that can handle nonlinearities at the Biot or effective poroelastic scale. We will outline the general methodology of the Generalized Multiscale Finite Element method (GMsFEM) and its application to poroelastic problems. We then present its application on some interesting test cases.
AN ANISOTROPIC A PRIORI ERROR ANALYSIS FOR A CONVECTION DIFFUSION PROBLEM USING AN HDG METHOD

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In this talk we present an a priori error analysis for a convection diffusion problem, considering an HDG method and a family of anisotropic triangulation. As result, we deduce that when diffusion is dominant, the behaviour of the method (considering $k$ as degree of approximation for every unknown) is such that the global $L^2$-norm of the error of the scalar and vector unknowns converge with order $k + 1$, while the unknown related to the trace of scalar unknown, on the skeleton of the mesh, does with order $k + 2$. For convection dominated diffusion equation, isotropic triangulations are not suitable. However, the use of anisotropic meshes let us to recover the convergence of the method, once the boundary or inner layer is solved. Numerical examples confirm these theoretical results.

References


EFFICIENT QUADRATURE FOR HIGH DEGREE ISOGEOMETRIC ANALYSIS

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In this talk we present a result on the assembly of the linear system arising in the Galerkin isogeometric method. The main interest are the cases where the degree of the approximation is raised, so that the computational cost in assembling become challenging.

Key ingredients are the application of weighted quadrature and sum-factorization. These modifications demand for a change of paradigm the existing fem-based codes. The resulting method is more efficient compared to the other approaches known in literature.

Paper in preparation:

Related litterature:
Calabrò F., Manni C., Pitoli F.; Computation of quadrature rules for integration with respect to refinable functions on assigned nodes, App Num 90 (2015), 168–189.
Mantzaflaris A., Juttler B., Integration by interpolation and look-up for galerkin-based isogeometric analysis, CMAME 284 (2015), 373–400
The aim of this work is to analyze the numerical approximation of the eigenvalue problem for the curl operator on a multiply connected domain. In order to obtain a well-posed eigenvalue problem, additional constraints must be imposed (see [3]). A combination between two type of constraints related to the homology of the domain have been added in order that the problem has a discrete spectrum (see [2]). A mixed variational formulation of the resulting problem and a finite element discretization are introduced and shown to be well-posed. Optimal-order spectral convergence is proved, as well as a priori error estimates, by using classical spectral approximation results (see [1]). It is described how to implement this numerical method taking care of these additional constraints. Finally the results of some numerical tests are also reported.

References


CONTINUUM DAMAGE MECHANICS IN SPH BASED ON PARTICLE INTERACTION AREA

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The underlying concept of continuum damage mechanics is that mechanical damage is that the effect of damage within a material, occurring at a length scale too small to resolve in the numerical model, is averaged over a volume. Numerically this is represented by a continuous variable that is related to the density of the defects within the material and reduces the effective area over which stresses apply. Meshless methods such as Smooth Particle Hydrodynamics (SPH) are well suited to the application of large material deformation and failure. The SPH momentum equation can be rewritten in terms of a particle-particle interaction area. Damage acts to reduce this area and ultimately lead to material fracture. An implementation of this approach will be presented and discussed. The concept is demonstrated on a 1D flyer plate impact test and the results were compared to experimental data. Numerical results show that the model can recreate the phenomena associated with uniaxial spall to a high degree of accuracy.

ADAPTIVITY AND BLOW-UP DETECTION FOR NONLINEAR EVOLUTION PROBLEMS

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We present reliable space-time adaptive algorithm, based on a rigorous a posteriori error bound, for a semilinear convection-diffusion problem which may exhibit blow-up in finite time. More specifically, the a posteriori bound is derived for a first order in time implicit-explicit (IMEX) interior penalty discontinuous Galerkin (dG) in space discretization of the problem, although the theory presented is directly applicable to the case of conforming finite element approximations in space. A key novelty of the approach is the use of a local-in-time continuation argument in conjunction with a space-time reconstruction. A useful by-product of the local continuation argument used in this work is that it gives a natural stopping criterion for approach towards the blow-up time. The new adaptive algorithm is shown to accurately estimate the blow-up time of a number of problems, including one which exhibits regional blow-up.
A VERTEX-BASED SCHEME ON POLYHEDRAL MESHES FOR ADVECTION-REACTION EQUATIONS WITH SUB-MESH STABILIZATION

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We devise and analyze vertex-based schemes on general meshes, \textit{i.e.} with polyhedral cells or with non-matching interface, to approximate advection-reaction equations. Error estimates of order $O(h^{3/2})$ in the discrete inf-sup stability norm are established. The two key ingredients are a local polyhedral reconstruction map leaving affine polynomials invariant, and a local design of stabilization whereby gradient jumps are only penalized across some sub-faces in the interior of each mesh cell. Numerical results are presented on three-dimensional general meshes.

SMOOTH PARTICLE HYDRODYNAMICS ANALYSIS OF HIGH-SPEED IMPACT INCLUDING FRACTURE CRITERIA AND FE CONVERSION

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Smooth Particles Hydrodynamics (SPH) is a very popular method for the numerical modelling of high speed impact (ballistics), where materials are usually subjected to high strain rates and large deformations. In this work, a newly developed SPH method is integrated with different high strain rate constitutive models for the numerical prediction of material degradation and failure during high speed ballistic simulations. The simulations will include contact between particles and finite elements as well as conversion of finite elements into SPH particles for the efficient modelling of large deformations and material separation. The SPH method is based on the use of the Moving Least Square (MLS) method for the SPH basis functions so that at least linear reproducibility is guaranteed. Different fracture criteria and element conversion thresholds will be presented for the conversion of finite elements into SPH particles.
A POSTERIORI ERROR ANALYSIS FOR A FULLY-MIXED FORMULATION OF THE NAVIER–STOKES/DARCY COUPLED PROBLEM WITH NONLINEAR VISCOITY

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We present an a posteriori error analysis for an augmented fully-mixed finite element method for the coupling of fluid flow with porous media flow in 3D. The flows are governed by a class of nonlinear Navier–Stokes and linear Darcy equations, respectively, and the corresponding transmission conditions are given by mass conservation, balance of normal forces, and the Beavers–Joseph–Saffman law. We consider dual-mixed formulations in both domains, and the nonlinearity involved in the Navier–Stokes region is handled by setting the strain and vorticity tensors as auxiliary unknowns. In turn, since the transmission conditions become essential, they are imposed weakly, which yields the introduction of the traces of the porous media pressure and the fluid velocity as associated Lagrange multipliers. A feasible choice of finite element subspaces includes piecewise constants, Raviart–Thomas elements of lowest order, continuous piecewise linear elements, and piecewise constants for the strain tensor, stress, velocity, and vorticity in the fluid, Raviart–Thomas elements of lowest order and piecewise constants for the velocity and pressure in the porous medium, together with continuous piecewise linear elements for the traces. We derive a reliable and efficient residual-based a posteriori error estimator for the coupled problem. By making use of the global inf-sup condition, Helmholtz decompositions in both media, and local approximation properties of the Clément interpolant and Raviart–Thomas operator, the reliability of the estimator is obtained under a suitable smallness data assumption. On the other hand, inverse inequalities, the localization technique based on tetrahedron-bubble and face-bubble functions and known results from previous works, are the main tools for proving the efficiency of the estimator. Finally, some numerical results confirming the properties of the estimator and illustrating the performance of the associated adaptive algorithm are also reported.
A RECOVERY BASED LINEAR FINITE ELEMENT METHOD FOR 4TH ORDER PROBLEMS

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We analyze a gradient recovery based linear finite element method to solve string equations and the corresponding eigenvalue problems. Our method uses only \( C^0 \) element, which avoids complicated construction of \( C^1 \) elements and nonconforming elements. Optimal error bounds under various Sobolev norms are established. Moreover, after a post-processing the recovered gradient is superconvergent to the exact one. Finally, some numerical experiments are presented to validate our theoretical findings.

AN INTERFACE-FITTED MESH GENERATOR AND VIRTUAL ELEMENT METHODS FOR ELLIPTIC INTERFACE PROBLEMS

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In this work, we develop a simple interface-fitted mesh algorithm which can produce an interface-fitted mesh in two and three dimension quickly. Elements in such interface-fitted mesh are not restricted to simplex but can be polygon or polyhedron. We thus apply virtual element methods to solve the elliptic interface problem in two and three dimensions. We present some numerical results to illustrate the effectiveness of our method.
A block diagonal preconditioner with the minimal residual method and a block triangular preconditioner with the generalized minimal residual method are developed for Hu-Zhang mixed finite element methods of linear elasticity. They are based on a new stability result of the saddle point system in mesh-dependent norms. The mesh-dependent norm for the stress corresponds to the mass matrix which is easy to invert while the displacement it is spectral equivalent to Schur complement. A fast auxiliary space preconditioner based on the $H^1$ conforming linear element of the linear elasticity problem is then designed for solving the Schur complement. For both diagonal and triangular preconditioners, it is proved that the conditioning numbers of the preconditioned systems are bounded above by a constant independent of both the crucial Lamé constant and the mesh-size. Numerical examples are presented to support theoretical results. As a byproduct, a new stabilized low order mixed finite element method is proposed and analyzed and a superconvergence of Hu-Zhang element is obtained.
We introduce a Bassi-Rebay type discontinuous Galerkin method for both stationary and time-dependent third-order linear equations. This method is the first discontinuous Galerkin method which conserves the mass and the $L^2$-norm of the approximations of the solution and that of its first and second derivatives. For the stationary case, $L^2$-projections of the errors (in the approximation of the solution, its first and second derivatives) are proven to have optimal convergence rates when the polynomial degree $k$ is even and the mesh is uniform, and to converge sub-optimally, but sharply, with order $k$ when $k$ is odd or the mesh is non-uniform. We show that suitably defined projections of the errors superconverge with order $k + 1 + \min\{k, \frac{1}{2}\}$ on uniform meshes and converge optimally on non-uniform meshes. The numerical traces are proven to superconverge with order $2k$ if $k$ is odd or the mesh is non-uniform. For even $k$ and uniform meshes, we show that the numerical traces superconverge with order $2k + \frac{3}{2}$. If in addition, the number of intervals is odd, the convergence order is improved to $2k + \frac{3}{2} + \min\{k, \frac{1}{2}\}$. This allows us to use an element-by-element postprocessing to construct new approximations that superconverge with the same orders as the numerical traces. For the time-dependent case, the errors are proven to be of order $k + 1$ for even $k$ on uniform meshes, and of order $k$ when $k$ is odd or the mesh is nonuniform. Numerical results are displayed which verify all of the above-mentioned theoretical orders of convergence as well as the conservation properties of the method. We also show that the orders of convergence of the the stationary case also hold for the time-dependent case.
A MATHEMATICAL MODEL FOR INDUCTION HARDENING
INCLUDING NONLINEAR MAGNETIC FIELD
AND CONTROLLED JOULE HEATING

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We provide a derivation and an analysis of the mathematical model for induction hardening. We assume a non-linear relation between the magnetic field and the magnetic induction field.

The coupling between the electromagnetic and the thermal part is provided through the temperature-dependent electric conductivity and the joule heating term, the most crucial element, considering the mathematical analysis of the model. It functions as a source of heat in the thermal part and leads to the increase in temperature. Therefore, in order to be able to control it, we apply a truncation function.

Using the Rothe’s method, we prove the existence of the global solution to the whole system. The nonlinearity in the electromagnetic part is overcome by utilizing the theory of monotone operators and the technique of Minty-Browder.
Discontinuous Galerkin (DG) methods have been the subject of numerous research activities in the last 15 years and have been successfully developed for various physical contexts modeled by elliptic, mixed hyperbolic-parabolic and hyperbolic systems of PDEs. Despite many advantages, one major drawback of high order DG methods is their intrinsic cost due to the very large number of globally coupled degrees of freedom as compared to classical high order conforming finite element methods. This is particularly the case when one consider the possibility of using an implicit scheme for the time integration of an hyperbolic system of equations such as the system of Maxwell equations in the time-domain. Different attempts have been made in the recent past to improve this situation and one promising strategy has been recently proposed by Cockburn et al. [1] in the form of so-called hybridizable DG (HDG) formulations. The distinctive feature of these methods is that the only globally coupled degrees of freedom are those of an approximation of the solution defined only on the boundaries of the elements of the discretization mesh. Since then, this kind of methods has been developed for various physical models [3, 4]. In the case of Maxwell’s equations, HDG methods have been mainly developed for time-harmonic problems [2, 5]. Thereby, the present work is concerned with the study of such a HDG method for the solution of the three-dimensional Maxwell equations in time-domain. On one hand, we are interested in designing a high order HDG method that can handle efficiently locally refined unstructured meshes by considering the possibility of using a fully implicit time scheme or a hybrid implicit-explicit (IMEX) time scheme. On the other hand, we are concerned with applications involving the interaction of light with matter at the nanoscale which possibly requires solving the system of time-domain Maxwell PDEs coupled to a system of ODEs modeling the dispersive properties of metallic nanostructures.

References


BEST APPROXIMATION ERROR ESTIMATES FOR THE ALLEN-CAHN EQUATION

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Fully-discrete approximations of the Allen-Cahn equation are discussed. In particular, we consider schemes of arbitrary order based on a discontinuous Galerkin (in time) approach combined with standard conforming finite elements (in space). We prove best approximation a-priori error estimates, with constants depending polynomially upon $(1/\epsilon)$. We also prove that these schemes are unconditionally stable under minimal regularity assumptions on the given data. The key feature of our approach is an appropriate duality argument, combined with a boot-strap technique.
A PRIORI ERROR ESTIMATES FOR AN OPTIMAL CONTROL PROBLEM RELATED TO QUASI-LINEAR PARABOLIC PDES

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We consider an optimal control problem related to quasi-linear parabolic pdes. The standard tracking type of functional is minimized and the controls are of distributed type satisfying point-wise constraints. After presenting some results regarding existence and regularity of solutions, first and second order conditions, we focus on the discretization of the control to state mapping. A-priori error estimates for a fully-discrete scheme are presented. The scheme is based on the lowest order discontinuous Galerkin time stepping scheme combined with standard conforming finite elements (in space). We present estimates at the natural energy norm, as well as improved error estimates in $L^2(0,T;L^2(\Omega))$ norm. Using these estimates, as well as similar estimates for the discrete adjoint mapping, we discuss error estimates for the optimal control problem.
The behaviour of a nuclear core reactor depends on the nuclear chain reaction, which is described by the neutron transport equation. This equation is a balance statement that conserves neutrons. It governs the neutron flux density, which depends on 7 variables: 3 for the space, 2 for the motion direction, 1 for the energy (or the speed), and 1 for the time. In the steady-state case, one must solve an eigenvalue problem. The energy variable is discretized using the multigroup theory ($G$ groups). Concerning the motion direction, an inexpensive approach to approximate the transport equation is to solve the simplified $PN$ equations ($N+1$ coupled diffusion equations). It can be shown that the basic building block which allows to solve the general multigroup simplified $PN$ equations, is the so-called neutron diffusion equation set in a bounded domain $\Omega$ of $\mathbb{R}^3$ ($G = 1$, $N = 1$), which reads:

Find $\phi \in H^1(\Omega) \setminus \{0\}, \lambda \in \mathbb{R}^+$ such that:

$$
\begin{cases}
-\text{div} \, D \nabla \phi + \Sigma_a \phi &= \lambda \nu \Sigma_f \phi & \text{in } \Omega \\
\phi &= 0 & \text{on } \partial \Omega.
\end{cases}
$$

(1)

Above, $D$, $\Sigma_a$, $\nu$ and $\Sigma_f$ denote respectively the diffusion coefficient, the macroscopic absorption cross section, the fission yield and the fission cross section. More precisely, we look for the criticality factor: $1/\min_1 \lambda$, together with the associated $\phi$ which corresponds to the averaged neutron flux density. Special attention is paid to the case where the solution $\phi$ to problem (1) is of low regularity. Such a situation commonly arises in the presence of three or more intersecting material components with different characteristics. As a matter of fact, the reactor cores often have a Cartesian geometry and the cross sections are averaged in every cell of the discretization. They may be constant or piecewise polynomial, and can differ from one cell to its neighbor by a factor of order 10.

We analyze matching and non-matching domain decomposition methods for the numerical approximation of the dual-mixed equations. The domain decomposition method can be non-matching in the sense that the meshes of the subdomains, and more generally the finite elements spaces, may not fit at the interface between subdomains. We prove well-posedness of the discrete problems with the help of a uniform discrete inf-sup
condition, and we provide optimal a priori convergence estimates. To improve the convergence rate, one can use a coarse grid correction based on the singular complement method. Numerical experiments illustrate the accuracy of the method.

A-POSTERIORI ERROR ESTIMATION OF DISCONTINUOUS GALERKIN METHODS FOR APPROXIMATELY-DIVERGENCE-FREE CONVECTION-DIFFUSION PROBLEMS

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Mantle convection is often modelled by a stationary Stokes system coupled to a time-dependent convection-diffusion equation for the temperature variable. Given the size of the resulting models, \textit{a posteriori} error estimators are highly desirable for the control of adaptive FE schemes in order to reduce the solution cost.

In a system containing some reaction, the reaction can typically be used to handle the convective term. However, since we have no reactive term, this option is not open to us, and so we proceed with an exponential-fitting method. Meanwhile, the numerical solution of the Stokes system may yield a convection field that is only approximately divergence-free. We present a derivation of an \textit{a posteriori} error estimator for the discontinuous Galerkin discretisation of a time-dependent convection-diffusion equation with varying, nearly-divergence-free convection, based on an exponential-fitting method, along with numerical experiments to show the suitability of the error estimator.
The ideal DPG method [2] reproduces the stability of the continuous problem and guarantees optimal convergence for any well posed problem. The broken test spaces methodology makes it computationally efficient and can be applied to any well posed variational formulation [2]. The practical DPG method approximates the Riesz (error) representation function $\psi$ using an enriched test space. Needless to say, the ultimate success of the practical DPG method hinges on controlling the error in resolving $\psi$.

For standard, "mathematician" test norms, the resolution of $\psi$ is relatively easy and the damage due to the error in $\psi$ can be estimated via the construction of appropriate Fortin operators [2,3]. For challenging singular perturbation problems, and test norms involving the perturbation parameter, resolution of $\psi$ is challenging but not because of stability (as for the original problem) but rather approximability issues. The double adaptivity idea of Cohen, Dahmen and Welper [1] calls for introducing an inner adaptivity loop to control the error in $\psi$. The adaptively determined enriched test space is “custom made” for the particular load and the trial space, and it does not imply the discrete stability. And yet the ultimate method converges.

I will present a series of 1D and 2D double adaptivity experiments for convection dominated diffusion. Out of many possible variational formulations, the ultraweak formulation stands out as the corresponding optimal test norm is known explicitly, and it is robustly equivalent to the adjoint graph norm (with a properly scaled $L^2$-term). Consequently, the DPG method delivers an orthogonal projection in an energy norm robustly equivalent to the trial $L^2$-norm. The adjoint graph norm, however, is difficult to resolve, and the double adaptivity comes in as a natural means to cope with the problem.

The inner adaptivity loop requires a robust a-posteriori error estimate for the discretization of Riesz representation function $\psi$. A residual estimate seems to be a natural (if not the only possible) option. For a broken test space, the residual is equal to the sum of element residuals, so the residual estimation is naturally reduced to a single element $K$. Cumbersome construction of Clement-like interpolation operators, necessary for standard conforming methods, reduces to a simple orthogonal projection in the test norm. The element residual estimate leads to a number of multiscale generalized eigenvalue problems involving the test norm, $L^2(K)$, $L^2(\partial K)$ and $H^{-1}(\partial K)$ norms. The eigenvalue problems are solved off line, harvesting appropriate “interpolation” constants for different values of diffusion $\epsilon$, element size $h$, enriched element order $r$, and advection vector components. The precomputed constants enter then the residual estimate.
Ideally, one should use two independent meshes, one for the original unknown $u$, and the second for Riesz representation $\psi$. The dynamically determined mesh for $\psi$ depends upon approximate solution $u_h$ (and, therefore, the first mesh). For practical reasons, we attempt to use the same mesh for both unknowns, enriching only the order of approximation for $\psi$. If the maximum order is reached, we force $h$-refinements and restart the whole problem. 1D and 2D numerical experiments indicate that, for small diffusion, the adaptivity process is driven entirely by the resolution of $\psi$, i.e. the inner adaptivity loop. This is rather disappointing as we would like to see a robust solution for very coarse meshes (which is critical for nonlinear problems).

In the end, we will present experiments based on the ideas of Broersen and Stevenson [4] based on evolving a pure convection to a convection-diffusion problem. With a proper selection of a variational formulation, the underresolved Riesz representation function $\psi$ for the confusion problem, represents a perfect approximation for the corresponding Riesz representation function for the pure convection problem. The game involves also relaxing the full stop outflow boundary conditions which must evolve with the mesh. The numerical results are promising but, at the moment, we do not have a full understanding of the underlying mathematics. We hope to understand it better by the time of the conference.

References


Research towards predicting the failure and fragmentation growth of explosively and electromagnetically driven metal cylinders and rings explosively within a meshless framework is described. Smoothed Particle Hydrodynamics (SPH), a meshless method, is of particular interest for the accurate prediction of fragmentation and fracture at high strain rate in metals. In combination with a damage model to initiate and propagate damage, the SPH method is able to treat the initiation, propagation, bifurcation and coalescence of cracks in a relatively straightforward manner. A modified Johnson-Cook material model combined with the Lemaitre damage model was used to describe the constitutive behaviour of the metals, while the explosive was modelled using a high explosive burn constitutive model and a JWL equation of state. Contact between explosive and casing are treated using a node to node contact algorithm based on a contact potential. The SPH method was used with both Eulerian and total Lagrangian interpolation kernels. In order to correctly model fracture in the total Lagrangian SPH formulation a visibility criterion based on a truncated cone has been developed to stop particles obscured by a failed particle from interacting with other particles. The detailed data from electromagnetically driven ring experiments and fragment mass statistics from explosively driven fragmentation tests is used to evaluate the accuracy of the model predictions. The results demonstrate that this type of model is capable of predicting to good degree of accuracy the number of fragments as well as the fragment mass distribution.

THEORETICAL ANALYSIS FOR CAPILLARY RISE BETWEEN A FLEXIBLE FILM AND A SOLID WALL

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We study the dynamics of meniscus rise of a liquid contained in a narrow gap between a flexible film and a solid wall. In this talk we will show that the meniscus rises indefinitely expelling liquid from the gap region, and that the height of the rising front $h(t)$ increases with time as $h(t) \propto t^{2/7}$, while the gap distance $e(t)$ decreases as $e(t) \propto t^{-3/7}$. These results are consistent with the experiments of Cambau et al.
BRIDGING HYBRID HIGH-ORDER METHODS
AND HYBRIDIZABLE
DISCONTINUOUS GALERKIN METHODS

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We consider here the application of the recently introduced Hybrid High-Order (HHO) method \cite{HHO} to the model problem: Find $u \in H^1_0(\Omega)$ such that
\begin{equation}
\int_{\Omega} \kappa \nabla u \cdot \nabla v = \int_{\Omega} fv \quad \forall v \in H^1_0(\Omega),
\end{equation}
where $\Omega \subset \mathbb{R}^d$ is a bounded, connected polyhedral domain and $\kappa$ a piecewise constant, bounded, symmetric, uniformly positive-definite matrix-valued function.

The HHO method supports general polyhedral meshes and delivers an arbitrary-order accurate approximation using face-based discrete unknowns that are polynomials of degree at most $k \geq 0$ on each face. The construction hinges on two key ingredients: (i) a polynomial reconstruction of the potential of degree $(k+1)$ in each mesh cell and (ii) a face-based stabilization consistent with the high-order provided by the reconstruction. The design relies on intermediate cell-based discrete unknowns in addition to the face-based ones (hence, the term hybrid), which can be locally eliminated by static condensation. Besides the original method with cell-based unknowns of degree $k$, we consider here some new variants with cell unknowns of degree $(k-1)$ and $(k+1)$.

The main contribution of this work is to recast the HHO method into an equivalent mixed formulation and to identify the corresponding conservative numerical flux. We show, in particular, how the solution provided by the HHO method can be characterized as the solution of local problems which are then matched by a single global equation. Such equation can be interpreted as a discrete version of a transmission condition.

This new reformulation enables a comparison to Hybridizable Discontinuous Galerkin (HDG) methods within the general framework introduced in \cite{HDG}. We show, in particular, that both the local spaces and numerical trace of the flux are novel, distinctive choices which enrich the family of HDG methods. In particular, the spaces for the flux are much smaller than the ones previously known, and the stabilization function displays a rich structure that allows for optimal convergence of both the potential $u$ (with order $(k+2)$) and its flux $q := -\kappa \nabla u$ (with order $(k+1)$) on general meshes composed of polyhedral cells. We also show that one of the novel variants of the method bears relations with the recently introduced High-Order Mimetic method.
A HYBRID HIGH-ORDER METHOD FOR LERAY–LIONS ELLIPTIC EQUATIONS ON GENERAL MESHES

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We study here the numerical approximation of the steady Leray–Lions equation

$$-\nabla \cdot (a(\cdot, u, \nabla u)) = f \text{ in } \Omega, \quad u = 0 \text{ on } \partial \Omega,$$

where $\Omega \subset \mathbb{R}^d$, $d \geq 1$, is a polytopal bounded connected domain of boundary $\partial \Omega$, while $a : \Omega \times \mathbb{R} \times \mathbb{R}^d \to \mathbb{R}^d$ is a (possibly nonlinear) function of its arguments. This model, which contains the $p$-Laplace equation, appears, e.g., in the modelling of glacier motion, of incompressible turbulent flows in porous media, and in airfoil design. Our goal is to design and analyze a discretization method for problem (1) inspired by the Hybrid High-Order (HHO) of [2] in the context of a linear diffusion model problem. The proposed method offers several assets: (i) The construction is dimension-independent; (ii) general polytopal meshes are supported; (iii) arbitrary polynomial orders (including $k=0$) can be considered; (iv) it is efficiently parallelisable (the local stencil only connects a mesh element with its faces), and it has reduced computational cost (the element-based unknowns can be eliminated locally).

The design of a HHO method for the nonlinear problem (1) entails several new ideas. A first difference with respect to the linear case is that a more natural choice is to seek the gradient reconstruction in the full space of vector-valued polynomials of degree $\leq k$ (as opposed to the space spanned by gradients of scalar-valued polynomials of degree $\leq (k+1)$). The main consequence of this choice is that, when applied to the interpolates of smooth functions, the discrete gradient operator commutes with the $L^2$-projector, and can thus be shown to enjoy $L^p$-stability properties. A second important point is the design of a high-order stabilization term with appropriate scaling. Here, we propose a generalization of the stabilization term of [2] which preserves the property of
vanishing for polynomials of degree \( \leq (k + 1) \). As in the linear case, the construction hinges on the solution of small local linear problems inside each element, and the possibility of statically condense element-based DOFs remains available.

The convergence analysis is carried out using a compactness argument in the spirit of [3], and under minimal regularity assumptions on the exact solution. Adapting the compactness argument has prompted us to develop discrete functional analysis tools (stability and approximation properties for the \( L^2 \)-orthogonal projector, discrete Sobolev embeddings, compactness for bounded sequences of discrete functions, etc.) whose interest goes beyond the specific method and problem considered in this work.

References


AN UPDATE ON THE MAXIMUM STRATEGY
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The adaptive finite element method, with an automatic refinement driven by error estimators, allows to resolve singularities at minimal computational costs. One strategy of refinement is to split those triangles, where the error indicators are almost maximal (maximum strategy). We show optimality of the corresponding adaptive finite element loop. The original result is restricted to the two-dimensional case with linear elements. In this talk we present extensions of these arguments, which include the use of higher order elements.

OPTIMIZED SCHWARZ METHODS
FOR THE STOKES-DARCY PROBLEM
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In this talk we present optimized Schwarz methods for the coupled system formed by the Stokes and the Darcy equations. Transmission conditions of Robin type are introduced and the coupled problem is reduced to a suitable interface system that can be solved using Krylov methods. A practical strategy to compute optimal Robin coefficients is considered which takes into account both the physical parameters of the problem and the size of the mesh. Numerical results show the effectiveness of our approach.
THE INTERFACE CONTROL DOMAIN DECOMPOSITION (ICDD) METHOD FOR THE STOKES-DARCY COUPLING

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In this talk we present a new technique called Interface Control Domain Decomposition (ICDD) method to couple the Stokes and the Darcy equation to model the filtration of a free fluid through a porous medium. According to this approach the coupled problem is reformulated as an optimal control problem with control variables corresponding to the traces of the velocity and of the pressure on the boundary of an overlapping region between the Stokes and the Darcy subdomains. The coupling between the two models is practically achieved by imposing the continuity of velocity and pressure across the boundary of the overlapping region in a least-squares sense. No additional modelling is required. We will show that the optimal control problem is well-posed, we will present some numerical tests to illustrate the behaviour of the method, and we will compare it with the more classical approach based on the Beavers-Joseph-Saffman condition.
Bird strike is one of the major hazards for aircraft structures, particularly for the jet engines, where the strike can lead to significant power loss and fatal outcomes. Consequently, the key requirement of the damage tolerant design is to ensure survivability of aircraft components under bird strike, which must not lead to immediate loss of performance. The performance of a new jet engine design in the bird strike needs to be demonstrated experimentally in the certification process, but significant part of the blade design process is nowadays dominated by more cost effective numerical simulation tools. The main aim of the work presented here was simulation of bird strikes on lightweight engine blades. The simulations were performed with an in-house developed Smoothed Particle Hydrodynamics (SPH) code coupled with a transient nonlinear Finite Element (FE) code (Lawrence Livermore National Laboratory - DYNA3D), where the bird was modelled with SPH particles and the blades with the FE mesh. The key aspect of the analysis was modelling of contact between the bird and the blade, including the particle to node and the particle to surface contact algorithms, which are both available in the code. This was followed by parametric studies of the bird shape, the impact location and the impact timing. Two lightweight blade designs were considered in this work: a titanium-metallic blade and a carbon fibre composite blade. Simulation results were compared and validated in terms of the extent of damage induced in the blade and its final deformed shape recovered from the bird strike test.
Sometimes the partial differential equations with random coefficients can be better formulated on moving domains, especially in biological applications. We will introduce and analyse the advection-diffusion equations with random coefficients on moving hypersurfaces. Since we will consider evolving domains, for the definition of the Bochner type solution space we will use the approach that transforms the equation onto a fixed reference domain. Under suitable regularity assumptions, using Banach-Nečas-Babuška theorem, we will prove existence and uniqueness of the weak solution and also we will give some regularity results about the solution. For discretization in space, we will apply the evolving surface finite element method to the weak form of the equation for which we approximate the hypersurface by an evolving interpolated polyhedral surface. Numerical approximation of uncertainty is performed by the Monte-Carlo-Method. We plan to illustrate our theoretical findings by numerical computations.
Over the past decades there has been a strong effort on developing a family of filters, Smoothness-Increasing-Accuracy-Conserving (SIAC) filters, designed to extract superconvergence from discontinuous Galerkin (DG) solutions. The filtering technique is a point-wise convolution of a B-Spline kernel with the DG solution at final time, resulting in a smoother solution and in many cases, of higher order accuracy. These advantages can be exploited during flow visualization of Partial Differential Equations (PDEs). For example, introducing the filter between the underlying Discontinuous Field and a streamline solver, produces locally a high order smooth solution, allowing implementation of relatively simple schemes whilst obtaining satisfactory curves. In this work we concentrate on the filter geometry and its impact on error. We introduce the rotated SIAC filter and discuss the kernel orientation, support size and splines configuration to obtain maximum error reduction from the DG solution for its applications to streamline visualization.
SUPERCONVERGENT HDG METHODS FOR THIRD-ORDER EQUATIONS IN ONE-SPACE DIMENSION

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We design and analyze the first hybridizable discontinuous Galerkin methods for stationary, third-order linear equations in one-space dimension. The methods are defined as discrete versions of characterizations of the exact solution in terms of local problems and transmission conditions. They provide approximations to the exact solution \( u \) and its derivatives \( q := u' \) and \( p := u'' \) which are piecewise-polynomials of degree \( k_u \), \( k_q \), and \( k_p \), respectively. We consider the methods for which the difference between these polynomial degrees is at most two. We prove that all these methods have superconvergence properties which allows us to prove that their numerical traces converge at the nodes of the partition with order at least \( 2k + 1 \), where \( k \) is the minimum of \( k_u, k_q, k_p \). This allows us to use an element-by-element post-processing to obtain new approximations for \( u, q \) and \( p \) converging with order at least \( 2k + 1 \) uniformly. Numerical results validating our error estimates are displayed.
We present a new $hp-$version space-time discontinuous Galerkin (dG) finite element method for the numerical approximation of parabolic evolution equations on general spatial meshes consisting of general polygonal/polyhedral (polytopic) elements, giving rise to prismatic space-time elements. A key feature of the proposed method is the use of space-time elemental polynomial bases of total degree, say $p$, defined in the physical coordinate system, as opposed to standard dG-time-stepping methods whereby spatial elemental bases are tensorized with temporal basis functions. This approach leads to a fully discrete $hp-$dG scheme using less degrees of freedom for each time step, compared to standard dG time-stepping schemes employing tensorized space-time, with acceptable deterioration of the approximation properties. A second key feature of the new space-time dG method is the incorporation of very general spatial meshes consisting of possibly polygonal/polyhedral elements with arbitrary number of faces or shape irregular elements with finite number of face. A priori error bounds are shown for the proposed method in various norms. An extensive comparison among the new space-time dG method, the (standard) tensorized space-time dG methods and the classical dG-time-stepping and conforming finite element method in space, is also presented in a series of numerical experiments.
MULTIGRID METHODS FOR BOUNDARY 
CONTROL OF ELLIPTIC EQUATIONS

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The purpose of this project is to devise and analyze efficient multigrid algorithms 
for boundary control of elliptic equations. Using a reduced formulation, our focus is 
on designing optimal order multigrid preconditioners for the Hessian of the reduced 
cost functional. Ideally, the preconditioners should approximate the reduced Hessian 
with optimal order with respect to the discretization of the elliptic equation. We 
show that for Dirichlet boundary control of elliptic equations the preconditioner is of 
suboptimal quality, though still efficient. Instead, for Neumann boundary control, the 
preconditioner proves to be of optimal order. We contrast these two problems with the 
case of distributed optimal control, where similarly defined multigrid preconditioners 
are of optimal order.
Mimetic Finite Difference (MFD) methods are numerical schemes developed for diffusion equations on generic grids. Recently, it was understood that the lowest order mixed/hybrid MFD method is similar to two finite volume methods, and that all three could be gathered into a same family of methods, the Hybrid Mimetic Mixed (HMM) family.

The construction of an HMM scheme requires one to select a point inside each cell (a “cell centre”). Usually, the centre of gravity is chosen and, in that case, an HMM method is an MFD scheme. Super-convergence, that is an $O(h^2)$ rate of convergence in the $L^2$ norm (compared with an $O(h)$ in an $H^1$-equivalent norm), is then known.

For flexibility of the methods, cell centres other than the centres of gravity can also be considered. This allows for example to include, for triangular meshes, the Two Point Flux Approximation (TPFA) finite volume in the HMM family. In this talk, we will analyse the super-convergence of HMM methods when the cell centres are moved away from the centres of gravity. Specifically, we will:

1. show that, for specific meshes, super-convergence can fail;
2. describe a modification of HMM methods (which only consists in modifying the quadrature rule for the source term) to recover the super-convergence for any choice of cell centres;
3. describe a local compensation phenomenon, occurring for many meshes, that ensure the super-convergence of HMM schemes even if the cell centres are not the centres of gravity.

Finally, using the flexibility of the choice of cell centres, we will solve a 20+-year old conjecture on TPFA schemes, namely their super-convergence on (almost all) triangular grids.
Tertiary oil recovery is modelled by a system of an elliptic and a parabolic partial differential equations. This system has dominating convection terms, which require special treatment in numerical implementations.

The Eulerian–Lagrangian Localised Adjoint Method (ELLAM) combines the method of characteristics, for an exact solution of the convective terms, and finite element methods, for diffusion terms. It has been implemented in Wang et al. for the tertiary oil recovery model, with apparently good numerical outcomes.

We will however show that, under the conditions described in the numerical tests, the ELLAM method of Wang et al. cannot provide the claimed results and gives rise to concentrations that explode beyond 25 after only one time step. We will show how to modify and properly implement the method in order to recover acceptable numerical results.

References

CVD-MPFA Darcy Flux Approximation on Unstructured Grids

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Control-volume distributed multi-point flux approximations (CVD-MPFA) are presented for unstructured grids in two and three dimensions. These schemes are designed to be consistent and flux-continuous while maintaining the reservoir simulation standard of only employing a single degree of freedom per control-volume, per flow variable, and are consequently proving popular in reservoir simulation. Both cell-centred and cell-vertex approximations are considered. Cell-vertex approximation proves to be advantageous, and requires appropriate grids for Darcy flux approximation, grid generation issues are discussed. The methods are applied to problems involving strong full-tensor permeability fields, faults and layers. The talk will touch on a number of topics (depending on time) including convective and elliptic flux approximation together with fracture models and the resulting flow in fractures.

References


Parametric PDEs have gained a lot of attention in recent years, especially in the context of uncertainty quantification (UQ) where the parameters are random variables. Since in practice often many parameters of the considered problems cannot be determined precisely or are stochastic by nature, modelling and simulation with uncertain data is particularly relevant with engineering applications.

Spectral methods for PDEs with random data are based on the functional representation of the solution manifold in some polynomial chaos basis, including all dependencies on the stochastic parameters of the model. While the implementation of these numerical methods can be more involved than popular sampling techniques such as Monte Carlo and its more advanced variants, they potentially lead to optimal convergence rates with respect to the regularity of the considered problem, i.e. higher regularity can be fully exploited. As another advantage, they allow for the computation of a posteriori error indicators or estimators based on a hierarchical discretisation or on the residual. In case of a Galerkin method, the latter even leads to reliable a posteriori error estimation similar to what has become standard in deterministic FEM. When using equilibration error estimators, the error bound of the mean energy error is even guaranteed.

While sampling techniques solely rely on the evaluation of single realisations, the full discretisation of the stochastic problem in a Galerkin approach usually results in very high-dimensional algebraic problems which easily become unfeasible for numerical computations because of the dense coupling structure of the stochastic differential operator. Recently, an adaptive SGFEM based on a residual a posteriori error estimator was presented and the convergence of the adaptive algorithm was shown [Eigel, Gittelson, Schwab, Zander 2014]. This approach leads to a drastic reduction of the complexity of the problem due to the iterative discovery of the sparsity of the solution and a subsequent quasi optimal discretisation. To allow for larger and more general problems, in [Eigel, Pfeffer, Schneider 2015] we exploit the tensor structure of the parametric problem by representing operator and solution iterates in the modern tensor train (TT) format. The (successive) compression carried out with such a (linearised) hierarchical representation can be seen as a generalisation of some other model reduction techniques, e.g. reduced basis methods. The suggested approach facilitates the efficient computation of different error indicators related to the computational mesh, the active polynomial chaos index set, and the TT rank. Most notably, the curse of dimension is circumvented despite the use of a full stochastic tensor space.
In this presentation, we will consider the second order elliptic boundary value problem in 2D and 3D with highly varying and heterogeneous coefficients, and present variants of the harmonically enriched multiscale coarse space for the additive Schwarz preconditioner for the problem. The preconditioner is based on the abstract Schwarz framework. For the coarse space we propose to use the standard multiscale finite element function or its variants, and show how to enrich the coarse space in order to construct preconditioners that are robust with respect to any variations and discontinuities in the coefficients. The harmonic enrichment is based on solving certain, simple, but carefully chosen, lower dimensional generalized eigenvalue problems on the interfaces between subdomains. Convergence analysis and the numerical results supporting the analysis will be presented.
A DEFLATION TECHNIQUE FOR DETECTING MULTIPLE LIQUID CRYSTAL EQUILIBRIUM STATES

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Multiple equilibrium states arise in many physical systems, including various types of liquid crystal structures. Having the ability to reliably compute such states enables more accurate physical analysis and understanding of experimental behavior. In this talk, we consider adapting and extending a deflation technique for the computation of multiple distinct solutions arising in the context of modeling equilibrium configurations of nematic and cholesteric liquid crystals. The deflation method is applied as part of an overall free-energy variational approach and is modified to fit the framework of optimization of a functional with pointwise constraints. It is shown that multigrid methods designed for the undeflated systems may be applied to efficiently solve the linear systems arising in the application of deflation. For the numerical algorithm, the deflation approach is interwoven with nested iteration, creating a dynamic and efficient method that further enables the discovery of distinct solutions. Finally, we present numerical simulations demonstrating the efficacy and accuracy of the algorithm in detecting important physical phenomena, including bifurcations and disclination behaviors.
ON $\omega$-NONLINEAR EIGENVALUE PROBLEMS
WITH APPLICATIONS IN ELECTROMAGNETICS

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On a macroscopic scale dielectric materials are described by the polarization $P$ of the material. Electromagnetic problems with time-dependent material parameters can then be expressed as a coupled problem in $P$ and the electric field $E$. The corresponding eigenvalue problem is in general nonlinear in the frequency $\omega$. Another example of a $\omega$-nonlinear eigenvalue problem is a resonance problem with a Dirichlet-to-Neumann map on an artificial boundary. These two $\omega$-nonlinear eigenvalue problems have similar structure, but the behaviour of eigenvalues close to a pole is completely different.

Let $M_\ell, \ell = 1, 2, \ldots, L$ denote bounded linear operators in a Hilbert space $\mathcal{H}$ and denote by $A$ a self-adjoint operator with compact resolvent that is bounded from below. In this talk we consider operator functions of the form

$$S(\omega) = A - \omega^2 - \sum_{\ell=1}^L f_\ell(\omega)M_\ell, \quad \text{dom} S(\omega) = \text{dom} A, \quad \omega \in \Omega,$$

which include operator functions that describe problems with $\omega$-dependent material coefficients and resonance problems. We prove spectral properties of $S$ and propose a new enclosure of the numerical range. Finally, we discuss convergence results for Galerkin approximations and computation of eigenvalues of matrix-valued functions.

The talk is based on joint works with Juan Carlos Araujo-Cabarcas, Luka Grubišić, Elias Jarlebring, Heinz Langer, Axel Torshage, and Christiane Tretter.
A classical finite volume method (FVM) describes numerically a conservation law of an underlying model problem. It naturally preserves local conservation of the numerical fluxes. Therefore, FVMs are well-established in the engineering community (fluid mechanics).

We consider an adaptive vertex-centered finite volume method with first-order conforming ansatz functions. The adaptive mesh-refinement is driven by the local contributions of the weighted-residual error estimator. We prove that the adaptive algorithm leads to linear convergence with generically optimal algebraic rates for the error estimator and the sum of energy error plus data oscillations. While similar results have been derived for finite element methods and boundary element methods, the present work appears to be the first for adaptive finite volume methods, where the lack of the classical Galerkin orthogonality leads to new challenges.

Sequel to losses of sheet surface integrity and dimensional accuracy observed in the cold rolling of aluminium A1200, the effect of tensional forces at mandrel, roll velocity and contact angle; on the stress distribution of the Aluminium strips and roll torque were investigated. Experiments were conducted at a four-high reversible Aluminium rolling mill. Thereafter, the Aluminium sheet was modelled for three passes of cast coil reduction from 7.0mm to 2.2mm using the Elastoplastic model with Von-Mises yield criteria and Perfectly Plastic model for hardening. The geometries were finely meshed using free quadrilateral. The roll velocities were applied as prescribed velocities and the tensional force as boundary load. The models were run on the COMSOL GUI to determine stress distributions and hence the roll force and roll torques. Simulation results compared favourably with that of experiments. Results showed that tensional forces applied at the mandrels during rolling, were higher than required as the sheet inlet thickness got smaller. The magnitudes of the roll torque were found to be strongly dependent on the amounts of draft and roll velocity for each passes. The arc length of contact was also found to be a strong leading parameter. Finite element analysis was effectively used to determine the effect of tensional forces at mandrel and roll velocities on the stress distribution in the Aluminium model during cold rolling.
In this talk we present our recent work [Führer, Heuer: Robust coupling of DPG and BEM for a singularly perturbed transmission problem, arXiv:1603.05164], in which we consider a transmission problem consisting of a singularly perturbed reaction diffusion equation on a bounded domain and the Laplacian in the exterior, connected through standard transmission conditions. We establish a DPG scheme coupled with Galerkin boundary elements for its discretization, and prove its robustness for the field variables in so-called balanced norms. Our coupling scheme is the one from [Führer, Heuer, Karkulik: On the coupling of DPG and BEM, arXiv:1508.00630], adapted to the singularly perturbed case by using the scheme from [Heuer, Karkulik: A robust DPG method for singularly perturbed reaction diffusion problems, arXiv:1509.07560]. Essential feature of our method is that optimal test functions have to be computed only locally. We report on various numerical experiments in two dimensions.

**A MIXED FORMULATION FOR LARGE DEFORMATION CONTACT PROBLEM USING ISOGEOOMETRIC ANALYSIS**

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IsoGeometric Analysis (proposed by T. Hughes and coauthors in [2]) uses B-Splines and Non-Uniform Rational B-Splines (NURBS) as basis functions to solve partial differential equations.

In this talk, we will consider rigid-deformable contact problems in large deformations. The contact constraints are treated with a mortar like approach combined with an interpolation of gap (to see [1] on a second order elliptic equations and [3] using a augmented Lagrangian method). These constraints are satisfied with a Lagrangian formulation to impose the Signorini contact conditions and an Active Set Strategy [4] ensures the complementary conditions. Some numerical results will be presented showing the good convergence properties of our algorithms.
LOCAL ERROR ESTIMATES AND CONVERGENCE OF THE GALERKIN BOUNDARY ELEMENT METHOD ON POLYGONAL DOMAINS

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We consider the local behavior of the Galerkin error of a quasi-uniform boundary element discretization of Symm’s equation on polygonal (or polyhedral) Lipschitz domains. It is well-known that the convergence of the global Galerkin error is limited by the regularity of the solution, i.e., singularities (both in the data and geometry) may reduce the global order of convergence. However, on smooth parts of the boundary away from the singularities the behavior of the Galerkin error is much better. For the finite element method this has, e.g., been observed by \cite{1}, and for the boundary element method on smooth screens by \cite{2}. In fact, the convergence of the FEM is locally optimal in the energy norm on polygonal domains. For the boundary element method, the local estimates of \cite{2} imply that the local error in the energy norm is at least better than the global error by a factor of square root of the mesh width.

In this talk, we provide local estimates for the $L^2$- and $H^{-1/2}$-error on a polygonal domain and show that the local error in the $L^2$-norm converges with the rate of $O(h^{1/2+\alpha+\alpha_D})$, where $\alpha$ is the global regularity of the solution, and $\alpha_D$ denotes the additional regularity of the dual problem on polygonal domains. The numerical observations also confirm that this rate is optimal. However, the rate of convergence can be improved if the singularities of the data and the dual problem are separated.
References


FINITE ELEMENT-DISCONTINUOUS GALERKIN METHOD FOR THE NUMERICAL SIMULATION OF TWO-PHASE FLOW

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The subject of the contribution is the numerical simulation of two-phase flow of immiscible fluids. Their motion is described by the incompressible Navier-Stokes equations with piecewise constant density and viscosity. The interface between the fluids is defined with the aid of the level-set method using a transport first-order hyperbolic equation. The Navier-Stokes system equipped with initial and boundary conditions and transmission conditions on the interface between the fluids is discretized by the Taylor-Hood \( P2/P1 \) conforming finite elements in space and the second-order BDF method in time. The transport level-set problem is solved with the aid of the space-time discontinuous Galerkin method (DGM). Numerical experiments demonstrate the applicability, accuracy and robustness of the developed method.

The results were obtained in cooperation with E. Bezechlebová, V. Dolejší and P. Sváček.
DISCONTINUOUS GALERKIN METHOD
FOR THE SOLUTION OF ELASTO-DYNAMIC AND
FLUID-STRUCTURE INTERACTION PROBLEMS

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This contribution will be concerned with the numerical solution of dynamic elasticity by the discontinuous Galerkin (DG) method. We consider the linear case as well as the nonlinear St. Venant-Kirchhoff model. The space discretization is carried out by the DG method. For the time discretization several techniques are applied and tested. As the best method the DG discretization both in space and time appears. The applicability of the developed technique is demonstrated by several numerical experiments. Then the developed method is combined with the space-time DG method for the solution of compressible flow in a time dependent domain and used for the numerical simulation of fluid-structure interaction.

The results were obtained in cooperation with M. Balázsová, M. Hadrava, A. Kosík and J. Horáček.

The contribution will be presented in the minisymposium "Higher order space-time finite element methods".
We present a Nitsche-XFEM method for fluid-structure interaction problems involving a thin-walled elastic structure (Lagrangian formalism) immersed in an incompressible fluid (Eulerian formalism). The fluid domain is discretized with an unstructured mesh not fitted to the solid mid-surface mesh. Weak and strong discontinuities across the interface are allowed for the velocity and pressure, respectively. The kinematic-dynamic interface coupling is enforced consistently using a variant of Nitsche’s method involving cut elements. Robustness with respect to arbitrary interface/element intersections is guaranteed through suitable stabilization. For the temporal discretization, we introduce a semi-implicit scheme which overcomes strong coupling without compromising stability and accuracy. Numerical examples, involving static and moving interfaces, illustrate the performance of the methods.
ON THE ANALYSIS OF LPS ON S-TYPE MESHES FOR CONVECTION-DIFFUSION PROBLEMS

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Let us consider the singularly perturbed convection-diffusion problem given by

\[-\varepsilon \Delta u - bu_x + cu = f \quad \text{in } \Omega = (0,1)^2,\]

\[u = 0 \quad \text{on } \Gamma = \partial \Omega,\]

under the usual smoothness assumptions and \(b \geq \beta > 0\) on \(\Omega\), while \(0 < \varepsilon \ll 1\) is a small perturbation parameter.

We use a-priori adapted meshes like S-type meshes as the layer structure for this kind of problem is known. Still, it is preferable to add a stabilisation to the standard Galerkin formulation in order to increase the stability of the numerical solution. One such a stabilisation method is the local stabilisation projection – LPS.

In the convergence analysis for this method one can usually prove for the discrete error in the LPS-norm and \(p\)-th order elements

\[\|Iu - u_h\|_{LPS} \leq C(N^{-1} \max |\psi'|)^p,\]

see e.g. [1, 2], while the actual error can only be bounded uniformly in the energy norm

\[\|u - u_h\|_\varepsilon \leq C(N^{-1} \max |\psi'|)^p.\]

We present a minor modification of the LPS-method such that we can prove in the stronger LPS-norm

\[\|u - u_h\|_{LPS} \leq C(N^{-1} \max |\psi'|)^p.\]

The idea goes back to a paper from Zhang and Liu [3], where the convergence of a modified SDFEM was investigated.

References


DISCONTINUOUS GALERKIN METHODS: TIME EVOLUTION OF SUPERCONVERGENCE PROPERTIES

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Numerical solutions to hyperbolic conservation laws whose spatial discretisation is completed by the discontinuous Galerkin (DG) method often profit from a superconvergence property. In this talk, we consider how superconvergence properties are affected by the pairing of DG with a time-stepping method as well as by the choice of flux function. We present theoretical results for linear equations and illustrate nonlinear cases with numerical experiments.

This is joint work with Jennifer Ryan.

ACCURATE SPATIAL AND TEMPORAL DISCRETISATION TECHNIQUES FOR INTERFACE PROBLEMS AND FLUID-STRUCTURE INTERACTIONS IN EULERIAN COORDINATES

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Interface problems pose several challenges for discretisation, especially in the case of moving interfaces. If the interface is not resolved by the discretisation, one obtains a reduced order of convergence and possibly stability issues.

In this talk, we present discretisation schemes in both space and time in order to avoid these issues. The proposed finite element discretisation in space corresponds to a fitted finite element method that uses a fixed patch mesh that is independent of the interface location in combination with an interior refinement that resolves the interface. For time discretisation, we use a modified time-stepping scheme that is based on a space-time continuous Galerkin approach (cG(1)). Instead of using polynomials in direction of time that cross the interface, we define Galerkin spaces on trajectories that stay within each subdomain. Similar techniques have been used within the fixed-mesh ALE method by Codina et al. We show second-order convergence for both discretisation in space and time and give a bound on the condition of the system matrix. Finally, we illustrate the capability of our approach in the context of fluid-structure interaction problems.
We study inf-sup stable finite element discretizations of the evolutionary Navier–Stokes equations with grad-div type stabilization. The analysis covers both the case in which the solution is assumed to be smooth and consequently has to satisfy nonlocal compatibility conditions as well as the practically relevant situation in which the nonlocal compatibility conditions are not satisfied. As we are interested in the case of high Reynolds number, we derive error bounds that do not depend on negative powers of the viscosity. Taking into account the loss of regularity at the initial time suffered by the solution of the Navier–Stokes equations, second order error bounds are obtained. The analysis is optimal for quadratic/linear inf-sup stable pairs of finite elements.
We present an HDG formulation for a model diffusion equation on a polygonal/polyhedral mesh. We then show how to obtain optimal and superconvergent HDG methods by carefully choosing the approximate finite element spaces; see [1, 2, 3]. We also briefly discuss another approach to superconvergence by carefully choosing the stabilization operator; see [4, 5].

References


The motivation is to study void formation inside thermoset polymers used as matrices for composite materials that act as electrical insulators inside form-wound coils of large medium-voltage electromachinery. A full derivation of the linear first order system of thermoviscoelastic equations in the time and frequency domain is presented. Compatible variational formulations with unbroken test spaces and broken test spaces are deduced for the thermoviscoelasticity equations in the frequency domain. A minimum residual method with broken test spaces, i.e. the discontinuous Petrov-Galerkin (DPG) methodology, is applied to the “broken” variational formulation to solve the equations. Expected convergence rates for $p = 1, 2, 3$ are observed for a manufactured setting with a smooth solution. Preliminary results used to validate experimental data are also shown.
REDUCED BASIS APPROXIMATION AND A POSTERIORI ERROR ESTIMATES FOR PARAMETRIZED ELLIPTIC EIGENVALUE PROBLEMS

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In many applications, ranging from optics and electronics to acoustics and structural mechanics, the solution of eigenproblems plays a crucial role. Moreover, repeated solutions are required, for different physical or geometrical settings, as soon as optimal control issues or inverse problems are addressed. In this framework, the reduced basis (RB) method can represent a suitably efficient technique to contain the demanded computational effort, especially in a many-query context. Starting from the pioneering work [1], in the last fifteen years the RB method has been applied to linear and nonlinear eigenproblems, also depending on a high number of parameters [2]. Nevertheless, few results on the \textit{a posteriori} error estimation of the reduced order solution have been published.

In [3], we develop a new RB method for the approximation of a parametrized eigenproblem for the Laplacian. This method hinges upon dual weighted residual type \textit{a posteriori} error indicators, which give rigorous upper bounds, for any value of the parameters, of the error between the high-fidelity finite element approximation of the first eigenvalue and eigenfunction and the corresponding RB approximations. The proposed error estimators are exploited not only to certify (\textit{online}) the RB approximation, but also to set up a greedy algorithm for the \textit{offline} construction of the RB space. Furthermore, a computationally inexpensive approximation of the inf-sup coefficient on which the error bounds depend is provided, addressing an issue that often represents a bottleneck in the efficient application of reduced order approximations. Several numerical experiments assess the overall reliability and efficiency of the proposed RB approach, both for affine and non-affine parametrizations.

References


It is shown that the $h$-adaptive mixed finite element method for the discretization of eigenvalue clusters of the Laplace operator produces optimal convergence rates in terms of nonlinear approximation classes. The results are valid for the typical mixed spaces of Raviart–Thomas or Brezzi–Douglas–Marini type with arbitrary fixed polynomial degree in two and three space dimensions. The talk is based on the work [1].

References

A stable splitting of $2m$-th order elliptic partial differential equations into $2(m-1)$ problems of Poisson type and one generalized Stokes problem is established for any space dimension $d \geq 2$ and any integer $m \geq 1$. This allows a numerical approximation of high-order partial differential equations like

$(-1)^m \Delta^m u = f$  \hspace{1cm} (1)

(possibly with lower-order terms) with standard finite elements that are suited for the Poisson equation and the Stokes system, respectively. In contrast to certain conventional splittings of (1), the approach presented in this talk does not require any additional regularity of the solution to (1). For some fourth- and sixth-order problems in two and three space dimensions, precise finite element formulations along with a priori error estimates and numerical experiments are presented. The talk is based on the work [1].

References

Once an evolution problem has been discretized in space-time, it is of interest due to its size to solve it on a large scale parallel computer. Several recent time parallel methods have been developed only for linear problems, and they use linearity in essential ways, for example the ParaExp algorithm, or the parallelization method based on diagonalization of the time stepping matrix. I will use the latter to explain how one can use such an essentially linear method also in the context of a non-linear evolution problem. I will first explain the method for a scalar model problem, and then give a formulation for a non-linear partial differential equation based on tensorization. I will also illustrate the methods with numerical experiments.
Many engineering and industrial processes are modeled using partial differential equations (PDEs) in time-dependent domains. Since the analytical solutions of these PDEs are almost impossible to obtain, the numerical approximation of these solutions is the only viable option, especially when the deformation of the domain is large. Apart from other challenges associated with the simulations of industrial processes, the presence of moving boundaries/interfaces makes the computation more complex. Moreover, the computational domain becomes a part of the numerical solution. Even though several approaches have been proposed to track/capture the moving boundaries, arbitrary Eulerian-Lagrangian (ALE) approach is preferred when the application demands accurate numerical solution and/or sharp moving boundaries/interfaces.

In ALE finite element schemes, PDEs can be written in (i) conservative and (ii) non-conservative ALE forms. Although both forms are equivalent in continuous level, these forms are no longer equivalent in (time) discrete level. Further, appropriate quadrature formula needs to be used for the time integration when different time discretizations are applied. This quadrature requirement is imposed as the geometric conservation law (GCL) in numerical schemes. Moreover, standard time discretizations, e.g., implicit Euler, Crank-Nicolson, discontinuous Galerkin, need to be modified in order to satisfy the GCL condition and to derive stability estimates. In this talk, an assessment of these modified time-discretizations applied to both conservative and non-conservative ALE forms will be presented. Further, the application of these methods in practical applications will also be discussed.
A LOCALLY MODIFIED FITTED FINITE ELEMENT METHOD FOR INTERFACE PROBLEMS IN SHAPE AND TOPOLOGY OPTIMIZATION

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We consider the design optimization of an electric motor by means of PDE-constrained topology and shape optimization. The goal is to find the optimal distribution of ferromagnetic material within a design subregion of the computational domain. In the course of the optimization procedure, the interface between ferromagnetic material and air regions evolves.

In every iteration of the optimization procedure, the interface between different subdomains is updated. On the updated geometry, which is in general not resolved by the finite element discretization, the state and adjoint equations have to be solved. We present an easy to implement numerical method that allows us to resolve a piecewise linear interface exactly in every iteration by only locally modifying the underlying triangular mesh. Moreover, the chosen mesh adaptation strategy ensures a maximum angle condition which yields optimal order of convergence independent of the location of the interface relative to the mesh. The presented method is based on \cite{1}.

References

OPTIMAL CONVERGENCE FOR ADAPTIVE
IGA BOUNDARY ELEMENT METHODS

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A posteriori error estimation and optimal adaptive mesh-refinement are well-established
for the Galerkin boundary element method (BEM) with piecewise polynomial ansatz
functions on polygonal boundaries. In contrast to that, the mathematically reliable
a posteriori error analysis for isogeometric BEM (IGABEM) is still in its infancy.
In our talk, we discuss recent results on reliable a posteriori error estimators (see
\cite{1} for Galerkin IGABEM resp. \cite{2} for collocation IGABEM in 2D) and on optimal
convergence of corresponding adaptive IGABEM algorithms in 2D (see e.g. \cite{3}).

As model example, we consider the weakly-singular as well as the hyper-singular
integral equation for the 2D Laplacian and the corresponding weighted-residual error
estimator which controls the (in general, non-computable and unknown) discretization
error in the $\tilde{H}^{-1/2}$ resp. $\tilde{H}^{1/2}$ norm. Its local contributions are used for adaptive
IGABEM computations to steer an adaptive algorithm of the form

\[
\text{Solve $\rightarrow$ Estimate $\rightarrow$ Mark $\rightarrow$ Refine}
\]

for which optimal convergence behaviour is proved. Unlike available results in the lit-
erature, the adaptive algorithm steers the local mesh-refinement as well as the local
smoothness of the ansatz functions across nodes of the boundary partition. The algo-
rithm automatically detects and resolves jumps and singularities of the exact solution
as well as possible smooth parts. If compared to uniform mesh-refinement as well as
adaptive standard BEM based on piecewise polynomials, this dramatically reduces the
storage requirements and the computing time needed to achieve a certain prescribed
accuracy.

References

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ANALYSIS OF FIRST ORDER PROJECTION METHODS FOR THE NAVIER-STOKES EQUATIONS

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We present convergence results of a projection method for the Navier-Stokes equations with non inf-sup stable elements recently analyzed in \cite{2}. The method depends on a parameter $\delta$ that, when taken equal to the time step $\Delta t$, gives rise to the Chorin-Temam projection method, but we do not restrict ourselves to this particular case. The method, with different restrictions on $\delta$ has been partially analyzed in the literature (see e.g. \cite{1}, \cite{3}, \cite{4}) with different techniques and obtaining partial convergence results. We present a unified analysis that improves the existing error bounds in the literature. The analysis covers first steady and evolutionary Stokes problems, where the restrictions of the method and the relations that must be satisfied by $\Delta t$ and $\delta$ for convergence are established. In particular, no restriction $\Delta t > Ch^2$ ($h$ being the spatial mesh size and $C$ a positive constant) is needed for convergence. The analysis in the case of the Navier-Stokes equations will (almost) be a direct consequence of the previous analysis in simpler linear problems.

References


We present some recent results on the finite element approximation of fluid-structure interaction problems. In particular, we consider a new variational formulation of the Immersed Boundary Method (IBM) based on the introduction of a suitable Lagrange multiplier. We prove that a semi-implicit time advancing scheme is unconditionally stable. At each time step we have to solve a saddle point problem. We study existence and uniqueness of the continuous solution and analyze the finite element discretization providing optimal error estimates. For more details see [2] [1].

References


AN ADAPTIVE $P_1$ FINITE ELEMENT METHOD FOR TWO-DIMENSIONAL MAXWELL’S EQUATIONS

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We extend the Hodge decomposition approach for the cavity problem of two-dimensional time harmonic Maxwell’s equations to include the impedance boundary condition, with anisotropic electric permittivity and sign changing magnetic permeability. We derive error estimates for a $P_1$ finite element method based on the Hodge decomposition approach and develop a residual type $a$ posteriori error estimator. We show that adaptive mesh refinement leads empirically to smaller errors than uniform mesh refinement for numerical experiments that involve metamaterials and electromagnetic cloaking. The well-posedness of the cavity problem when both electric permittivity and magnetic permeability can change sign is also discussed and verified for the numerical approximation of a flat lens experiment.

References


This talk presents a residual-based a posteriori error estimator for the Arnold–Winther mixed finite element that utilises a post-processing for the skew-symmetric part of the strain. Numerical experiments verify the proven reliability and efficiency for suitable approximation of the skew-symmetric deformation gradient. Numerical evidence supports that the $L^2$-stress error estimator is robust in the Poisson ratio and allows stable error control even in the incompressible limit.

References

A POSTERIORI ERROR ESTIMATES FOR A DISCONTINUOUS GALERKIN METHOD FOR INTERFACE PROBLEMS ON GENERAL DOMAINS

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An interior-penalty discontinuous Galerkin (dG) method for an elliptic interface problem involving, possibly, curved, interfaces, modelling the mass transfer of solutes through semi-permeable membranes, is considered. The method allows for extremely general curved element shapes employed to resolve the interface geometry exactly. A residual-type a posteriori error estimator for this dG method is proposed and respective upper and lower bounds of the error in the respective dG-energy norm are proven. The a posteriori error bounds are subsequently used to prove a basic a priori convergence result. The theory presented is complemented by a series of numerical experiments.

HYBRID NUMERICAL ASYMPTOTIC BOUNDARY ELEMENT METHOD FOR MULTIPLE SCATTERING PROBLEMS

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Standard numerical schemes for scattering problems have a computational cost that grows at least in direct proportion to the frequency of the incident wave. For many problems of scattering by single obstacles, it has been shown that a careful choice of approximation space, utilising knowledge of high frequency asymptotics, can lead to numerical schemes whose computational cost is independent of frequency. Here, we extend these ideas to multiple scattering configurations, focusing in particular on the case of two scatterers, with one much larger than the other.
GENERALIZED BARYCENTRIC COORDINATES FOR DEGENERATE GEOMETRY IN FEM

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Generalized barycentric coordinates are an essential tool in the growing area of polytopal element methods. Recent applications of the coordinates include the construction of stability matrices for virtual element methods and gradient correction for nonlinear elasticity problems. The success of these and other applications rely on the fact that the coordinates can provide a good-quality interpolation scheme even on elements with (seemingly) poor geometry. We will present recent numerical experiments that exhibit how various types of generalized barycentric coordinates behave under a variety of degenerate geometry scenarios.

THE SERENDIPITY PYRAMID FINITE ELEMENT

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Pyramid geometries are used to bridge between tetrahedral and hexahedral meshing regimes. While pyramid finite elements with tensor product degrees of freedom on the base have been well-studied, pyramid finite elements with ‘serendipity’ degrees of freedom on the base have not. In the process of describing the scalar serendipity pyramid element, we will see definitions of shape functions and degrees of freedom related to finite element exterior calculus, as well as unisolvence and polynomial reproduction results.

References


We consider well-posedness, convergence and a posteriori error estimates for fluid-structure interaction and contact problems in time-domain.

In the case of an elastic body immersed in a fluid, a Galerkin time-domain boundary element method (TDBEM) for the wave equation in the exterior is coupled to a finite element method for the Lamé equation. Based on ideas from the time–independent coupling formulation and its a posteriori error analysis, we give a priori and a posteriori error estimates, which demonstrate the convergence and give rise to adaptive mesh refinement procedures.

We then discuss a first error analysis for dynamic Signorini problems with flat contact area, a variational inequality involving the Dirichlet-Neumann operator for the wave equation. Here refined information about the Dirichlet-Neumann operator allows to prove well-posedness as well as a priori and a posteriori error estimates for the TDBEM solutions.

The talk concludes with a survey of recent computational work on TDBEM in our group.
Parametrized parabolic problems often occur in industrial or financial applications, e.g. as pricing of options on the stock market. If we want to calibrate an option pricing model, we need several evaluations for different parameters. Fine discretizations, that are needed for these problems, resolve in large scale problems and thus in long computational times. To reduce the size of those problems, we use the Reduced Basis Method (RBM) \cite{2,1}. The ambition of the RBM is to efficiently reduce discretized parametrized partial differential equations given in a variational form. Using space-time formulations, we do not use a time-stepping scheme, but take the time as an additional variable in the variational formulation of the problem.

Well-posedness for the space-time variational approach has been shown for a wide range of problems. For the general case of a parabolic variational equation, see \cite{3}.

Combining the RBM with the space-time formulation, we derive a possibly noncoercive Petrov–Galerkin problem, where improved error estimators for parabolic equations could be achieved \cite{4}.

In this talk we consider a comparison between space-time methods and the often used time-stepping scheme for the RBM. We conclude with an overview where the space-time methods has been successfully applied to RBM.

References


A DTN FINITE ELEMENT METHOD FOR AXISYMMETRIC ELASTICITY IN SEMI-INFINITE DOMAINS

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In some problems arising in geophysical applications, the solid earth is mathematically modelled as an elastic semi-infinite domain. In general, to solve numerically a boundary-value problem formulated in an elastic domain, the finite element method appears to be very convenient. However, it cannot be directly applied if the involved domain is unbounded. A good alternative to overcome this drawback is to use the Dirichlet-to-Neumann (DtN) map in order to deal with the unboundedness. The DtN map provides, on an artificial boundary of regular shape, exact boundary conditions, which may be combined with a finite element discretisation of the bounded computational domain lying inside the artificial boundary. Such a procedure is known as the DtN finite element method, and it has been successfully applied to different problems formulated in infinite exterior domains, since in this case it is usually possible to compute an explicit closed-form expression for the DtN map. However, in the case of a semi-infinite elastic domain this is not, in general, possible. For this reason, the use of the DtN finite element method in geophysical applications has been rather limited.

In this work, we present a DtN finite element method for solving boundary-value problems of elasticity formulated in a locally perturbed half-space with axisymmetry about the vertical axis. The lack of a closed-form expression for the DtN map is remedied by employing an approximation procedure that combines numerical and analytical computation techniques. Firstly, the locally perturbed half-space is truncated by means of a semi-spherical artificial boundary, dividing it into a bounded computational domain and a semi-infinite residual domain. Then, a finite element formulation of the elasticity problem is established in the computational domain, taking into account the exact boundary conditions on the artificial boundary provided by the DtN map. As it is not possible to obtain a closed-form expression for the DtN map, we approximate only those boundary integral terms occurring in the finite element formulation that involve precisely the DtN map. To do so, the boundary-value problem in the residual domain is solved by a semi-analytical technique, just for the required Dirichlet data on the artificial boundary. By applying Boussinesq potentials and separation of variables, the solution is expressed as a series with unknown coefficients, which are approximated by minimising a quadratic energy functional appropriately chosen. The minimisation yields a symmetric and positive definite linear system of equations for a finite number of coefficients, which are efficiently solved by exploiting its particular block-structure, in such a way that the coefficients of the series are in practice computed by mere forward and backward substitution. This procedure allows an approximate but very effective coupling of the DtN map with the finite element method for the semi-infinite elastic problem under study. The procedure is validated by solving a particular case where an
exact solution is available, using structured triangular meshes of different sizes. The relative error between the numerical and the exact solution is calculated for each mesh size considered, corroborating the effectiveness and accuracy of the proposed procedure. Indeed, the numerical evidence shows that it achieves second-order accuracy.

A MULTILEVEL CORRECTION METHOD FOR OPTIMAL CONTROLS OF ELLIPTIC EQUATION

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In this talk we present a multilevel correction method to solve optimal control problems constrained by elliptic equations with the finite element method on both uniform and adaptive meshes. In this scheme, solving an optimization problem on the finest finite element space is transformed into a series of solutions of linear boundary value problems by the multigrid method on multilevel meshes and a series of solutions of optimization problems on the coarsest finite element space. Our proposed scheme, instead of solving a large scale optimization problem in the finest finite element space, solves only a series of linear boundary value problems and the optimization problems in a very low dimensional finite element space, and thus can improve the overall efficiency of the solution of optimal control problems governed by PDEs.
We develop a posteriori error estimates of optimal order in time for the wave equation in the fully discrete situation discretized with the Newmark scheme in time and with finite elements in space. We look for a posteriori upper bounds in the $L^\infty$-in-time-energy-in-space norm of the error. We adopt a particular choice for the parameters in the Newmark method, namely $\beta = 1/2$, $\gamma = 1/4$. This is a popular choice since it provides a conservative method with respect to the energy norm. Another interesting feature of this variant of the method, which is in fact essential for analysis, is the fact that the method can be reinterpreted as the Crank-Nicolson discretization of a reformulation of the governing equation as a first-order in time system of equations as in [C. Bernardi, E. Suli, Time and space adaptivity for the second-order wave equation, Math. Models Methods Appl. Sci. 15, 2 (2005), pp. 199–225]. We are thus able to use the techniques from [A. Lozinski, M. Picasso, V. Prachittham, An anisotropic error estimator for the Crank-Nicolson method: application to a parabolic problem, SIAM J. Sci. Comput. 31, 4 (2009), pp. 2757–2783], i.e. a piecewise quadratic polynomial in time reconstruction of the numerical solution, which leads to optimal a posteriori error estimates in time and also allows us to recover the estimates in space easily as well. We shall present the technical proofs and illustrate them by numerical results.
Endovascular stents are thin metallic structures which are used for treating a narrowing of blood vessels (stenosis). Stents are typically modeled as an assembly of struts and since they are a metallic structure their small deformations are sufficiently well described by 3D linearized elasticity. However, a direct numerical treatment of such model would lead to considering equations of 3D linearized elasticity in a thin domain. This is a very challenging and time consuming numerical task. As an alternative we start from a simpler analytical approximation – a reduced model – which can be obtained using a one-dimensional model of a curved elastic rod. As a result we obtain a system of ordinary differential equations on a graph. Note that our chosen model has been obtained as a limit – in an appropriate Sobolev space – of the 3D elasticity as the diameter of the strut goes to zero. Associated eigenvalue problem is discretized using a mixed finite element method. As model problems for our approach we consider four different coronary stents which are commercially available at the market and present numerical results.

This is a joint work with Josip Tambaca and Josip Ivekovic.
BOUNDARY-DOMAIN INTEGRAL FORMULATION OF BOUNDARY VALUE PROBLEMS ON SURFACES

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A boundary value problem for the Laplace-Beltrami operator on a smooth two-dimensional surface embedded in $\mathbb{R}^3$ is considered. As in the case of an inhomogeneous heat transfer, a suitable parametrix (Levi function) is found and an integral formulation of the problem is derived. This formulation involves geometrical properties of the surface. Furthermore, besides the usual boundary integrals the integration along the surface is present. The developed approach is also employed to obtain new boundary-domain integral equations for the mean curvature vector.

A numerical method of finding the approximate solution is derived similarly to the corresponding case in $\mathbb{R}^3$. Several key differences and similarities to the popular finite element methods are discussed. Some aspects of implementation are commented on and several numerical examples are presented.

References


AN INTERIOR PENALTY METHOD WITH $C^0$ FINITE ELEMENTS FOR THE APPROXIMATION OF THE MAXWELL EQUATIONS IN HETEROGENEOUS MEDIA: CONVERGENCE ANALYSIS WITH MINIMAL REGULARITY

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The present paper proposes and analyzes an interior penalty technique using $C^0$-finite elements to solve the Maxwell equations in domains with heterogeneous properties. The convergence analysis for the boundary value problem and the eigenvalue problem is done assuming only minimal regularity in Lipschitz domains. The method is shown to converge for any polynomial degrees and to be spectrally correct.

AN EXPLICIT INVARIANT DOMAIN PRESERVING CONTINUOUS FINITE ELEMENT TECHNIQUE FOR HYPERBOLIC SYSTEMS

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We propose a numerical method to solve general hyperbolic systems in any space dimension using forward Euler time stepping and continuous finite elements on non-uniform grids. The properties of the method are based on the introduction of an artificial dissipation that is defined so that any convex invariant sets containing the initial data is an invariant domain for the method. Our technique extends to continuous finite elements the work of Hoff (1985), and Frid (2001). The invariant domain property is proved for any hyperbolic system provided a CFL condition holds. The solution is also shown to satisfy a discrete entropy inequality for every admissible entropy of the system. The method is formally first-order accurate in space and can be made high-order in time by using any Strong Stability Preserving technique. Extensions to Arbitrary-Lagrangean-Eulerian formulations for general hyperbolic systems will also be discussed in the talk.
Given $f \in L^2(\Omega)$, we consider adaptive FEM for problems of the type
\[
a(u, v) + \langle K u, v \rangle_{L^2(\Omega)} = \langle f, v \rangle_{L^2(\Omega)} \quad \text{for all } v \in H^1_0(\Omega),
\]
where $a(\cdot, \cdot)$ is an elliptic and symmetric bilinear form on $H^1_0(\Omega)$ and $K : L^2(\Omega) \to L^2(\Omega)$ is a continuous linear operator. We suppose that (1) is well-posed and hence admits a unique solution $u \in H^1_0(\Omega)$. This setting is met, e.g., for the Helmholtz equation or second-order linear elliptic problems with reaction and/or convection. For a standard conforming FEM discretization of (1) by piecewise polynomials, usual duality arguments show that the underlying triangulation has to be sufficiently fine to ensure the existence and uniqueness of the Galerkin solution.

Extending the abstract approach of [1], we prove that adaptive mesh-refinement is capable of overcoming this preasymptotic behavior and eventually leads to convergence with optimal algebraic rates. Unlike previous works [2, 3, 4], one does not have to deal with the \textit{a priori} assumption that the initial mesh is sufficiently fine. The overall conclusion of our results thus is that adaptivity has stabilizing effects and can, in particular, overcome preasymptotic and possibly pessimistic restrictions on the meshes.

References


Surface active agents (surfactants) are of importance in numerous practical applications. The influence of surfactants on the deformation of droplets and the structure of the surrounding flow field is an active research area. Local accumulation of surfactants and the resulting Marangoni forces may lead to a destabilization of the interface with essential consequences on the flow structure. This is a complex process whose tailored use in applications requires a fundamental understanding of the mutual interplay.

We present a finite element method for the flow of two immiscible, incompressible fluids in two and three dimensions. Thereby the presence of soluble and insoluble surfactants is considered. The finite element method uses the Arbitrary Lagrangian Eulerian (ALE) technique, which tracks the interface by moving grids. We use second order finite elements and a second order interface approximation, which allows precise incorporation of surface tension forces and Marangoni forces.

We consider a bounded domain $\Omega \subset \mathbb{R}^d$, $d = 2, 3$, filled with two incompressible, immiscible fluids, which occupy at time $t$ the domains $\Omega^1(t), i = 1, 2$. Let $\partial \Omega^i(t)$ denote the boundary of $\Omega^i(t), i = 1, 2$, and $\Gamma(t) = \partial \Omega^1(t) \cap \partial \Omega^2(t)$ the interface of $\Omega^1(t)$ and $\Omega^2(t)$. Our model consists of the Navier-Stokes equations for the flow fields $u^i$ and pressure fields $p^i$ in the phases $\Omega^i(t), i = 1, 2$, a convection diffusion equation for the bulk surfactant $c^i$ in $\Omega^i(t), i = 1, 2$ and a convection diffusion equation on the moving manifold $\Gamma(t)$ for the surface surfactant $c_\Gamma$, completed with various coupling terms. The full model reads:

\begin{align*}
\rho_i (\partial_t u^i + (u^i \cdot \nabla) u^i) - \nabla \cdot S(u^i, p^i) &= f, \quad \nabla \cdot u^i = 0 \quad \text{in } \Omega^i(t), \\
[-S]n &= \sigma(c_\Gamma) \kappa n + \nabla_\Gamma \sigma(c_\Gamma), \quad [u] = 0, \quad V = u \cdot n \quad \text{on } \Gamma(t), \\
\partial_t c^i - D^i \Delta c^i + (u^i \cdot \nabla) c^i &= 0 \quad \text{in } \Omega^i(t), \\
[D \partial_n c] &= -S(c^1, c^2, c_\Gamma) \quad \text{on } \Gamma(t), \\
\partial_t c_\Gamma - D_\Gamma \Delta_\Gamma c_\Gamma + \nabla_\Gamma \cdot (c_\Gamma u_\Gamma) &= S(c^1, c^2, c_\Gamma) \quad \text{on } \Gamma(t).
\end{align*}

for $i = 1, 2$. Where, $S$ is the usual stress tensor for Newtonian fluids, $f$ describes gravitational forces, $[h] := h^1 - h^2$ denotes a jump of quantity $h$ across the interface, $\sigma(c_\Gamma)$ is the surface tension coefficient, $\kappa$ denotes the mean curvature of the interface, $D^i$ is the diffusion coefficient for the bulk $\Omega^i(t)$, $D_\Gamma$ is the surface diffusion coefficient, $\nabla_\Gamma$ and $\Delta_\Gamma$ are the surface version of the corresponding differential operators, $S$ describes ad- and absorption of surfactant at the interface and while $\partial_t$ denotes the time derivative, $\partial_n$ denotes the spatial derivative in normal direction and $\rho_i$ the fluid density in phase $i$. 
We study the influence of surfactants on the dynamics of two-phase flows. In several numerical tests we compare cases of soluble and insoluble surfactants and surfactant free settings.

**POSITIVITY PRESERVING DISCONTINUOUS GALERKIN METHOD FOR DRIFT-DIFFUSION SYSTEM**

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We consider drift-diffusion models describing the classical transport of charge carriers in a semiconductor coupled with a Poisson equation for electric potential. The difficulties of solving this problem numerically are that the numerical scheme should conserve the total charge inside the device, any negative numerical density is unphysical, and the numerical scheme should respect monotonicity of the solution. Here we present a method for solving the drift-diffusion system uses a Discontinuous Galerkin (DG) finite element algorithm, which combines features of both finite element and finite volume methods, and it is particularly suitable for problems satisfying the conservation laws. Furthermore, we have applied a post-processing technique with a bound preserving limiter [1] to insure that the solution satisfies a global positivity. To demonstrate the capabilities of this new method combined with the adaptive mesh refinement technique, and evaluate the trade-offs in computational speed, cost and solution accuracy we also present results for the same test using the Finite Element Method (FEM) which uses the artificial entropy viscosity stabilization scheme.

References

A NATURAL FRAMEWORK FOR ISOGEOMETRIC FLUID-STRUCTURE-INTERACTION: COUPLING BEM AND SHELL MODELS

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The interaction between thin structures and incompressible Newtonian fluids is ubiquitous both in nature and in industrial applications. We present an isogeometric formulation of such problems which exploits a boundary integral formulation of Stokes equations \cite{1} to model the surrounding flow, and a non linear Kirchhoff-Love shell theory \cite{2,3} to model the elastic behaviour of the structure. We propose three different coupling strategies: a monolithic, fully implicit coupling, a staggered, elasticity driven coupling, and a novel semi-implicit coupling, where the effect of the surrounding flow is incorporated in the non-linear terms of the solid solver through its damping characteristics. The novel semi-implicit approach is then used to demonstrate the power and robustness of our method, which fits ideally in the isogeometric paradigm, by exploiting only the boundary representation (B-Rep) of the thin structure middle surface.

References


A STUDY ON THE ACCURACY OF IMMERSED FINITE ELEMENT METHODS

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Immersed Finite Element Methods (IFEM) are an evolution of the original Immersed Boundary Element Method (IBM) developed by Peskin \([6]\) in the early seventies for the simulation of complex Fluid Structure Interaction (FSI) problems. In the IBM, the coupled FSI problem is discretised using a single (uniformly discretised) background fluid solver, where the presence of the solid is taken into account by adding appropriate forcing terms in the fluid equation. Dirac delta distributions are used to interpolate between the Lagrangian and the Eulerian framework in the original formulation by Peskin, while a variational formulation was introduced by Boffi et al. \([1]\), and later generalised in Heltai and Costanzo \([4]\) that does not require any Dirac delta approximation.

One of the key issues that kept people from adopting IBM or IFEM techniques is related to the loss in accuracy attributed to the non-matching nature of the discretisation between the fluid and the solid domains, leading to only formally optimal solvers (see, for example, Lai and Peskin \([5]\)). In this work we exploit some techniques introduced by D’Angelo and Quarteroni \([2, 3]\), to show that, for variational formulations, the loss in accuracy is only restricted to a thin layer of elements around the solid-fluid interface, and optimal error estimates in all norms are recovered if one uses appropriate weighted norms, or by removing the layer of non-matching cells from the error estimates.

References


EFFICIENT PRECONDITIONING OF $hp$-FEM MATRICES
BY HIERARCHICAL LOW-RANK APPROXIMATIONS

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During the last decade, substantial advances have enabled the efficient construction and application of low-rank approximations to large matrices. Among many examples, matrices arising as discretizations of compact operators such as boundary integral operators, have been shown to enable very efficient compression, thus allowing for both compression and solution in linear complexity.

However, for matrices arising from unbounded operators, e.g., finite element discretizations of differential operators, progress has been slower and is often more challenging. These difficulties are further enhanced when considering linear operators originating from the $hp$-FEM discretizations of non-trivial operators such as highly anisotropic problems with high contrast and the wave Helmholtz problem.

In this talk, we discuss two different attempts to take advantage of low rank approximations to develop efficient preconditioners for a variety of problems arising as $hp$-finite element discretizations of linear problems. We discuss the development of efficient hierarchical techniques, utilizing efficient compression of the Schur complement on a hierarchical skeleton, and consider scaling behavior of the compression for both $h$- and $p$-refinement.

We illustrate the performance of the techniques of a number of challenging test cases, including highly anisotropic problems and the wave Helmholtz problem, and discuss a few open problems towards an efficient black-box preconditioner.
We present and analyse a time-stepping DPG method for the heat equation. Motivation of this work is to develop a DPG framework that can lead to robust approximations of singularly perturbed parabolic problems.

We use the backward Euler scheme as time discretisation and propose a DPG space approximation of the time-discrete scheme. Well-posedness and stable approximation properties are obtained from a precise analysis of the underlying time-discrete variational formulation at every time step. Appropriate convergence properties for field variables are proved. We present numerical experiments that underline our theoretical results.

This work has been partially supported by CONICYT-Chile through Fondecyt grants 1150056, 3150012, and Anillo ACT1118 (ANANUM).
SECOND-KIND SINGLE TRACE
BOUNDARY INTEGRAL EQUATIONS

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For second-order linear transmission problems involving a single closed interface separating two homogeneous materials, a well-posed second-kind boundary integral formulation has been known for a long time. It arises from a straightforward combination of interior and exterior Calderón identities. Apparently, this simple approach cannot be extended to “composite” settings involving more than two materials.

The key observation is that the same second-kind boundary integral equations (BIE) can also be obtained through a multi-potential representation formula. We can attach a potential to each boundary of a material sub-domain, add them all up to a multi-potential, and then we notice that, thanks to a null-field property, the sum provides a representation of the field solution, when its traces are plugged into the potentials. Taking traces yields a BIE on the skeleton of the sub-domain partition. The skeleton traces of the unknown field will solve it.

Using the fact that multi-potentials for a single homogeneous material must vanish, the BIE can be converted into second-order form: for the scalar case (acoustics) its operator becomes a compact perturbation of the identity in $L^2$. Galerkin matrices arising from piecewise polynomial Galerkin boundary element (BEM) discretization will be intrinsically well-conditioned.

The new second-kind boundary element method has been implemented both for acoustic and electromagnetic scattering at composite objects. Numerical tests confirm the excellent mesh-size independent conditioning of the Galerkin BEM matrices and the resulting fast convergence of iterative solvers like GMRES. Furthermore, by simple postprocessing, we obtain discrete solutions of competitive accuracy compared to using BEM with the standard first-kind BIE.

Well-posedness of the new second-kind formulations is an open problem, as is the compactness of the modulation of the identity in the case of Maxwell’s equations. Reassuringly, computations have never hinted at a lack of stability.

References


DISPERSION ANALYSIS OF
PLANE WAVE DISCONTINUOUS GALERKIN METHODS

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We conduct a partly empirical dispersion analysis of the method in a discrete translation-invariant setting by studying the mismatch of wave numbers of discrete and continuous plane waves travelling in the same direction. We find agreement of the wave numbers for directions represented in the local trial spaces. For other directions the PWDG methods turn out to incur both phase and amplitude errors. This manifests itself as a pollution effect haunting the \(h\)-version of the method. Our dispersion analysis allows a quantitative prediction of the strength of this effect and its dependence on the wave number and number of plane waves.

References

ROBUST MULTIGRID FOR ISOGEOMETRIC ANALYSIS USING SUBSPACE CORRECTION

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We present a robust and efficient multigrid method for isogeometric discretizations using tensor product B-splines of maximum smoothness. Our method is based on a stable splitting of the spline space into a large subspace of “interior” splines which satisfy a robust inverse inequality, as well as one or several smaller subspaces which capture the boundary effects responsible for the spectral outliers known to occur in Isogeometric Analysis. We then construct a multigrid smoother based on a subspace correction approach, applying a different smoother to each of the subspaces. For the interior splines, we use a mass smoother, whereas the remaining components are treated with suitably chosen Kronecker product smoothers or direct solvers.

The resulting multigrid method exhibits iteration numbers which are robust with respect to the spline degree and the mesh size. Furthermore, it can be efficiently realized both for two- and three-dimensional problems. Our numerical examples show further that the iteration numbers also scale relatively mildly with the problem dimension.

MODELLING TRANSIENT FLUID LOADING ON FLEXIBLE STRUCTURES

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Structural response to water impact is important for several areas, including the aerospace and automotive industries. Liquid sloshing in tanks is important of manoeuvre and crash behaviour and additionally aircraft must be designed to cope with ditching. The goal is a reliable technique for predicting the structural response to transient and extreme fluid loading. This is a complex problem, potentially involving the interaction of non-linear fluid behaviour with non-linear structural behaviour. This paper discussed the coupled FE/SPH approach for modelling fluid interaction with structures. The capabilities of the method are illustrated through comparison of model results with experimental data for sloshing and impact on water. Current challenges with respect to engineering application of this approach will be discussed.
Hamilton-Jacobi-Bellman (HJB) equations characterise the value functions of optimal control problems. For a wide range of control problems one can compute optimal control policies from the partial derivatives of the value function.

An important tool in the analysis of HJB equations and their numerical approximations is the concept of viscosity solutions. Its definition is based on sign information on function values of candidate solutions, leading typically to proofs of uniform convergence of numerical methods. It is more difficult to prove convergence in other norms if solely viscosity solutions are used.

The use of weak solutions, familiar from semilinear differential equations, in the context of Hamilton-Jacobi-Bellman equations is delicate because often uniqueness cannot be ensured. However, we believe that combining the notions of viscosity and weak solution is attractive for numerical analysis: the former to deal with uniqueness and the later to study convergence of partial derivatives.

In a previous work the uniform convergence of P1 finite element approximations to the viscosity solutions of isotropic, degenerate parabolic HJB equations was shown. In addition $L^2(H^1)$ convergence was demonstrated, under the assumption that the HJB equation is uniformly parabolic.

Yet in a wide range of applications, in particular from financial mathematics, the resulting Bellman equations are only degenerately parabolic. In this talk I will explain how to remove the assumption of uniform parabolicity and verify that strong convergence in weighted $L^2(H^1_\gamma)$ spaces can be maintained. This provides the basis to recover optimal control policies for degenerate optimal control problems.
In this talk I will present a semi-Lagrangian discretisation of the Monge-Ampère operator on P1 finite element spaces. The wide stencil of the scheme is designed to ensure uniform stability of numerical solutions.

Monge-Ampère type equations, along with Hamilton-Jacobi-Bellman type equations are two major classes of fully nonlinear second order partial differential equations (PDEs). They arise from many scientific and technological applications such as antenna design, astrophysics, differential geometry, image processing, optimal mass transport, semi-geostrophic fluids, optimal control, finance, electrical and mechanical engineering as well as calculus of variations just name a few. From the PDE point of view, Monge-Ampère type equations are well understood. On the other hand, from the numerical point of view, the situation is far from ideal. Very few numerical methods, which can reliably and efficiently approximate viscosity solutions of Monge-Ampère type PDEs on general convex domains.

There are two main difficulties which contribute to the situation. Firstly, the fully nonlinear structure and nonvariational concept of viscosity solutions of the PDEs prevent a direct formulation of any Galerkin-type numerical methods (such as finite element, discontinuous Galerkin and spectral methods). Secondly, the Monge-Ampère operator is not an elliptic operator in generality, instead, it is only elliptic in the set of convex functions and the uniqueness of viscosity solutions only holds in that space. This convexity constraint, imposed on the admissible space, causes a daunting challenge for constructing convergent numerical methods; it indeed screens out any trivial finite difference and finite element analysis because the set of convex finite element functions is not dense in the set of convex functions.

The goal of our work is to develop a new approach for constructing convergent numerical methods for the Monge-Ampère Dirichlet problem, in particular, by focusing on overcoming the second difficulty caused by the convexity constraint. The crux of the approach is to first establish an equivalent (in the viscosity sense) Bellman formulation of the Monge-Ampère equation and then to design monotone numerical methods for the resulting Bellman equation on general triangular grids. The foundation of the rigorous convergence analysis is the proof of a comparison principle for the Bellman operator. An aim in the design of the numerical schemes was to make Howard’s algorithm available, which is a globally superlinearly converging semi-smooth Newton solver. This allows us to robustly compute numerical approximations on very fine meshes of non-smooth viscosity solutions. The proposed approach also bridges the gap between advances on numerical methods for these two classes of second order fully nonlinear PDEs.

The contents of the presentation is based on joint work with X Feng from the
We consider a non-self-adjoint fourth order eigenvalue problem using a discontinuous Galerkin (DG) method. For high order problems, DG methods are competitive since they use simple basis functions and have less degrees of freedom. We propose an interior penalty discontinuous Galerkin method using C0 Lagrange elements (C0IP) for the transmission eigenvalue problem and prove the optimal convergence. We also consider invisibility cloaking in acoustic wave scattering. The proposed cloaking device takes a three-layer structure with a cloaked region, a lossy layer and a cloaking shell. This is mainly based on studying a novel type of interior transmission eigenvalue problems and their connection to invisibility cloaking.

Fractional diffusion arises in a number of practical applications, e.g., flow in heterogeneous media, thermal diffusion in fractal domains. One mathematical model to describe the physical process is the subdiffusion equation, which involves a Caputo fractional derivative in time. The nonlocality of the fractional derivative leads to limited smoothing property, which poses significant challenge in the design and analysis of robust numerical schemes. In this talk, I shall discuss some recent progresses, e.g., the convolution quadrature and L1 scheme, for discretizing such equations in time. Error estimates and qualitative properties will be discussed.
NUMERICAL SOLUTION OF THE PLANAR DIRICHLET PROBLEM FOR AN ELLIPTIC EQUATION WITH VARIABLE COEFFICIENTS BY AN INTEGRAL EQUATIONS APPROACH

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We present a numerical approximation to the Dirichlet problem for elliptic equations of second-order in divergence form with spacewise dependent coefficients, in planar bounded smooth domains. In the proposed method, the problem is reduced, with the use of the Levi function (parametrix), to a system of domain-boundary integral equations. Making a change of variables involving shrinkage of the boundary curve of the domain, and employing numerical integration, an efficient Nyström scheme is derived for the construction of an approximation of the solution to the obtained system of integral equations. Numerical examples are included showing the feasibility of the proposed approach.
Various realizations of variational multiscale (VMS) methods for simulating turbulent incompressible flows have been proposed in the past fifteen years. All of these realizations obey the basic principles of VMS methods: They are based on the variational formulation of the incompressible Navier-Stokes equations and the scale separation is defined by projections. However, apart from these common basic features, the various VMS methods look quite different. In this review, the derivation of the different VMS methods is presented in some detail and their relation among each other and also to other discretizations is discussed. Another emphasis consists in giving an overview about known results from the numerical analysis of the VMS methods.

This is joint work with Naveed Ahmed (WIAS Berlin), Tomás Chacón Rebollo (Seville), and Samuele Rubino (Seville).

References

On the one hand, block iterative methods may be useful when solving systems with multiple right-hand sides, for example when dealing with time-harmonic Maxwell’s equations. They indeed offer higher arithmetic intensity, and typically decrease the number of iterations of Krylov solvers. On the other hand, recycling also provides a way to decrease the time to solution of successive linear solves, when all right-hand sides are not available at the same time. I will present some results using both approaches, as well as their implementation inside the open-source framework HPDDM (https://github.com/hpddm/hpddm). Combined with efficient preconditioners based on domain decomposition or algebraic multigrid methods, linear systems with tens of millions of unknowns are solved to assess the efficiency of the framework.

In Free Material Optimization, the design variable is the full material tensor of an elastic body. Written in matrix notation one obtains a control-in-the-coefficients problem for the material tensor.

In this talk we discuss recent results in the finite element analysis in Free Material Optimization. We employ the variational discretization approach, where the control, i.e., the material tensor, is only implicitly discretized. Using techniques from the identification of matrix-valued diffusion coefficients, we derive error estimates depending on the coupling of the discretization and Tikhonov regularization parameters. Furthermore, this approach allows to also take into account a noise level on the measured data. Numerical examples supplement our analytical findings.
We investigate the problem of finding optimal topologies of fluid domains. In a given hold all domain $\Omega$ we search for a topology of a fluid domain, filling at most a given proportion of the container, such that an objective is minimized that might depend on the velocity field and the pressure field inside the domain an the topology itself. Here the velocity and pressure owe to the Navier–Stokes system. This especially contains the problem of minimizing the drag of an obstacle in free flow.

Our approach consists of using a phase field description and a porosity approach. Thus we describe the distribution of the material inside the domain $\Omega$ by a phase field variable $\varphi \in H^1(\Omega) \cap L^\infty(\Omega)$ that encodes the obstacle by $\varphi(x) = -1$ and the fluid domain by $\varphi(x) = 1$, while values between -1 and +1 encode a small diffuse region between the fluid domain and the obstacle. By the porosity approach we assume that the obstacle itself is part of the fluid domain, but contains a very dense material with low porosity, that results in an additional Darcy term in the equation. Introducing an interpolation function that interpolated between the dense material and the void we can then extend the fluid equation to the complete domain.

The structure of the final problem is an optimal control problem of a Navier–Stokes equation where the control is given as the phase field and appears as coefficient in the Navier–Stokes equation.

Due to the inherent regularity of the optimization variable, which is $H^1(\Omega) \cap L^\infty(\Omega)$ we can not apply classic descent methods like steepest descent to solve the optimality conditions. Therefore, we apply the variable metric projection type method proposed in [L. Blank and C. Rupprecht, An extension of the projected gradient method to a Banach space setting with application in structural topology optimization, arXiv:1503.03783].

In earlier work also a gradient flow approach was used, see [H. Garcke, C. Hecht, M. Hinze, C. Kahle, Numerical approximation of phase field based shape and topology optimization for fluids, SISC 2015, 37(4), 1846–1871] [H. Garcke, C. Hecht, M Hinze, C. Kahle, K.F. Lam, Shape optimization for surface functionals in Navier–Stokes flow using a phase field approach, IFB 2016, 18(2)]
A CONSERVATIVE DISCRETIZATION OF BIOT’S MODEL FOR SOIL CONSOLIDATION

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We present a finite element discretization of Biot’s linear consolidation model. The method couples the volumetric part of displacement with the fluid in a projection free and thus strongly conservative way. We discuss a priori error estimates and present numerical results.

UNDERPENALIZED DISCONTINUOUS GALERKIN METHODS FOR RADIATION TRANSPORT

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While discontinuous Galerkin (DG) methods had been developed and analyzed in the 1970s and 80s with applications in radiative transfer and neutron transport in mind, it was pointed out later in the nuclear engineering community, that the upwind DG discretization by Reed and Hill may fail to produce physically relevant approximations, if the scattering mean free path length is smaller than the mesh size. Mathematical analysis reveals, that in this case, convergence is only achieved in a continuous subspace of the finite element space. By choosing a weighted upwinding, the conditions on the diffusion limit can be weakened; by choosing the stabilization carefully, the DG method can yield either the LDG method or the method by Ern and Guermond in its diffusion limit.
A class of efficient preconditioners for discretized elliptic problems can be obtained via equivalent operator preconditioning. This means that the preconditioner is chosen as the discretization of a suitable auxiliary operator that is equivalent to the original one, see, e.g., [1,2,3]. Under proper conditions one can thus achieve mesh independent convergence rates. Hence, if the discretized auxiliary problems possess efficient optimal order solvers (e.g. of multigrid type) regarding the number of arithmetic operations, then the overall iteration also yields an optimal order solution, i.e. the cost $O(N)$ is proportional to the degrees of freedom.

The talk is based on the joint work of the authors, see, e.g., [4,5,6]. First some theoretical background is summarized, including both linear and superlinear mesh independent convergence, then various applications are shown. The results can be applied, among other things, for parallel preconditioning of transport type systems, for streamline diffusion preconditioning of convection-diffusion problems, and to achieve superlinear convergence under shifted Laplace preconditioners for Helmholtz equations.

References


TWO LEVEL NON-OVERLAPPING AND OVERLAPPING
SCHWARZ METHODS FOR DISCONTINUOUS GALERKIN
APPROXIMATIONS OF SECOND AND
FOURTH ORDER ELLIPTIC PROBLEMS

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We present some two-level non-overlapping and overlapping additive Schwarz domain
decomposition methods for the solution of the linear systems resulting from symmetric
interior penalty discontinuous Galerkin discretizations of second and fourth order el-
liptic problems. In particular we investigate the influence of the penalty terms as well
as the choice of the coarse mesh spaces on the condition numbers of the preconditioned
linear systems. We identify significant differences between the two methods as far as
such dependences are concerned. The numerical experiments conducted are largely in
agreement with the theoretical results.

A POSTERIORI ERROR ESTIMATES FOR FULLY
DISCRETE FRACTIONAL-STEP \( \vartheta \)-APPROXIMATIONS
FOR PARABOLIC EQUATIONS

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We derive optimal order a posteriori error estimates for fully discrete approximations
of initial and boundary value problems for linear parabolic equations. For the discreti-
sation in time we apply the fractional-step \( \vartheta \)-scheme and for the discretisation in space
the finite element method with finite element spaces that are allowed to change with
time. Optimal order a posteriori error estimates for the norms of \( L^\infty(0,T;L^2(\Omega)) \) and
\( L^2(0,T;H^1(\Omega)) \) are derived by applying the reconstruction technique.
We provide a posteriori error estimates in the $L^\infty(L^2)$--norm for fully discrete approximations for a class of evolution Schrödinger equations, including nonlinear Schrödinger equations up to the critical exponent. For the discretisation in time we use the relaxation Crank-Nicolson scheme, introduced by Besse in [Ch. Besse, A relaxation scheme for the nonlinear Schrödinger equation, SIAM J. Numer. Anal. 42 (2004) 934–952].

For the spatial discretisation we use finite element spaces that are allowed to change from one time-step to another.

For the derivation of estimates we use the reconstruction technique and nonlinear stability arguments as in the continuous problem. More precisely, key ingredients for our analysis include the time-space reconstruction for the relaxation Crank-Nicolson finite element scheme; the conservation laws available for the continuous problem; and appropriate bounds of the $L^\infty(L^2)$--norm of the gradient of the exact solution of the continuous problem.

Various numerical experiments verify and complement our theoretical results. The numerical implementations are performed using uniform partitions in time and space, and verify that the a posteriori estimator converges with the same rate as the exact error.

Based on the a posteriori estimator, we further design and analyse a time-space adaptive algorithm. The adaptive algorithm is shown to perform satisfactorily. More precisely, it drastically reduces the computational cost for Schrödinger equations in the semiclassical regime and nonlinear Schrödinger equations on the critical exponent.

This work will be presented in two linked talks (as Part A and Part B), with this talk (Part A) being concentrated more on the theoretical aspects.
A POSTERIORI ERROR CONTROL & ADAPTIVITY FOR EVOLUTION SCHRÖDINGER EQUATIONS (PART B)

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We provide a posteriori error estimates in the $L^\infty(L^2)$—norm for fully discrete approximations for a class of evolution Schrödinger equations, including nonlinear Schrödinger equations up to the critical exponent. For the discretisation in time we use the relaxation Crank-Nicolson scheme, introduced by Besse in [Ch. Besse, A relaxation scheme for the nonlinear Schrödinger equation, SIAM J. Numer. Anal. 42 (2004) 934–952].

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Based on the a posteriori estimator, we further design and analyse a time-space adaptive algorithm. The adaptive algorithm is shown to perform satisfactorily. More precisely, it drastically reduces the computational cost for Schrödinger equations in the semiclassical regime and nonlinear Schrödinger equations on the critical exponent.

This work will be presented in two linked talks (as Part A and Part B), with this talk (Part B) being concentrated more on numerical aspects.
Simulations of fluid flow through deformable porous media are of increasing importance in subsurface applications. While the flow equations are usually discretized by a finite volume method, it is common practice to apply finite elements to the elasticity equation. This situation has the disadvantage that finite volume and finite element methods inherently use different data structures, and are best adapted to different grid types.

Recently, a finite volume method for elasticity, termed multi-point stress approximations (MPSA) has been proposed, and extended to poro-elastic systems. The schemes have been proven convergent both for for elastic and poro-elastic problems. The proof highlights the role of local coercivity conditions, which are functions of the local geometry, material parameters and discretization scheme. These conditions can be verified for many classes of grids, but for simplex grids the situation is less clear, and existing MPSA-type may fail unless strict conditions are placed on the grid geometry.

Here we offer a resolution to these issues by the introduction of a new MPSA-method. The key tool is to enforce symmetry of the stress tensor weakly, motivated by similar approaches in mixed finite elements for elasticity. This removes the issues with local coercivity, and also significantly reduces the computational cost of discretization. The resulting method is stable also on simplex grids, and we verify its convergence for heterogeneous and nearly incompressible media. Furthermore, we discuss the extension of the weakly symmetric MPSA method to fractured media, and show applications of the method for hydraulic stimulation of geothermal systems.
SOME RECENT PROGRESS WITH THE DPG METHOD

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A growing interest for the DPG method is developing in our community. In this talk we reformulate the method as the approximate solution of a convex optimization problem. We then demonstrate some recent discoveries which stem from the generality of this formulation.

Topics discussed for the linear theory will include the solution of problems with more than one variational formulation in the same domain (e.g. mixed + primal + ultraweak), inequality constraints, and optimal test norms of primal linear elasticity. We will also illustrate the built-in adaptivity and stability of the method with a nonlinear viscoelastic fluid flow benchmark problem.
Filtering plays a crucial role in various scientific and engineering applications where the accuracy and efficiency of the filtering scheme varies based on the application. SIAC filtering is a class of B-spline-based techniques that aim to increase the smoothness of the DG approximation while conserving the inherent order of accuracy of the DG solution (i.e., superconvergence). Accuracy-order conserving properties of SIAC filter have been well-established in the literature. From an approximation theory perspective, accuracy preserving properties of SIAC kernel has a close connection with spline approximation of polynomial spaces that has not been thoroughly investigated before. In this talk, we summarize our theoretical results that establish this connection and provide a unified view of SIAC filtering. This unified view bridges the analysis gap between accuracy-order conserving properties of SIAC filtering and its accuracy-preserving properties. Our results broaden the mathematical analysis tools available for analyzing and designing new filtering schemes using variations of B-splines with desirable approximation properties and kernel supports.
A POSITIVITY AND LINEARITY PRESERVING AFC SCHEME ON GENERAL MESHES

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We consider an algebraic flux correction (AFC) finite element scheme for general steady-state linear boundary values problems. We formulate general assumptions on the limiters such that the scheme satisfies the discrete maximum principle on general simplicial meshes. Based on these assumptions, we propose a new flux limiter that also makes the scheme linearity preserving. Numerical results for convection-diffusion problems demonstrate advantages of the new limiter.
We present recent existence and uniqueness results in Sobolev and Besov spaces for boundary value problems of Dirichlet, Neumann or Robin type for a nonlinear Brinkman system with variable coefficients in Lipschitz domains in $\mathbb{R}^3$. Such a PDE system plays a main role in fluid mechanics and porous media. First, we analyze the corresponding boundary value problem for the linear Brinkman system with variable coefficients, and show that this problem is equivalent to a system of Boundary-Domain Integral Equations (BDIEs). Mapping properties of Brinkman Newtonian and layer potential operators are presented in appropriate Sobolev and Besov spaces. We show the existence and uniqueness of the solution of the BDIE system, and accordingly the well-posedness of the boundary value problem for the variable coefficient linear Brinkman system. Then this well-posedness result and a fixed point theorem provide the existence of a solution in $L^p$-based Sobolev spaces for a boundary value problem corresponding to the variable-coefficient nonlinear Brinkman system. Boundary value problems involving variable coefficient Brinkman and Darcy-Forchheimer-Brinkman systems are also discussed.
Our goal is to obtain residual-type a posteriori error estimates on reasonably general anisotropic meshes for the semilinear reaction-diffusion equation
\[- \varepsilon^2 \Delta u + f(x, u) = 0, \tag{1}\]
as well as the convection-dominated convection-diffusion equation
\[- \varepsilon \Delta u + \bar{a} \cdot \nabla u + bu = f(x). \tag{2}\]

Both (1) and (2) are posed in a polygonal domain \( \Omega \subset \mathbb{R}^n \), \( n = 2, 3 \). Here \( 0 < \varepsilon \leq 1 \).

In (1), we also assume that \( f \) is continuous on \( \Omega \times \mathbb{R} \) and satisfies \( f(\cdot; s) \in L_\infty(\Omega) \) for all \( s \in \mathbb{R} \), and the one-sided Lipschitz condition
\[ f(x, y; v) - f(x, y; w) \geq C_f[v - w] \]
whenever \( v \geq w \), with some constant \( C_f \geq 0 \).

For (1), residual-type a posteriori error estimates in the maximum norm were recently given in [1] in the case of shape-regular triangulations. In [2], the consideration was restricted to \( \Omega \) in \( \mathbb{R}^2 \) and linear finite elements, but the focus shifted to more challenging anisotropic meshes, i.e. we allowed mesh elements to have extremely high aspect ratios. More recently, the analysis of [2] was extended to the error estimation in the energy norm [3]. To give a flavour of the results of [3], assuming that all mesh elements are anisotropic and almost non-obtuse, our first estimator reduces to
\[ \varepsilon \| \nabla (u_h - u) \|_{L_2(\Omega)} + \| u_h - u \|_{L_2(\Omega)} \leq C \left\{ \sum_{z \in \mathcal{N}} \min \{ h_z H_z, \varepsilon H_z^2 h_z^{-1} \} \| \varepsilon J_z \|_{L_\infty(\gamma_z)}^2 + \sum_{z \in \mathcal{N}} \left( \min \{ 1, H_z \varepsilon^{-1} \} \left\| f_h \right\|_{L_2(\omega_z)}^2 + \left\| f_h - f_h \right\|_{L_2(\Omega)}^2 \right) \right\}^{1/2}, \]
where \( C \) is independent of the diameters and the aspect ratios of elements in \( \mathcal{T} \), and of \( \varepsilon \). Here \( f_h := f(\cdot, u_h) \), \( \mathcal{N} \) is the set of nodes in \( \mathcal{T} \), \( J_z \) is the standard jump in the normal derivative of the computed solution \( u_h \) across an element edge, \( \omega_z \) is the patch of elements surrounding any \( z \in \mathcal{N} \), \( \gamma_z \) is the set of edges in the interior of \( \omega_z \), \( H_z = \text{diam}(\omega_z) \), and \( h_z \sim H_z^{-1}|\omega_z| \).

In this talk, we shall review [1, 2, 3], and then present more recent work in which the approach of [3] is extended to finite-element approximations of (2) on anisotropic meshes.

References

Consider a semilinear parabolic equation in the form
\[ \mathcal{M} u := \partial_t u + Lu + f(x, t, u) = 0 \quad \text{for } (x, t) \in Q := \Omega \times (0, T], \]

with a second-order linear elliptic operator \( L = \mathcal{L}(t) \) in a spatial domain \( \Omega \subset \mathbb{R}^n \) with Lipschitz boundary, subject to \( u(x, 0) = \varphi(x) \) for \( x \in \Omega \) and \( u(x, t) = 0 \) for \( (x, t) \in \partial \Omega \times [0, T]. \) We assume that \( f \) satisfies \( 0 \leq \gamma^2 \leq \partial_z f(x, t, z) \leq \bar{\gamma}^2 \) for \( (x, t, z) \in \bar{\Omega} \times [0, T] \times \mathbb{R}. \) We are particularly interested in the case \( \mathcal{L} := -\varepsilon^2 \Delta \) in the regular (\( \varepsilon = 1 \)) and singularly perturbed (\( \varepsilon \ll 1 \)) regimes.

For this equation, we give computable a posteriori error estimates in the maximum norm. Semidiscrete and fully discrete versions of the backward Euler, Crank-Nicolson and discontinuous Galerkin dG(\( r \)) methods are addressed. For their full discretizations, we employ elliptic reconstructions that are, respectively, piecewise-constant, piecewise-linear and piecewise-quadratic for \( r = 1 \) in time. We also use certain bounds for the Green’s function of the parabolic operator.

To give a flavour of our results, in the case of semi-discretizations (in time only) with the discrete solutions \( U^j \in H^1_0(\Omega) \cap C(\Omega) \) associated with \( t = t_j \), one gets
\[
\|U^m - u(\cdot, t_m)\|_{\infty, \Omega} \leq C_1(\kappa_1 \ell_m + \kappa_2) \max_{j=1, \ldots, m-1} \|\chi^j\|_{\infty, \Omega} + C_2 \kappa_0 \|\chi^m\|_{\infty, \Omega}
\]
\[ + \kappa_0 \sum_{j=1}^m \int_{t_{j-1}}^{t_j} e^{-\gamma^2(t_m-s)} \|\theta(\cdot, s)\|_{\infty, \Omega} ds. \]

Here \( \kappa_p, p = 0, 1, 2, \) depend on \( \mathcal{M} \) (they appear in the bounds for the parabolic Green’s function), \( \ell_m = \ell_m(\gamma) := \int_{t_m}^{t_m} s^{-1} e^{-\frac{1}{2}\gamma^2 s} ds \leq \ln(t_m/\tau_m). \) The remaining quantities can be summarized as follows:
For the evaluation of $\chi^{j+1}$ and $\theta$ we use

$$\psi^{j+\alpha} := L(t_{j+\alpha}) U^{j+\alpha} + f(\cdot, t_{j+\alpha}, U^{j+\alpha}), \quad \tilde{\psi} := L(t) \tilde{U} + f(\cdot, t, \tilde{U}),$$

where $\alpha \in (0, 1]$ is any value for which the approximate solution $U^{j+\alpha}$ at time $t_{j+\alpha} := t_j + \alpha \tau_{j+1}$ is available from the definition of the semidiscrete method. Also, $\tilde{U}$ is a piecewise-polynomial interpolant of the computed solution of degree $p - 1$, while $I_{p-1,t}\tilde{\psi}$ is a piecewise-polynomial interpolant of $\tilde{\psi}$ of the same degree using the same interpolation points.


MULTILEVEL APPROACHES IN SPACE AND TIME

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Time parallel algorithms are more and more a promising strategy to extend the scalability of PDEs solvers. In fact the sequential time integration limits the parallelism of a solver to the spatial variables.

In this context, firstly we present a space-time multilevel algorithm for the nonlinear systems arising from the discretization of Navier-Stokes (N-S) equations with finite differences. In particular we study the incompressible, unsteady N-S equations with periodic boundary condition in time.

Time periodic flows, that we find, for example, in biomechanics or engineering, can be conveniently discretized in space-time, where adding parallelism in the time direction is natural.

To achieve fast convergence, we used a multigrid algorithm with parallel box smoothing, the properties of which are studied using local Fourier analysis. We used numerical experiments to analyze the scalability and the convergence of the solver, focusing on the case of a pulsatile flow in three dimensions.

We also present some recent results for an iterative time integrator based on Discontinuous Galerkin (DG) and the Spectral Deferred Correction method (SDC). The DG approach can improve stability, convergence and flexibility of SDC, preserving its structure. This algorithm may find application as a smoother in time-parallel multilevel solvers, as the popular PFASST [Emmett, M. and Minion, M., Toward an Efficient Parallel in Time Method for Partial Differential Equations, Comm. in App. Math. and Comp. Science, 2012, v. 7, pp. 105–132].
THE ROLE OF OSCILLATION IN A POSTERIORI ERROR ANALYSIS

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In a posteriori analysis, it is a common believe that the so-called oscillation is the prize to pay for the ‘computability’ of the estimator, in particular, for estimating local \(H^{-1}\)-norms by scaled \(L^2\)-norms.

It is the merit of Cohen, DeVore, and Nochetto [CoDe:2012] to uncover that there is a catch: They presented an example, where the error is vanishing faster than the estimator. This implies that even asymptotically, the oscillation cannot be bounded by the error. Interestingly, in this example, the local \(H^{-1}\)-norms can be computed exactly and thus computability is not be the reason for the asymptotic overestimation.

In this talk, we shall present a posteriori bounds, where the oscillation appears only because of the computability requirement. In contrast to previous a posteriori analyses, we derive oscillation terms that are dominated by the error irrespective of mesh fineness and regularity of the exact solution. As a consequence, the estimator and the oscillation converge at least as fast as the error.

We present a convergence analysis for an adaptive finite element method for implicit power-law-like models for viscous incompressible fluids. In the considered class of models, the Cauchy stress and the symmetric part of the velocity gradient are related by a, possibly multi-valued, maximal monotone $r$-graph, with $\frac{2d}{d+1} < r < \infty$.

We establish upper and lower bounds on the finite element residual, as well as an estimator for an approximation of the maximal monotone graph. We then present an adaptive strategy and show, under suitable assumptions, the weak convergence of the adaptive algorithm to a weak solution of the boundary-value problem. The argument is based on a variety of weak compactness techniques, including Chacon’s biting lemma and a finite element counterpart of the Acerbi-Fusco Lipschitz truncation of Sobolev functions, introduced by L. Diening, C. Kreuzer and E. Süli [Finite element approximation of steady flows of incompressible fluids with implicit power-law-like rheology. SIAM J. Numer. Anal., 51(2), 984-1015].
ITERATIVE METHODS FOR COUPLED FLOW AND GEOMECHANICS PROBLEMS IN POROUS MEDIA

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Coupling of geomechanics and flow in a poroelastic porous medium has several energy and environmental applications including subsidence events and groundwater remediation. The geomechanical effects account for the influence of deformations in the porous media caused due to the fluid pore pressure whereas the changes in the pore structure due to mechanical stresses affect the flow field. Single phase quasi-static Biot model is typically used to model these coupled flow and deformation processes. The model consists of quasi-static elliptic linear elastic equation coupled to a parabolic flow equation.

We report here some of the developments in suitable iterative schemes for such models and their extensions. Our work has two components: 1. Developing suitable iterative schemes for the extensions of the Biot model to include more physics such as fractures and non-linearities, 2. Developing multirate schemes by exploiting the different time scales of mechanics and flow solve by taking coarser time step for mechanics and smaller time steps for flow. The iterative multirate schemes combine the advantages of both implicit and explicit approaches. They are efficient, allow larger time steps, are robust, and the decoupling allows us to solve the linear systems efficiently. We analyse these iterative and explicit multirate schemes and rigorously analyse the convergence and stability properties of these schemes. The flow equation is discretised using the mixed method whereas the mechanics equation is solved using conformal Galerkin. Our approach can deal with a wide variety of discretizations.
Schwarz waveform relaxation (WR) methods, as well as the related Neumann-Neumann WR method, are domain decomposition methods for solving time-dependent PDEs in parallel. By dividing the computational domain into many subdomains, one can solve the time-dependent PDE in each subdomain separately, and in parallel, over a given time window. The subdomains then exchange interface data, and we iterate until a consistent global solution is obtained. Unlike classical parallelization approaches where the same time step is used for the whole domain and domain decomposition is only applied to the spatial problem, WR methods permits the use of different spatial and time discretizations for different subdomains. Moreover, WR methods have been shown to converge superlinearly to the single domain solution over finite time windows, although convergence deteriorates as the time window size increases.

In this talk, we first show how WR methods can be parallelized naturally in time by running several iterations simultaneously. This allows an additional direction of parallelization, after saturation in the spatial direction. Next, we observe that because of the superlinear convergence of WR methods, the error in fact decreases much faster to zero at the beginning of the time window than at later times. Thus, with the help of a posteriori error estimates, it is possible to detect when the error has dropped below a given tolerance over some part of the time window. This allows us to stop iterating in the parts where the solution has converged and reduce the effective time window size, and hence the overall computational time. Finally, we show numerical examples to illustrate our approach.
Kinetic schemes developed in the late nineties and throughout the noughties to include source terms coming from gravitational pull. These have proved crucial in modeling river flows over long (physical) times.

In flood risk assessment models Saint-Venant’s shallow water equation must be coupled with other equations describing interacting meteorological and hydrogeological phenomena such as rain and groundwater flows. The SWE must therefore be appropriately modified to accommodate source and sink terms, so “classical kinetic schemes” as described above are no longer valid. While modifications of SWE in this direction have been recently proposed, e.g., [1, eq.(1.3–4)], in our approach we depart from the extant literature by proposing a model that is, to the best of our knowledge, novel in that it is both “entropy-consistent” and “naturally extends” the SWE by respecting its kinetic formulation connections. This allows the derivation of fluxes that make finite volume, and possibly discontinuous Galerkin schemes able to tackle long time integration.

We call our extension “natural” as we approached it via matched asymptotic expansions from the Navier–Stokes model with infiltration–recharge boundary conditions similar to the Beavers–Joseph–Saffmann conditions appearing in fluid-solid interaction problems. This puts our model on very solid “first-principle” bases.

Of course, only validation against physical data will decide whether our model is more accurate than previous ones; we are working in this direction with our industrial partners, in view of this, we have identified many numerical examples where our model exhibits features such as shocks and waves (captured thanks to the kinetic formulation) that are quite different from competing models. I will conclude by showing these modes.


HYBRID NUMERICAL-ASYMPTOTIC METHODS
FOR WAVE SCATTERING PROBLEMS

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Linear wave scattering problems (e.g. for acoustic, electromagnetic and elastic waves) are ubiquitous in science and engineering applications. However, conventional numerical methods for such problems (e.g. FEM or BEM with piecewise polynomial basis functions) are prohibitively expensive when the wavelength of the scattered wave is small compared to typical length scales of the scatterer (the so-called “high frequency” regime). This is because the solution possesses rapid oscillations which are expensive to capture using conventional approximation spaces. In this talk we outline recent progress in the development of “hybrid numerical-asymptotic” methods. These methods use approximation spaces containing oscillatory basis functions, carefully chosen to capture the high frequency asymptotic behaviour, leading to a significant reduction in computational cost.

SPACE-TIME ISOGEOMETRIC ANALYSIS
OF PARABOLIC EVOLUTION PROBLEMS

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We present and analyze a new stable space-time Isogeometric Analysis (IgA) method for the numerical solution of parabolic evolution equations in fixed and moving spatial computational domains. The discrete bilinear form is elliptic on the IgA space with respect to a discrete energy norm. This property together with a corresponding boundedness property, consistency and approximation results for the IgA spaces yields an a priori discretization error estimate with respect to the discrete norm. The theoretical results are confirmed by several numerical experiments with low- and high-order IgA spaces including experiments on large-scale distributed memory computers with several thousand cores.
A FUNCTIONAL ANALYTIC APPROACH TO THE ANALYSIS OF HOMOGENIZATION PROBLEMS

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This talk is devoted to the homogenization of boundary value problems in a periodically perforated domain by an approach which is alternative to those of asymptotic analysis and of classical homogenization theory.

In particular, we consider a certain linear or nonlinear boundary value problem on a domain with a periodic structure determined by two positive parameters $\epsilon$ and $\delta$ and we analyze the behaviour of a family of solutions and of the corresponding energy integral as $(\epsilon, \delta)$ tends to $(0, 0)$. 
In this work, we develop variational formulations of Petrov-Galerkin type for one-dimensional fractional boundary value problems involving either a Riemann-Liouville or Caputo derivative of order $\alpha \in (3/2, 2)$ in the leading term and both convection and potential terms. This type of problems arise in mathematical modeling of asymmetric super-diffusion processes in highly heterogeneous media. The well-posedness of the formulations and sharp regularity pickup of the weak solutions are established.

A novel finite element method is developed, which employs continuous piecewise linear finite elements and “shifted” fractional powers for the trial and test space, respectively. The new approach has a number of distinct features as it allows deriving optimal error estimates in both $L^2$- and $H^1$-norms and produces well conditioned linear systems, since the leading term of the stiffness matrix is diagonal matrix for uniform meshes. Further, in the Riemann-Liouville case, an enriched FEM is proposed to improve the convergence. Extensive numerical results are presented to verify the theoretical analysis and robustness of the numerical scheme.
AN EXACT DIVERGENCE-FREE RECONSTRUCTION OPERATOR FOR THE TAYLOR-HOOD ELEMENT

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In this talk we focus on a well-known issue of discretization techniques for the incompressible Navier Stokes equations. The numerical solution is only discrete divergence-free, which may have a major impact on quantitative and qualitative properties of the solution.

In recent years Alexander Linke and cooperators (see [2] and [3]) developed a methodology to reconstruct exactly divergence-free solutions from discrete divergence-free ones, and use this operator within the Navier Stokes solver.

In this work we extend this approach from discontinuous pressure elements to continuous pressure elements including the popular Taylor-Hood element. While for discontinuous pressures the reconstruction operator is given by element-wise local procedures, we have to extend the construction to vertex or element patches. The reconstruction leads to non conforming methods, where the consistency error is estimated in dual norms. Convergence of optimal order is proven (see [1]).

The method is implemented in NGS-Py which is based on the finite element library Netgen/NGSolve. Several examples are presented.

References


The low cost characterisation and detection of conducting, dielectric and magnetic objects is important for a range of applications including security screening, landmine detection, medical imaging, archeological searches, ensuring food safety and non-destructive testing. In these applications, the ability to describe an object in terms of a small number of parameters using polarization/polarizability tensors hold great promise for the low-cost solution of electromagnetic inverse problems based on magnetic induction, ground penetrating radar, electrical impedance tomography and optical tomography modalities.

Asymptotic expansions, which describe the perturbation in electromagnetic fields caused by the presence of an object as its size tends to zero, have been obtained for the full Maxwell system [3], the eddy current model [1, 4] and electrical impedance tomography [2]. These expansions describe the shape and material properties of an object in terms of polarizability tensors, which are independent of an object’s position. We have recently obtained new results that describe the interrelationship between classes of (magnetic) polarizability tensors for different problems and the role the topology of an object has on its coefficients [5]. In the presentation we will summarise these recent developments.

In order to compute the polarizability tensor coefficients (vectorial) transmission problems must be solved. In the presentation we will also describe how the hp finite element can be applied to the solution of the transmission problems and the computation of the tensor coefficients thus allowing for the generation of a library for the characterisation potential objects and inclusions.

References


Biot’s consolidation model describes behaviors of a poroelastic solid saturated by a Newtonian fluid. The model has wide applications from geophysics to computational biomechanics, so there is a strong need of good numerical methods for it.

For the studies with numerical simulations, it is important to develop efficient numerical methods for problems with realistic parameter ranges. In this talk, we discuss finite element discretization and preconditioners for the problem, which are robust for realistic ranges of the elastic moduli and the permeability.
This work presents phase field modeling of fluid-filled fracture propagation in a poroelastic medium. Here lower-dimensional fracture surface is approximated by using the phase field function. The two-field displacement phase-field system solves fully-coupled constrained minimization problem due to the crack irreversibility. This constrained optimization problem is handled by using active set strategy. The pressure is obtained by using a diffraction equation where the phase-field variable serves as an indicator function that distinguishes between the fracture and the reservoir. Then the above system is coupled via a fixed-stress iteration. In addition, we couple with transport system for proppant filled fracture by using a power-law fluid system.

The numerical discretization in space is based on Galerkin finite elements for displacements and phase-field, and an Enriched Galerkin method is applied for the pressure equation in order to obtain local mass conservation. The concentration is solved with cell-centered finite elements. Nonlinear equations are treated with Newton’s method. Predictor-corrector dynamic mesh refinement allows to capture more accurate interface of the fractures with reasonable number for degrees of freedom.
OPTIMAL PRECONDITIONING OF A CUT FINITE ELEMENT METHOD FOR UNFITTED INTERFACE PROBLEMS

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In recent years unfitted finite element methods have drawn an increasing amount of attention. Handling complex geometries without complex and time consuming mesh generation is very appealing. We consider the model interface problem of the type:

$$-\text{div}(\alpha_i \nabla u) = f \text{ in } \Omega_i, \quad i = 1, 2, \quad [\alpha \nabla u]_\Gamma \cdot n_\Gamma = [u]_\Gamma = 0 \text{ on } \Gamma, \quad u = 0 \text{ on } \partial \Omega.$$  

Here, \(\Omega_1 \cup \Omega_2 = \Omega \subset \mathbb{R}^d, \quad d = 2, 3\), is a nonoverlapping partitioning of the domain, \(\Gamma = \bar{\Omega}_1 \cap \bar{\Omega}_2\) is the interface, \([\cdot]_\Gamma\) denotes the usual jump operator across \(\Gamma\) and \(\alpha_i, \quad i = 1, 2\) are positive constants. The methodology of unfitted finite element methods for this type of problem, i.e. methods which are able to cope with interfaces \(\Gamma\) which are not aligned to the grid, is often combined with a weak enforcement of interface conditions using Nitsche’s method. In the original paper \([1]\) the Nitsche-XFEM technique for interface problem has been introduced and analysed. Let \(V_h\) be the space of continuous piecewise linear finite elements with respect to the computational mesh. Then the Nitsche-XFEM method reads: Find \(u_h = V_h|_{\Omega_1} \oplus V_h|_{\Omega_2}\) such that

$$\int_{\Omega_1 \cup \Omega_2} \alpha \nabla u_h \cdot \nabla v_h \, dx - \int_{\Gamma} \llbracket \alpha \nabla u_h \cdot n \rrbracket [v_h] \, ds - \int_{\Gamma} \llbracket \alpha \nabla v_h \cdot n \rrbracket [u_h] \, ds + \frac{\bar{\alpha} \lambda}{h} \int_{\Gamma} [u_h][v_h] \, ds = \int_{\Omega_1 \cup \Omega_2} f v_h \, dx$$

for all \(v_h \in V_h^F\). Here we used the average \(\llbracket w \rrbracket := \kappa_1 w_1 + \kappa_2 w_2\) with an element-wise constant \(\kappa_i = \frac{|T \cap \Omega_i|}{|T|}\) as in \([1]\). In general, the resulting linear systems have very large condition numbers, which depend not only on the mesh size \(h\), but also on how the interface intersects the mesh.

Simple diagonal preconditioning circumvents these problems and achieves condition number bounds of the form \(c h^{-2}\) with a constant \(c\) that is independent of the location of the interface. The main ingredient in proving this result is the stable subspace splitting between standard degrees of freedom (corresponding to \(V_h\)) and extended degrees of freedom which is proven in \([2]\). Utilizing this property we are able to propose a preconditioner which is optimal in the sense that preconditioning actions have only \(O(N)\) costs (where \(N\) is the number of degrees of freedom) and the resulting condition number is independent of the mesh size \(h\) and the interface position.

We present the optimal preconditioner, numerical results and outline the main aspects of the analysis.
References


In this talk we consider the discretization of the unsteady incompressible Navier-Stokes equations in a velocity-pressure formulation:

\[
\begin{align*}
\frac{\partial u}{\partial t} + \text{div}(-\nu \nabla u + u \otimes u + pI) &= f \quad \text{in } \Omega \\
\text{div } u &= 0 \quad \text{in } \Omega
\end{align*}
\]  

(1)

with boundary conditions \( u = u_D \) on \( \Gamma_D \subset \partial \Omega \) and \( (\nu \nabla u - pI) \cdot n = 0 \) on \( \Gamma_{\text{out}} = \partial \Omega \setminus \Gamma_D \). Here, \( \nu = \text{const} \) is the kinematic viscosity, \( u \) the velocity, \( p \) the pressure, and \( f \) is an external body force. We present an efficient and high order accurate discretization method based on the following main ingredients:

First, we make a distinction between stiff linear parts and less stiff non-linear parts with respect to their temporal and spatial treatment. We exploit this using operator-splitting time integration schemes which rely only on efficient solution strategies for two simpler sub-problems: a corresponding hyperbolic transport problem and an unsteady Stokes problem.

Secondly, for the hyperbolic transport problem a spatial discretization with an Upwind Discontinuous Galerkin (DG) method and an explicit treatment in the time integration scheme is rather natural and allows for an efficient implementation.

Thirdly, the discretization of the Stokes problems is tailored with respect to two important challenges: efficient solution of arising linear systems and exactly divergence-free solution. In order to fulfill the incompressibility constraint exactly we use an \( H(\text{div}) \)-conforming discretization of the velocity combined with discontinuous pressures. To enforce continuity of the velocity (weakly) also in tangential direction we apply a Hybrid DG formulation. For the task of solving linear systems, a discretization with Hybrid DG methods is better suited than standard DG methods. To improve the efficiency even further we apply a projection operator in the Hybrid DG formulation which allows to reduce the unknowns on element interfaces and thereby the number of globally coupled unknowns.

We present the method, discuss aspects of the temporal and spatial discretization, implementational aspects and numerical results.

References

In this talk we discuss the following optimal control problem
\[
\min_{q,u} J(q, u) := \frac{1}{2} \int_0^T \|u(t) - \hat{u}(t)\|^2_{L^2(\Omega)} dt + \frac{\alpha}{2} \int_0^T |q(t)|^2 dt
\]
subject to the second order parabolic equation
\[
\begin{align*}
\partial_t u(t, x) - \Delta u(t, x) &= q(t) \delta_{\gamma(t)}, \quad (t, x) \in I \times \Omega, \\
u(t, x) &= 0, \quad (t, x) \in I \times \partial \Omega, \\
u(0, x) &= 0, \quad x \in \Omega
\end{align*}
\]
Here \( I = [0, T], \Omega \subset R^2 \) is a convex polygonal domain, and \( \delta_{\gamma(t)} \) is the Dirac delta function along the curve \( \gamma(t) \subset \Omega \). We assume that \( \gamma(t) \) satisfies the following:
\begin{itemize}
\item \( \gamma(t) \in C^1(0, T) \) and \( \max_t |\gamma'(t)| \leq C_\gamma \).
\item \( \gamma(t) \subset \Omega_0 \subset \subset \Omega \), for any \( t \in I \).
\end{itemize}
The parameter \( \alpha \) is assumed to be positive and the desired state \( \hat{u} \) fulfills \( \hat{u} \in L^2(I; L^\infty(\Omega)) \).

We discretize the problem with continuous Lagrange elements in space and discontinuous piecewise constant functions in time. Despite low regularity of the state equation we establish optimal (first order in time and the second order in space, modulo logarithmic terms) convergence rates for the fully discrete control variable. We will also discuss a new type of error estimates along the curve which are essential for our analysis.
ON POSITIVITY OF THE DISCRETE GREEN’S FUNCTION AND DISCRETE HARNACK INEQUALITY FOR PIECEWISE LINEAR ELEMENTS

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In this talk we discuss some recent results obtained for the finite element discrete Green’s function and its positivity. The first result shows that on smooth two-dimensional domains the discrete Green’s function with singularity in the interior of the domain must be strictly positive throughout the computational domain once the mesh is sufficiently refined. As an application of this result, we establish a discrete Harnack inequality for piecewise linear discrete harmonic functions. In contrast to the discrete maximum principle, the result is valid for general quasi-uniform shape regular meshes except for a condition on the layer near the boundary.

QUASI-OPTIMAL VARIATIONAL MULTISCALE STABILIZATION OF CONVECTION-DIFFUSION EQUATIONS

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We formulate a new stabilized quasi-optimal Petrov-Galerkin method for singularly perturbed convection-diffusion problems based on the Variational Multiscale method. A localized algorithm is derived and the energy error is estimated to converge to zero exponentially.
A TREFFTZ POLYNOMIAL SPACE-TIME DISCONTINUOUS GALERKIN METHOD FOR THE SECOND ORDER WAVE EQUATION

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A new space-time discontinuous Galerkin (dG) method utilising special Trefftz polynomial basis functions is proposed and fully analysed for the scalar wave equation in second order formulation. The dG method considered is motivated by the class of interior penalty dG methods, as well as by the classical work of Hulbert and Hughes \cite{4}. The choice of the penalty terms included in the bilinear form is essential for both the theoretical analysis and for the practical behaviour of the method for the case of lowest order basis functions. A best approximation result is proven for this new space-time dG method with Trefftz-type basis functions. Rates of convergence are proved in any dimension and verified numerically in spatial dimensions $d = 1$ and $d = 2$. Numerical experiments highlight the effectiveness of the Trefftz method in problems with energy at high frequencies.

References


TOWARDS PRESSURE-ROBUST MIXED METHODS FOR
THE INCOMPRESSIBLE NAVIER–STOKES EQUATIONS

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For more than thirty years it was thought that the construction of pressure-robust
mixed methods for the incompressible Navier–Stokes equations, whose velocity error
is pressure-independent, was practically impossible. However, a novel, quite universal
construction approach shows that it is indeed rather easy to construct pressure-robust
mixed methods. The approach repairs a certain $L^2$-orthogonality between gradient
fields and discretely divergence-free test functions, and works for families of arbitrary-order
mixed finite element methods, arbitrary-order discontinuous Galerkin methods,
and finite volume methods. Novel benchmarks for the incompressible Navier–Stokes
equations show that the approach promises significant speedups in computational prac-
tice compared to pure Galerkin discretizations or grad-div stabilization, whenever the
continuous pressure is complicated.

THE MIMETIC FINITE DIFFERENCE METHOD
FOR THE LANDAU-LIFSHITZ EQUATION

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The Landau-Lifshitz equation describes the dynamics of the magnetization inside ferro-
magnetic materials. This equation is highly nonlinear and has a non-convex constraint
(the magnitude of the magnetization is constant) which pose interesting challenges in
developing numerical methods. We develop and analyze explicit and implicit mimetic
finite difference schemes for this equation. These schemes work on general polytopal
meshes which provide enormous flexibility to model magnetic devices with various
shapes. A projection on the unit sphere is used to preserve the magnitude of the mag-
etization. The developed schemes are tested on general meshes that includes distorted
and randomized meshes. The numerical experiments include a test proposed by the
National Institute of Standard and Technology and a test showing formation of domain
wall structures in a thin film.
A framework of high-precision verified eigenvalue bounds for self-adjoint differential operators

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A universal framework is proposed to give high-precision explicit lower and upper bounds for the eigenvalues of self-adjoint differential operators [1]. In the case of the Laplacian operator, by applying Crouzeix–Raviart finite elements, an efficient algorithm is developed to bound the eigenvalues for the Laplacian defined in 1D, 2D and 3D spaces. For biharmonic operators, Fujino–Morley FEM is adopted to bound the eigenvalues. To obtain high-precision eigenvalue bounds, Lehmann–Goerisch’s theorem along with high-order finite element methods is adopted [3, 2]. See Table 1 for a sample computation result for eigenvalue of Laplacian with homogeneous boundary condition over square-minus-square domain, where there exist singularities of eigenfunction around the reentrant corners.

By further adopting the interval arithmetic, the explicit eigenvalue bounds from numerical computations can be mathematically correct. As a computer-assisted proof, the verified eigenvalue bounds have been used to investigate the solution existence of semi-linear elliptic differential equations; see, e.g., [4].

Bounds for the eigenvalues of Laplacian over square-minus-square domain [2] (with homogeneous Dirichlet boundary condition)

<table>
<thead>
<tr>
<th>( \lambda_i )</th>
<th>Eigenvalue bound</th>
<th>Relative Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>9.16021, 54 ( \times ) 10^{-37}</td>
<td>2.8E-7</td>
</tr>
<tr>
<td>2</td>
<td>9.17008, 89 ( \times ) 10^{-61}</td>
<td>2.9E-7</td>
</tr>
<tr>
<td>3</td>
<td>9.17008, 89 ( \times ) 10^{-61}</td>
<td>2.9E-7</td>
</tr>
<tr>
<td>4</td>
<td>9.18056, 90 ( \times ) 10^{-62}</td>
<td>3.0E-7</td>
</tr>
<tr>
<td>5</td>
<td>10.089843, 37 ( \times ) 10^{-14}</td>
<td>2.2E-8</td>
</tr>
</tbody>
</table>

References


DIRECT VISUALIZATION OF IGA SIMULATION MODELS ON MODERN GPUs

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Using isogeometric analysis, design problems can be formulated and investigated through simulation and optimization in a unified workflow without relying on intermediate discretization steps. Visualization, however, a key component in understanding results, still forces tesselation if typical software tools are used. In this talk, we will discuss a visualization software prototype that leverages the massively-parallel computational power of modern GPUs to achieve accurate and efficient visualization of IGA models without requiring pre-tesselation; rather, tesselation is performed on the fly, on-GPU at just the required level of detail for pixel-accurate representation. This approach eschews the data amplification inherent in tesselation and thus provides (near) realtime visualization capabilities even for complex models. Beyond the general architecture of our system, we illustrate how typical visualization algorithms can be adapted to our system. To conclude, we present examples from several practical applications.
Elliptic partial differential equations (PDE) describe the long-term evolution of a diffusion or heat problem. The medium through which the fluid is flowing is said to be heterogeneous if it consists of a mixture of several substances through which the fluid flows at varying rates. For example, a heterogeneous soil may consist of layers of sand (through which water flows quickly) and rock (through which water diffuses very slowly).

Domain decomposition is a method for solving elliptic PDEs in parallel in an efficient manner. The basic idea is to partition the overall domain $\Omega$ into many subdomains $\Omega = \bigcup_k \Omega_k$ and to solve the PDE iteratively on each subdomain in parallel. In optimized Schwarz methods (OSM) and 2-Lagrange multiplier methods (2LM), the boundary conditions on $\partial \Omega_k$ are of the Robin type. For a suitable choice of Robin parameter, one obtains a method that converges faster than a classical Schwarz iteration.

In order to obtain good parallel scaling, the Schwarz method must be combined with a “coarse grid correction”, which serves to accelerate the convergence of the low frequencies. However, when the PDE is heterogeneous, some “fast-moving modes” are indeed “low frequency” (e.g. it does not take very much energy for water to flow through sand). This means that the coarse space must contain some nontrivial fast-moving but low frequency modes.

One way to capture such modes in the coarse space is to find a few low-energy eigenvectors for the Dirichlet-to-Neumann map of each subdomain $\Omega_k$. We show how one can use such a coarse space for OSM and 2LM and thus obtain arbitrarily fast convergence for heterogeneous problems.
We propose a stabilized hybrid mixed method to solve the Stokes problem with velocity and pressure stabilization terms on the edges of the elements. The method is close related to a hybrid DG method proposed by Egger and Waluga but uses Lagrange multipliers associated with the traces of both velocity and pressure fields. As a consequence of this choice, the local problems are stable and all velocity and pressure degrees of freedom can be eliminated at the element level by static condensation. Continuous or discontinuous approximations for the multipliers are adopted. With continuous multiplier approximation, the proposed SHM method presents some similarities with classical Galerkin mixed methods that use $C^0$ continuous interpolation for both velocity and pressure fields. When discontinuous approximations are adopted for the multipliers, eliminating the Lagrange multiplier we recover a slightly modified version of a symmetric DG method.
STABILIZED CUTFEM FOR THE DISCRETIZATION OF TWO-PHASE INCOMPRESSIBLE FLOWS IN 3D

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For the mathematical modeling of two-phase flow phenomena we use the incompressible (Navier-)Stokes equations in each phase. The coupling of the phases is achieved by a surface tension force at the interface (coupling of the stresses normal to the interface) and a no-slip condition on the velocity tangential to the interface. For the evolution of the interface a level set technique is used.

For the finite element discretization of one-phase flow problems the Taylor-Hood $P_2-P_1$ pair is a popular choice due to the quadratic convergence and LBB-stability. For two-phase flow problems however, the $P_2-P_1$ discretization with unfitted meshes leads to a rather poor approximation quality of $O(\sqrt{h})$ as $P_1$ elements are not able to represent discontinuities in the solution. Enriching the $P_1$ space with Heaviside jump functions one can recover the optimal approximation property, but numerical experiments indicate that the $P_2-P_1 X$ velocity-pressure pair is not LBB stable.

In [3] the enriched pressure space has been reduced by omitting the extended basis functions with small supports, which cause the instability. Introducing the so-called ghost penalty stabilization [1] for the pressure space results in a discrete inf-sup stability for a modified bilinear form. As opposed to the reduced XFEM space, the ghost penalty method does not need to reduce the approximation space and thus may lead to smaller errors in the solution. The added stability terms lead to a modified Schur complement and therefore the preconditioners have to be adapted in order to solve the system matrix efficiently. New preconditioning strategies developed in [2] are presented here. For a constructed Stokes model problem with an analytical solution both stabilization methods are compared with respect to the discretization errors and convergence rates. For a realistic, fully coupled Navier-Stokes rising droplet problem the stabilization methods are compared with respect to the resulting droplet position and velocity.

References


SOLVING COMPRESSIBLE FLOW PROBLEMS
BY ISOGEOMETRIC ANALYSIS

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Isogeometric Analysis (IgA), introduced in [3], aims at bridging the gap between Finite
Element Analysis (FEA) and Computer-Aided Design (CAD) by extending classical
FEA towards ansatz functions such as B-splines or NURBS (non-uniform rational B-
splines), which enable the more precise or even exact representation of complex ge-
ometry objects. Since its introduction IgA gained popularity in many computational
mechanics and fluid dynamics applications but its use in compressible flow calculations
is very limited.

In this paper, we present our implementation of a positivity-preserving isogeomet-
ric high-resolution scheme for compressible flow problems in the open-source library
G+Smo [4]. It builds upon the generalization of the algebraic flux correction paradigm
[5] to multi-patch IgA as universal building block for the design of positivity-preserving
high-order discretizations. In particular, we analyze techniques for the positivity-
preserving multi-patch coupling and boundary treatment.

Our implementation adopts Fletcher’s group formulation [1] together with an ef-
cient edge-based formation of system matrices and vectors [2] from pre-computed
coefficients to overcome the high computational costs that are typically observed in
quadrature-based IgA-assembly algorithms. Finally, we extend our solution algorithm
to a space-time formulation that makes it possible to combine high-order approxima-
tions in space and time. The suggested approach is applied to several test problems
for compressible flows.

References


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ements, nurbs, exact geometry and mesh refinement. Computer Methods in Applied
This presentation considers a specific aspect of optimal control for partial differential equations, where the control is given by the shape of the domain of interest. The key point in shape optimisation is the definition of the shape derivative, which is needed for the standard optimisation procedure. Several approaches exist whereas we follow the ideas of Sokolowski and Zolesio who provide a method to derive an analytical gradient. The shape gradient can be deduced from the state equation by applying shape calculus and solving an auxiliary adjoint equation. This approach leads to two analytically equivalent representations of the shape gradient, i.e. the boundary and the domain form. However this equivalence property does not transfer to the discrete case. The pros and cons will be discussed and how each representation influences the optimisation procedure.

The discussion is part of a research project for turbulence reduction in water pipes by modifying their shape. Hence it is embedded in the framework of incompressible flow equations, i.e. the Navier-Stokes equations and their simplifications. These equations were solved within a finite element approach, which is implemented by the finite element software package FEniCS.
In this talk we study the transient scattering of acoustic waves by an obstacle in an infinite two dimensional domain, where the scattered wave is represented in terms of time domain boundary layer potentials. The problem of finding the unknown solution of the scattering problem is thus reduced to finding the unknown density of the time domain boundary layer operators on the obstacle’s boundary, subject to the boundary data of the known incident wave. Using a Galerkin approach, the unknown density is approximated by a piecewise polynomial function, the coefficients of which can be found by solving a linear system. The entries of the system matrix of this linear system involve, for the case of the two dimensional scattering problem under consideration, integrals over four dimensional space-time manifolds. An accurate computation of these integrals is crucial for the stability of this method.

Using piecewise polynomials of arbitrary order, the two temporal integrals can be evaluated analytically, leading to kernel functions for the spatial integrals with complicated domains of piecewise support.

These spatial kernel functions can be generalised into a class of admissible kernel functions which, as we prove, belong to countably normed spaces \[1\]. Therefore, a quadrature scheme for the approximation of the two dimensional spatial integrals with admissible kernel functions converges exponentially \[3\]. Similar results for the three dimensional case can be found in \[2, 4\].

This talk concentrates on an efficient scheme to evaluate the integrals with high order polynomials and stability results for the Galerkin scheme. We also show numerical experiments underlining the theoretical results, cf. \[1\].

References


Results on a posteriori error control of transport, convection-diffusion and nonlinear hyperbolic problems are discussed. We consider classical discretisation methods combined with self adapted meshes. The methods are compared to certain recently proposed “idealised” discretisations.
Generalized splines are smooth piecewise functions with sections in spaces more general than classical algebraic polynomials. Interesting examples are spaces comprising trigonometric or hyperbolic functions. Under suitable assumptions, generalized splines enjoy all the desirable properties of polynomial splines, including a representation in terms of basis functions (the so-called GB-splines) that are a natural extension of the polynomial B-splines.

Tensor-product GB-splines are an interesting problem-dependent alternative to tensor-product polynomial B-splines and NURBS in isogeometric analysis (IgA). Like any discretization method, the IgA paradigm requires to solve large linear systems. A deep understanding of the spectral properties of the related matrices is crucial for the design of fast solvers for these linear systems.

In this talk we focus on IgA discretizations based on trigonometric or hyperbolic GB-splines. In particular, we prove that the corresponding stiffness matrices possess an asymptotic eigenvalue distribution which can be compactly described by a function, the so-called symbol, see [2]. These results extend those obtained for IgA discretization methods based on polynomial B-splines, see [1], and strengthen the structural similarity between the polynomial and the generalized setting.

References


Isogeometric analysis (IGA), introduced by Hughes and collaborators in 2005, requires the seamless integration of Finite Element Analysis (FEA) and Computer-aided Design (CAD) software. Towards the realization of this revolutionary goal, we have initiated in the year 2012 the development of the open-source, collaborative software project “Geometry + Simulation Modules” (for short, gismo, see http://gs.jku.at/gismo), which aims at providing a unified development framework for IGA. In this talk we present some highlights of the on-going development of the library. G+Smo is an object-oriented, template C++ library, that implements a generic concept for IGA, based on abstract classes for geometry map, discretization basis, assemblers, solvers and so on. It makes use of object polymorphism and inheritance techniques in order to support a variety of different discretization bases, namely B-spline, Bernstein, NURBS bases, hierarchical and truncated hierarchical B-spline bases of arbitrary polynomial order, and so on. Our design allows the treatment of geometric entities such as surfaces or volumes through dimension independent code, realized by means of template meta-programming. Available features include simulations based on continuous and discontinuous Galerkin approximation of PDEs, over conforming and non-conforming multi-patch computational domains. The codebase, currently in beta stage, focuses on both the efficiency and ease of use, promotes code quality and cross-platform compatibility and encourages the exploration of new strategies potentially better suited for isogeometric analysis, before adopting known FEA practices.
ON THE USE OF NUMERICAL TENSOR METHODS IN ISOGEOMETRIC ANALYSIS

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The use of tensor methods in the field of numerical simulation was explored the last decade, with the aim to overcome the curse of dimensionality, i.e. the exponential complexity with respect to the spatial dimension of the computational domain. With the advent of Isogeometric Analysis (IGA) during the same period of time, the very same difficulty of dimensionality has appeared, in particular in the task of matrix assembly. Indeed, this task is more challenging than in the case of traditional finite element methods. This is due to factors such as the increased degree and the larger supports of the ansatz functions (tensor-product B-splines), that burden the sparsity pattern and bandwidth of the system matrix.

In an attempt to address this problem, we developed an interpolation-based approach that approximately transforms the integrands into piecewise polynomials and uses look-up tables to evaluate their integrals [1]. Shortly after, this led us to employ tensor methods to accelerate the assembly process further [2], focusing on the two-dimensional (bivariate) case.

In particular, we obtained a compact representation of the matrices that occur in IGA as sums of a small number of Kronecker products of auxiliary matrices, which are defined by univariate integrals. This representation, which is based on a low-rank tensor approximation of certain parts of the integrands, made it possible to achieve a significant speedup of the assembly process without compromising the overall accuracy of the simulation. The talk will describe our recent progress towards the extension of these methods to the multivariate case (i.e., to any dimension).

This is joint work with Bert Jüttler, Ulrich Langer and Boris Khoromskij.

References


AN ARBITRARY ORDER ACCURATE MIMETIC METHOD FOR ELLIPTIC PROBLEMS IN MIXED FORM

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We present an arbitrary-order accurate Mimetic Finite Difference (MFD) method for the approximation of diffusion problems in mixed form on unstructured polygonal and polyhedral meshes. As usual in the mimetic numerical technology, the method satisfies local consistency and stability conditions, which determines the accuracy and the well-posedness of the resulting approximation. The method also requires the definition of a high-order discrete divergence operator that is the discrete analog of the divergence operator and is acting on the degrees of freedom. The new family of mimetic methods is proved theoretically to be convergent and optimal error estimates for flux and scalar variable are derived from the convergence analysis. A numerical experiment confirms the high-order accuracy of the method in solving diffusion problems with variable diffusion tensor. It is worth mentioning that the approximation of the scalar variable presents a superconvergence effect.

SERENDIPITY NODAL VEM SPACES

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We introduce a new variant of Nodal Virtual Element spaces that mimics the “Serendipity Finite Element Methods” (whose most popular example is the 8-node quadrilateral) and allows to reduce (often in a significant way) the number of internal degrees of freedom. When applied to the faces of a three-dimensional decomposition, this allows a reduction in the number of face degrees of freedom: an improvement that cannot be achieved by a simple static condensation. On triangular and tetrahedral decompositions the new elements (contrary to the original VEMs) reduce exactly to the classical Lagrange FEM. On quadrilaterals and hexahedra the new elements are quite similar (and have the same amount of degrees of freedom) to the Serendipity Finite Elements, but are much more robust with respect to element distortions. On more general polytopes the Serendipity VEMs are the natural (and simple) generalization of the simplicial case.
THE HYBRID TOTAL FETI METHOD IN ESPRESO LIBRARY

Lubomír Říha\textsuperscript{a}, Tomáš Brzobohatý\textsuperscript{b}, Alexandros Markopoulos\textsuperscript{c}, Ondřej Meca\textsuperscript{d}, and Tomáš Kozubek\textsuperscript{e}.

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We present our hybrid variant of the Total FETI method, firstly proposed by A. Klawonn and O. Rheinbach \cite{2}. The original algorithm combines the FETI \cite{3} and FETI-DP \cite{4} method to treat the coarse problem in a more optimal way. Briefly said, the hybrid FETI method connects several neighbouring subdomains into clusters (using the FETI-DP approach), so each cluster behaves like one subdomain, and therefore the global coarse problem depends on the number of clusters and not on the number of subdomains. We present a slightly different variant of the algorithm \cite{5}, in which the FETI method is used on both levels. It allows the method to bond two or more subdomains into clusters differently, e.g., per the whole common face between each two neighbouring subdomains on average.

The numerical results presented in the talk were obtained via in-house developed ESPRESO library \cite{1}. This library is a highly efficient parallel solver containing several FETI method based algorithms including the HTFETI method able to solve problems over billions of unknowns. The solver is based on a highly efficient communication layer based on MPI, and it is able to run on massively parallel machines with thousands of compute nodes and hundreds of thousands of CPU cores. ESPRESO is also being developed to support modern many-core accelerators.

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\begin{enumerate}
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\end{enumerate}
THE HP VERSION OF VIRTUAL ELEMENT METHODS FOR THE POISSON PROBLEM: APPROXIMATION OF CORNER SINGULARITIES

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The Virtual Element Method (VEM) is a recent generalization of the Finite Element Method (FEM), see [1]. The main features of VEM are the employment of polygonal/polyhedral meshes (thus including non conforming meshes) and the possibility of building in an easy fashion global spaces of arbitrary regularity, see [4, 5].

In [3], the hp version of VEM was introduced; the basic idea of hp methods is that the convergence of the errors is achieved by means of mesh refinement and by increasing the dimension of local spaces.

In the present talk, based on [2], after recalling the notation and the results of [3], we discuss about the approximation properties of VEM in presence of corner singularities. As in FEM, it is possible to prove exponential convergence of the errors in terms of the number of degrees of freedom, by geometrically refining the mesh towards the singularity and increasing the local space dimension properly. VEM seems to be appropriate for solving this kind of approximation, since it allows for an automatic geometric refinement towards the corners of the domain, without the need of reshaping the mesh, as in FEM, in presence of hanging nodes. We also present a new stabilization of the method, on which explicit bounds on p are proven. Finally, numerical tests are shown.

References


LOCAL PROJECTION TYPE STABILISATION APPLIED TO INF-SUP STABLE DISCRETISATIONS OF THE OSEEN PROBLEM

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We consider inf-sup stable finite element discretisations of the Oseen equations. Hence, no pressure stabilisation is needed. However, the standard Galerkin method still suffers in general from global spurious oscillations in the velocity field which are caused by the dominating convection.

Local projection stabilisation methods will be used to overcome this difficulty. Based on a projection from the underlying approximation space onto a discontinuous projection space, the stabilisation is derived from additional weighted $L^2$-control on the fluctuation of the whole velocity gradient or only parts of it like the divergence and the derivative in streamline direction. This leads to different stabilisation terms.

A unified framework for analysing both the one-level and the two-level local projection stabilisation method applied to inf-sup stable discretisations of the Oseen equations will be presented.

We will discuss inf-sup stable pairs of finite element spaces which approximate both velocity and pressure by elements of order $r$. In contrast to the classical equal order interpolation, the velocity components and the pressure are discretised by different finite elements. The discrete inf-sup condition for these pairs of finite element spaces is fulfilled and a uniform error estimate of order $r + 1/2$ holds true for small viscosities. In the case of discontinuous pressure approximations, an additional term controlling the jumps of the pressure across inner cell faces becomes necessary.

Numerical tests which confirm the theoretical results will be given.
We discuss different time discretisations of variational type applied to time-dependent Oseen problems. As spatial discretisation, both inf-sup stable and equal-order pairs of finite element spaces for approximating velocity and pressure are considered.

Since Oseen problems are generally convection-dominated, a spatial stabilisation is applied. We will concentrate on local projection stabilisation methods which allow to stabilise the streamline derivative, the divergence constraint and, if needed, the pressure gradient separately.

To discretize in time, continuous Galerkin-Petrov methods (cGP) and discontinuous Galerkin methods (dG) as higher order variational time discretisation schemes are applied. These methods are known to be A-stable (cGP) or even strongly A-stable (dG). An adaption of the time postprocessing proposed by Matthies and Schieweck leads to numerical solutions which show for both velocity and pressure at the discrete time points a convergence rate of $2k + 1$ for dG($k$) and $2k$ for cGP($k$), respectively.

We consider the spatial discretisation of a time-fractional diffusion equation in a polygonal domain $\Omega$ using continuous, piecewise-linear finite elements. If $\Omega$ is convex, then the method is known to be second-order accurate in $L^2(\Omega)$, uniformly in time, but if the domain has a re-entrant corner then the error analysis breaks down because the associated Poisson problem is no longer $H^2$-regular. For a quasi-uniform family of triangulations with mesh parameter $h$, the error is of order $h^{2\beta}$ if largest re-entrant corner has angle $\pi/\beta$ with $1/2 < \beta < 1$, but a suitable local refinement strategy restores $h^2$ convergence.

Analogous results for the classical heat equation were proved in 2006 by Chatzi-panetelidis, Lazarov, Thomée and Wahlbin.
DISCONTINUOUS GALERKIN METHODS FOR NONLINEAR
SCALAR HYPERBOLIC CONSERVATION LAWS:
DIVIDED DIFFERENCE ESTIMATES
AND ACCURACY ENHANCEMENT

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In this talk, we present an analysis of the accuracy-enhancement for the discontinuous Galerkin (DG) method applied to one-dimensional scalar nonlinear hyperbolic conservation laws. This requires analyzing the divided difference of the errors for the DG solution. We therefore first prove that the $\alpha$-th order ($1 \leq \alpha \leq k + 1$) divided difference of the DG error in the $L^2$ norm is of order $k + \frac{3}{2} - \frac{\alpha}{2}$ when upwind fluxes are used, under the condition that $|f'(u)|$ possesses a uniform positive lower bound. By the duality argument, we then derive superconvergence results of order $2k + \frac{3}{2} - \frac{\alpha}{2}$ in the negative-order norm, demonstrating that it is possible to extend the Smoothness-Increasing Accuracy-Conserving filter to nonlinear conservation laws to obtain at least $(\frac{3}{2}k + 1)$th order superconvergence for post-processed solutions. As a by-product, for variable coefficient hyperbolic equations, we provide an explicit proof for optimal convergence results of order $k + 1$ in the $L^2$ norm for the divided differences of DG errors and thus $(2k + 1)$th order superconvergence in negative-order norm holds. Numerical experiments are given that confirm the theoretical results.

PARALLEL TIME-DOMAIN BOUNDARY ELEMENT METHOD
FOR 3-DIMENSIONAL WAVE EQUATION

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We present a boundary element method for 3-dimensional sound-hard scattering. It relies on an indirect formulation for the retarded double-layer potential introduced by Bamberger and Ha Duong in 1986 and on smooth time ansatz functions recently proposed by Sauter and Veit. The latter allows for an efficient use of Gauss quadrature within the assembly of the resulting boundary element system matrix. The assembling process is implemented in parallel and we numerically document its scalability. Further, a heuristical preconditioner, which accelerates flexible GMRES iterations, is presented. The efficiency of our approach is documented for a problem on a sphere with known analytical solution as well as for a scattering from a real-world geometry.
**hp-ADAPTIVE GALERKIN TIME STEPPING METHODS FOR NONLINEAR IVPs**

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In this talk, we will discuss the derivation of conditional a posteriori error estimates for continuous Galerkin approximations to nonlinear IVPs with an emphasis on those with solutions which exhibit finite-time existence. We then discuss the design of adaptive algorithms based on this error estimator with the goal of approximating the blow-up time. Numerical experiments complement the theoretical results.

**THE NITSCHE TRICK FOR THE OBSTACLE PROBLEM – A COUNTEREXAMPLE AND CONSEQUENCES FOR OPTIMAL CONTROL**

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We consider the Finite Element (FE) discretization of the obstacle problem using piecewise linear and continuous finite elements. While a priori error estimates in the energy space are standard and well known, the classical Nitsche trick for improved error estimates in \(L^2(\Omega)\) seems to fail due to a lack of regularity in the dual problem. This is demonstrated by two one-dimensional counterexamples, which provide a (rigorously computable) order of convergence of \(2 - 1/p\), if the obstacle is described by a function in \(W^{2,p}(\Omega)\). The \(L^2\)-a priori estimate directly affects the convergence analysis for an optimal control problem governed by the obstacle problem.
We consider an optimal control problem governed by a variational inequality of obstacle type. Problems of this type are known to be challenging due to the non-differentiable control-to-state mapping, which permits the use of standard techniques for the derivation of optimality conditions. Nevertheless it is possible to rigorously derive a priori error estimate for the finite element (FE) discretization of such problems which turn out to be optimal w.r.t. to the generic regularity of the optimal control problem. In addition we present a more heuristic a posteriori approach based on the dual weighted residual method. While a rigorous analysis of the error estimator is still lacking, the numerical tests show promising results.
For several years, the industry has brought the use of composite materials into focus, e.g. for the construction of wind turbines, aircrafts, and in the automotive industry. There exists a wide variety of possible applications due to the unbeatable advantages over conventional materials such as steel or aluminum; these are mainly the lower weight and an often significantly higher mechanical strength. In contrast to homogeneous materials, the modeling of composites is significantly more complex because of the fine structural features. We use a non linear strain- and stress-based continuum damage model, which was first introduced by Simon and Ju [2], and is well accepted throughout the engineering community [2]. The stress tensor $\sigma$ is defined by $\sigma(x) = (1 - d(\epsilon,x))C(x):\epsilon(x)$, where $\epsilon$ is the strain tensor, $d$ the internal damage variable and $C$ the stiffness tensor. Due to the model we make use of a multi domain Galerkin boundary element method for elasticity [3] coupled with a specific matrix valued radial basis function part to treat the non linear term. To reduce memory requirements of the fully populated matrices, we apply a low rank approximation for the matrices generated by the BEM and RBF parts. The resulting linear system is then solved by the use of specially developed preconditioner technique.


Segregated direct boundary-domain integral equations (BDIEs) based on a parametrix and associated with the Dirichlet and Neumann boundary value problems for the linear stationary diffusion partial differential equation with a variable coefficient are formulated. The PDE right hand sides belong to the Sobolev space $H^{-1}(\Omega)$ or $\tilde{H}^{-1}(\Omega)$, when neither classical nor canonical co-normal derivatives are well defined, which complicates the form of the conormal derivative of the third Green identity on the boundary and corresponding boundary-domain integral equations.

Equivalence of the BDIEs to the original BVP, BDIE solvability, solution uniqueness/non-uniqueness, as well as Fredholm property and invertibility of the BDIE operators are analysed in Sobolev (Bessel potential) spaces. It is shown that the BDIE operators for the Neumann BVP are not invertible, and appropriate finite-dimensional perturbations are constructed leading to invertibility of the perturbed operators. The contribution is based on and develops some results of [1-3].

References


Efficient and scalable algorithms are critical to deliver numerical PDE solutions fast for important scientific applications. This talk will discuss recent development on high-order spectral-element/spectral-element discontinuous Galerkin discretizations for solving wave, Poisson, and convection-diffusion type equations arising in electromagnetics and fluid systems \cite{1,2,3,4,5}. Discussion will include the algorithmic strategies on fast operator evaluations and minimizing communication cost that are key components to achieve a fast simulation on CPUs/GPUs on the advanced computing facilities.

References


SOLVING TIME-DEPENDENT HEAT TRANSFER PROBLEMS WITH ENRICHED FINITE ELEMENTS

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The finite element method (FEM) presents many advantages when solving time-dependent heat transfer problems, in comparison to other domain based methods. However, challenging aspects such as the presence of high heat gradients or multi-physics heat transfer may pose difficulties to efficiently solve practical problems. Enriching the FEM proved to be a successful approach to overcome this type of difficulties and leads to a significant reduction of the computational effort in spite of some numerical issues. In this presentation recent research progress in this area will be discussed.

SPACE–TIME TREFFTZ DISCONTINUOUS GALERKIN METHODS FOR WAVE PROBLEMS

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We present a space–time discontinuous Galerkin method for linear wave propagation problems. The special feature of the scheme is that it is a Trefftz method, namely that trial and test functions are solution of the partial differential equation to be discretised in each element of the (space–time) mesh. The method considered, described in [2] and [4], is a modification of the schemes of [3] and [5].

The DG scheme is defined for unstructured meshes whose internal faces need not be aligned to the space–time axes. The Trefftz approach can be used to improve and ease the implementation of explicit schemes based on “tent-pitched” meshes, cf. [1] and [5].

We show that the scheme is well-posed, quasi-optimal and dissipative, and prove a priori error bounds for general Trefftz discrete spaces. A concrete discretisation can be obtained using piecewise polynomials that satisfy the wave equation elementwise, for which we show high orders of convergence.
References


**PLANE WAVE DISCONTINUOUS GALERKIN METHODS FOR SCATTERING PROBLEMS**

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Plane Wave Discontinuous Galerkin (PWDG) methods can be used to approximate the Helmholtz equation on a bounded domain. To approximate a scattering problem, the PWDG can be used on a bounded region of free space around the scatterer provided a suitable truncation condition is imposed on the artificial boundary. I shall present error estimates for using the Dirichlet to Neumann map to supply the truncating boundary conditions and show numerical results that demonstrate the use of this approach.
Computing the electromagnetic field in a periodic grating due to light from the sun is critical for assessing the performance of thin film solar voltaic devices. This calculation needs to be performed for many angles of incidence and many frequencies across the solar spectrum. To compute at multiple frequencies one approach is to use a broadband incoming wave and solve the time domain scattering problem for a grating. The frequency domain response for a band of frequencies can then be computed by a Fourier transform.

In this presentation we discuss a two dimensional model problem derived from Maxwell’s equations by assuming that the fields and grating are translation invariant in one coordinate direction. This results in a wave equation with coefficients appearing as convolutions in the time domain. Assuming plane wave incidence, and a suitable space-time transformation we then arrive at a time dependent second order hyperbolic problem posed on an infinite strip with periodic boundary conditions. Two complications occur: first, materials used in practical devices have frequency dependent coefficients. In fact, at optical frequencies, commonly used metals have a frequency domain permittivity with negative real part but positive imaginary part which describes conductivity. Secondly the spatial domain for the problem is an infinite strip.

Using Laplace transform, we provide a proof of existence and uniqueness in the time domain for a general class of such frequency dependent materials. In the Laplace domain we can also derive a simple expression for the Dirichlet-to-Neumann map (D-t-N), and hence reduce the Laplace domain problem to a bounded domain containing the grating. Then using Convolution Quadrature we can construct a discrete D-t-N map to truncate the spatial computational domain after time discretization, and we prove fully discrete error estimates using a class of multistep methods in time and finite elements in space. Because of the use of Convolution Quadrature, the discrete time domain D-t-N map is perfectly matched to the time stepping scheme.

We end with some preliminary numerical results that demonstrate the convergence and stability of the scheme. We show that using the Backward Differentiation Formula-2 (BDF2) in time and finite elements in space we can compute the time dependent solution for a metal modeled by a Drude law, and for a dielectric modeled by the Sellmeier equation.
A MIXED FEM FOR A VORTICITY FORMULATION OF THE BRINKMAN EQUATIONS

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In this talk, we develop a mixed finite element method for the Brinkman equations formulated in terms of velocity, vorticity and pressure. Employing the Babuška–Brezzi theory, it is proved that the resulting continuous and discrete variational formulations are well-posed. In particular, we show that Raviart-Thomas elements of order $k \geq 0$ for the approximation of the velocity field, piecewise continuous polynomials of degree $k + 1$ for the vorticity, and piecewise polynomials of degree $k$ for the pressure, yield unique solvability of the discrete problem. We establish a priori error estimates in the natural norms. Finally, we report several numerical experiments illustrating the behavior of the proposed scheme and confirming our theoretical results.
THE NONLINEAR PETROV–GALERKIN METHOD IN
BANACH SPACES: YET ANOTHER IMPROVEMENT OF
BABUŠKA’S A PRIORI ERROR ESTIMATE

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In a recent 2015 paper by Stern [1], the author has sharpened the classical Babuška’s
\textit{a priori} error estimate for Petrov–Galerkin methods in Banach spaces (cf. [2], 1971).
The estimate had been previously sharpened only for the case of Hilbert spaces in a
2003 paper by Xu & Zikatanov [3] (more than 30 years after Babuška’s result!). All
of these estimates rely on a compatibility condition between the discrete trial and test
spaces, known as the dis\textit{crete inf–sup condition}.

From a different point of view, inspired in the residual minimization approach [4]
and the Hilbert-space theory of optimal Petrov-Galerkin methods [5], we address the
question of how to inherit discrete stability from continuous stability in a Banach space
setting. As a result, we deduce the nonlinear Petrov–Galerkin method in [6], whose
implementable (inexact) version consists in a monotone mixed method.

In this talk, we show in detail the error estimates of the method proposed in [6],
which depend explicitly on geometrical constants of the involved Banach spaces.

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Residual minimization, nonlinear Petrov–Galerkin, and monotone mixed meth-
The Galerkin (piecewise linear) finite element method is applied to approximate the solution of a time fractional diffusion equation with variable diffusivity. By a delicate energy analysis, a priori error bounds in $L^\infty(H^j), j = 0,1$ and $L^\infty(L^\infty)$-norms are derived for both smooth and nonsmooth initial data. Our analysis is based on a repeated use of an integral operator and use of a $t^m$ type of weights to take care of the singular behavior at $t = 0$. The generalized Leibniz formula for fractional derivatives is found to play a key role in our analysis. Numerical experiments are presented to illustrate the theoretical results.
A parametric finite element approximation of a fluidic membrane, whose evolution is governed by a surface Navier–Stokes equation coupled to bulk Navier–Stokes equations, is presented. The elastic properties of the membrane are modelled with the help of curvature energies of Willmore and Helfrich type. Forces stemming from these energies act on the surface fluid, together with a forcing from the bulk fluid.

We introduce a stable parametric finite element method to solve this complex free boundary problem. Local inextensibility of the membrane is ensured by solving a tangential Navier–Stokes equations, taking surface viscosity effects of Boussinesq–Scriven type into account. In our approach the bulk and surface degrees of freedom are discretized independently, which leads to an unfitted finite element approximation of the underlying free boundary problem. Bending elastic forces resulting from an elastic membrane energy are discretized using an approximation introduced by Dziuk. The obtained numerical scheme can be shown to be stable and to have good mesh properties.

Many problems in porous media science and geophysics comprise interactions of processes, and are typically formulated as a system of coupled PDEs. In most cases these systems are transient and often also non-linear. Developing efficient solvers is a delicate task, since one needs to must combine suitable schemes for (i) time integration, (ii) linearization, and (iii) (geometric and/or algebraic) multilevel solvers, finally being employed in a (iv) parallel computing environment. In this presentation, we take an application oriented approach, and focus on the problem classes of poroelasticity problems and density-driven-flow. For these two examples, we outline a common solution strategy, and provide numerical results.
Consider the von Kármán plates that describe the bending of thin elastic plates defined on polygonal domains. Conforming and non-conforming finite element methods are employed to approximate the displacement and Airy stress functions. Reliable and efficient \textit{a posteriori} error estimates are developed. The results of numerical experiments are presented.

We consider numerical approximation of a distributed optimal control problem governed by the von Kármán plate equations, defined on polygonal domains with pointwise control constraints. Conforming finite elements are employed to discretize the state and adjoint variables. The control is discretized using piece-wise constant approximations. \textit{A priori} error estimates are derived for the state, adjoint and control variables under minimal regularity assumptions.
Non-localized and localized nonlinear boundary-domain integral equation (NBDIE) formulations of the Dirichlet, Neumann and Robin boundary value problems are considered for some quasilinear partial differential equations of elliptic type. Using the properties of the non-localized and localized layer and volume potentials it is shown that the NBDIE systems are equivalent to the original classical and/or weak setting of the boundary value problems. For some special type of nonlinearities the corresponding NBDIE systems are analyzed in detail and an iterative method convergence is proved employing the Banach and Leray-Schauder fixed-point theorems. Some applications to the nonlinear problems of mathematical physics are presented.

A $C^0$ METHOD FOR THE BIHARMONIC PROBLEM
WITHOUT EXTRINSIC PENALIZATION.

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A symmetric $C^0$ finite element method for the biharmonic problem is presented and analyzed. In our approach, we introduce one-sided discrete second order derivatives and Hessian matrices to formulate our scheme. We show that the method is stable and converge with optimal order in a variety of norms. A distinctive feature of the method is that the results hold without extrinsic penalization of the gradient across inter-element boundaries. Numerical experiments are given that support the theoretical results, and the extension to Kirchhoff plates is also discussed.
DIVERGENCE-FREE-PRESERVING DISCRETIZATIONS OF INCOMPRESSIBLE FLOW

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We construct conforming finite element spaces for the Stokes and Navier–Stokes problem in two and three dimensions that yield divergence–free velocity approximations. The derivation of the finite element pairs is motivated by a smooth de Rham complex that is well–suited for the Stokes problem. We discuss the stability and convergence properties of the new elements and outline the construction of reduced elements that have fewer unknowns.

FINITE ELEMENT METHODS FOR PDES IN NON-DIVERGENCE FORM WITH APPLICATIONS TO HAMILTON-JACOBI-BELLMAN EQUATIONS

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In this talk, we describe a class of finite element methods for $W^{2,p}$ strong solutions of second-order linear elliptic PDEs in non-divergence form. The main novelty of the method is the inclusion of an interior penalty term, which penalizes the jump of the flux across the interior element edges/faces, to augment a nonsymmetric piecewise defined and PDE-induced bilinear form. Existence, uniqueness and error estimate in a discrete $W^{2,p}$ energy norm are proved for the proposed finite element method. This is achieved by establishing a discrete Calderon-Zygmund-type estimate and mimicking strong solution PDE techniques at the discrete level. We further discuss extensions to PDEs with discontinuous coefficients and to fully nonlinear Hamilton–Jacobi–Bellman equations.
This work considers the combined space-time discretization of time-dependent partial differential equations by using first order least square methods. We also impose an explicit constraint representing space-time mass conservation. To alleviate the restrictive memory demand of the method, we use dimension reduction via accurate element agglomeration AMG coarsening, referred to as AMGe upscaling. Numerical experiments demonstrating the accuracy of the studied AMGe upscaling method are provided.
Solving a PDE with heterogeneous coefficient is challenging. On the one hand, the size of the associated linear systems can be very large as a fine mesh is often required to represent all of the different scales in the coefficient. On the other hand, the high contrast and rapid variation of the coefficient can greatly increase the condition number of the associated linear system, making it difficult to solve by iterative solvers. Therefore, robust parallel preconditioners are needed for this type of problems. In this talk, we present a balancing domain decomposition by constraints (BDDC) method based on aggregation of elements with the same or “nearly” the same coefficient. Instead of imposing constraints on purely geometrical objects (faces, edges and vertices) of the partition interface, we use interface objects (subfaces, subedges and vertices) defined by the variation of the coefficients. When the coefficient is constant in each object, we can show both theoretically and numerically that the condition number does not depend on the contrast of the coefficient. In cases where the constant coefficient condition results in too many objects (a large coarse problem), we relax the condition and only require that the ratio of the minimal and maximal values of the coefficient in each object is larger than a predefined threshold. The threshold can be chosen so that the condition number is reasonably small while the size of the coarse problem is not too large. We emphasize that the new method is easy to implement and does not require to solve any eigenvalue or auxiliary problems. Numerical experiments are provided to support our findings.
NUMERICAL APPROXIMATION OF THE LAPLACE EIGENVALUES WITH MIXED BOUNDARY DATA

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Eigenfunctions of the Laplace operator with mixed Dirichlet-Neumann boundary conditions may possess singularities, especially if the Dirichlet-Neumann junction occurs at angles $\geq \frac{\pi}{2}$. This suggests the use of boundary integral strategies to solve such eigenproblems. As with boundary value problems, integral-equation methods allow for a reduction of dimension, and the resolution of singular behaviour which may otherwise present challenges to volumetric methods.

In this talk, we present a novel integral-equation algorithm for mixed Dirichlet-Neumann eigenproblems. This is based on joint work with Oscar Bruno and Eldar Akhmetgaliyev (Caltech).

For domains with smooth boundary, the singular behaviour of the eigenfunctions at Dirichlet-Neumann junctions is incorporated as part of the discretization strategy for the integral operator. The discretization we use is based on the high-order Fourier Continuation method (FC).

For non-smooth (Lipschitz) domains an alternative high-order discretization is presented which achieves high-order accuracy on the basis of graded meshes.

In either case (smooth or Lipschitz boundary), eigenvalues are evaluated by examining the minimal singular values of a suitably stabilized discrete system. This is in the spirit of the modification proposed by Trefethen and Betcke in the modified method of particular solutions.

The method is conceptually simple, and allows for highly accurate and efficient computation of eigenvalues and eigenfunctions, even in challenging geometries. If time permits, we also present results on the mixed Steklov-Neumann problem.
A BAYESIAN APPROACH TO EIGENVALUE OPTIMIZATION

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A celebrated conjecture by Polyá and Szegö asserts that amongst all n-sided polygons of a given area, the regular n-gon is the global optimizer of the first Dirichlet eigenvalue of the Laplacian. This conjecture has been shown to hold for triangles and quadrilaterals, but is open for pentagons.

In this talk, we present a novel framework for eigenvalue optimization combining finite element computations in a validated numerics setting, with a Bayesian optimization approach. We illustrate this approach for the specific case of the Polyá-Szegö conjecture on pentagons.

AN EIGENVALUE ANALYSIS BASED ON CONTOUR INTEGRALS FOR PERIODIC BOUNDARY VALUE PROBLEMS WITH THE BOUNDARY ELEMENT METHOD

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An eigenvalue analysis based on contour integrals for periodic boundary value problems for Helmholtz’ equation in 3D with the boundary element method (BEM) is proposed. The Sakurai-Sugiura method (SSM) is one of numerical methods for non-linear eigenvalue problems, which obtains eigenvalues inside a given contour in the complex plane by calculating an integral along the contour. In this paper, we extend integral operators in the BEM to complex phase factor in order to calculate the contour integrals used in the SSM. With the proposed method, we analyse behaviour of reasonance anomalies in some periodic boundary value problems for Helmholtz’ equation in 3D.
The great advances in computational engineering and sciences over the last half century, including profound advances in finite element methods initiated at the first MAFE-LAP conference over forty years ago, together with huge strides in high performance computing, have ushered in a new age in scientific discovery and engineering innovation. These advances have pushed computer simulation from a largely qualitative exercise to a source of quantitative information now used as a basis for important, often life-and-death decisions: predictive surgery, climate change, drug design, nano-manufacturing, design of materials, etc. However, as evidence of the true predictability of many time-honored models has come to light, the question of the reliability of all computer predictions has come under serious scrutiny. At the root of these concerns is the inevitable uncertainty in all phases of the predictive process, uncertainty in model selection, observational data, and model parameters, all contributing to the uncertainty of predictions of the target realities.

This lecture presents an overview of the foundations of predictive computational science, the discipline concerned with the quantification of uncertainty in computer predictions. It is argued that a Bayesian setting provides a logical and unifying framework for handling many of the uncertainties in model prediction. When coupled with tools from information theory, a powerful approach to predictive modeling can be formulated. We describe the Occam Plausibility Algorithm (OPAL) as an adaptive approach to model selection and validation. Applications to coarse-grained models of atomistic systems, phase-field models of tumor growth, and models of gamma wave radiation are presented as examples.
COMPUTATIONAL ASPECTS OF FAST ADAPTIVE BOUNDARY ELEMENT METHODS

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We will address computational aspects of fast methods in adaptive boundary element methods for 3d computations for the Laplace equation. In the computational examples we will use the (h - h/2)-error estimation strategy [M. Karkulik, G. Of, and D. Praetorius, Convergence of adaptive 3D BEM for weakly singular integral equations based on isotropic mesh-refinement. Numerical Methods for Partial Differential Equations, 29(6):2081-2106, 2013]. An important aspect is the automatic choice of parameters of the Fast Multipole method with respect to error estimation and in adaptive boundary element methods.

AUXILIARY SUBSPACE ERROR ESTIMATES FOR ELLIPTIC PROBLEMS

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Hierarchical basis methods are implicit schemes wherein global estimates and local indicators of the discretization error in a finite element space $V$ are obtained by solving a global residual equation in an appropriate auxiliary space $W$. Traditionally, the space $W$ has been chosen such that $V \oplus W$ is a natural finite element space, e.g. if $V$ is the degree $p$ Lagrange space on a given mesh, then $V \oplus W$ might be the degree $p + 1$ Lagrange space on the same mesh (a $p$-hierarchy), or the degree $p$ Lagrange space on a uniformly-refined mesh (an $h$-hierarchy). We provide a very different prescription for choosing the space $W$ that yields provably efficient and reliable error estimates at reasonable cost. We empirically demonstrate its robustness with respect to problem parameters (e.g. discontinuous and anisotropic diffusion with high contrasts) and polynomial degree.
A NYSTRÖM-BASED FINITE ELEMENT METHOD ON POLYGONAL ELEMENTS

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Standard forms of virtual element methods (VEM) and Boundary-Element based Finite Element Methods (BEM-FEM) employ local spaces that are defined implicitly in terms of solutions of Poisson problems with polynomial data. We here follow the path of BEM-FEM in terms of evaluating local basis functions for quadratures via integral equation techniques, but instead employ Nyström methods, which we believe provide several practical advantages. Among these are well-conditioned local linear systems that are trivial to set up even for high-order discretizations, better resolution of singular behavior in basis functions on non-convex elements, and flexibility to allow for elements having curved edges. We will describe the key details of the proposed approach and illustrate its performance in terms of interpolation and discretization errors.
We present an augmented mixed finite element method for the coupling of fluid flow with porous media flow. The flows are governed by a class of nonlinear Navier-Stokes and the linear Darcy equations, respectively, and the transmission conditions are given by mass conservation, balance of normal forces, and the Beavers-Joseph-Saffman law. We apply dual-mixed formulations in both domains, and the nonlinearity involved in the Navier-Stokes region is handled by setting the strain and vorticity tensors as auxiliary unknowns. In turn, since the transmission conditions become essential, they are imposed weakly, which yields the introduction of the traces of the porous media pressure and the fluid velocity as the associated Lagrange multipliers. Furthermore, since the convective term in the fluid forces the velocity to live in a smaller space than usual, we augment the variational formulation with suitable Galerkin type terms arising from the constitutive and equilibrium equations of the Navier-Stokes equations, and the relation defining the strain and vorticity tensors. The resulting augmented scheme is then written equivalently as a fixed point equation, so that the well-known Schauder and Banach theorems, combined with classical results on bijective monotone operators, are applied to prove the unique solvability of the continuous and discrete systems. In particular, given an integer $k \geq 0$, piecewise polynomials of degree $\leq k$, Raviart-Thomas spaces of order $k$, continuous piecewise polynomials of degree $\leq k + 1$, and piecewise polynomials of degree $\leq k$ are employed in the fluid for approximating the strain tensor, stress, velocity, and vorticity, respectively, whereas Raviart-Thomas spaces of order $k$ and piecewise polynomials of degree $\leq k$ for the velocity and pressure, together with continuous piecewise polynomials of degree $\leq k + 1$ for the traces, constitute feasible choices in the porous medium. Finally, several numerical results illustrating the good performance of the augmented mixed finite element method and confirming the theoretical rates of convergence are reported.
The mathematical theory and numerical analysis of non-local operators has been a topic of intensive research in recent years. One class of applications come from replacing Brownian motion diffusion by diffusion coming from a symmetric $\alpha$-stable Levy process, i.e., the Laplace operator is replaced by a fractional Laplacian.

In this talk, we propose a numerical approximation of equations with this type of diffusion terms posed on bounded domains. We focus on the simplest example of an elliptic variational problem coming from the fractional Laplacian on a bounded domain with homogeneous Dirichlet boundary conditions. Although it is conceptually feasible to study the Galerkin approximation based on a standard finite element space, such a direct approach is not viable as the exact computation of the resulting stiffness matrix entries is not possible (at least in two or more spatial dimensions).

Instead, we will develop a non-conforming method by approximating the action of the stiffness matrix on a vector (sometimes referred to as a matrix free approach). The bilinear form is written as an improper integral involving the solution of parameter dependent elliptic problems on $\mathbb{R}^d$. We compute an approximate action of stiffness matrix by applying a SINC quadrature rule to the improper integral, replacing the problems on $\mathbb{R}^d$ by problems on parameter dependent bounded domains, and the application of the finite element method to the bounded domain problems. The entire procedure can be implemented using standard finite element tools, e.g., the DEAL-II library. The analysis of the resulting algorithm is discussed. In addition, the results of numerical computations on a model problem with known solution are given.
A stabilized Lagrange multiplier method for second order elliptic interface problems is presented in the framework of mortar method. The requirement of LBB (Ladyzhenskaya-Babuška-Brezzi) condition for mortar method is alleviated by introducing penalty terms in the formulation. Optimal convergence results are established in natural norm which is independent of mesh. Error estimates are obtained with an assumption that: the multiplier space satisfies the strong regularity property in the sense of Babuška (see, [1]). Numerical experiments are conducted in support of the theoretical derivations.

References


We present a high-order Implicit Large-Eddy Simulation (ILES) approach for simulating transitional turbulent flows. The approach consists of hybridized Discontinuous Galerkin (DG) methods for the discretization of the Navier-Stokes (NS) equations and a parallel preconditioned Newton-GMRES method for the resulting nonlinear system of equations. The combination of hybridized DG methods with an efficient solution procedure leads to a high-order accurate NS solver that is competitive with finite volume codes in terms of computational cost. The proposed approach is applied to transitional turbulent flows over a NACA 65-(18)10 compressor cascade and an Eppler 387 wing at Reynolds numbers up to 300,000. Grid convergence studies are presented and the required resolution to capture transition at different Reynolds numbers is investigated. Numerical results show rapid convergence and excellent agreement with experimental data. This work aims to demonstrate the potential of high-order ILES for transition prediction and present a rationale for this approach through empirical observations.
Novel photonic materials such as photonic crystals and metamaterials are scientifically engineered to interact with and control electromagnetic waves in ways that cannot be achieved with conventional materials. Photonic crystals exhibit bandgap phenomena and have proven very important as an integrated component in many optical devices including waveguides, fibers, lasers, cloaks, superlenses. At sub-wavelength scales, the interaction between electromagnetic waves and conduction electrons at metallic interfaces leads to surface plasmon polaritons and to the confinement of electromagnetic fields over very small spatial dimensions with applications in heat transfer, energy harvesting and sensing. These and other novel applications have attracted considerable research interest. However, fundamental challenges abound about the design and fabrication of these photonic structures in order to yield a given set of prescribed properties. For instance, it is currently beyond the state-of-the-art to compute robust designs that exhibit prescribed properties subject to fabricability constraints. The issue of fabrication adaptivity (adapting a given computed design so that it is fabricable, without significantly deteriorating the design quality) is particularly important in practical applications since the desired length scales and material distributions are often at the limit of our fabrication capability and hence geometric design tolerances (in relative scale) need to be larger. Another important issue to address in the design optimization is the uncertainty arising in the mathematical model since physical phenomena can rarely be modeled with complete fidelity even under the best of circumstances. We will described a range of numerical simulation and optimization algorithms for the design of photonic structures. These will include our multi-scale high order Hybridized Discontinuous Galerkin method, including novel approaches for accurate wave propagation, our topology optimization approach via modern convex optimization techniques, particularly semi-definite programming (SDP) interior-point methods, and our fabrication adaptive optimization algorithm. We will illustrate our algorithms with examples in both photonic crystal design and plasmonics.
A PLANE WAVE VIRTUAL ELEMENT METHOD
FOR THE HELMHOLTZ PROBLEM

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The virtual element method (VEM) is a generalisation of the finite element method recently introduced in \cite{Beirao2013}, which takes inspiration from mimetic finite difference schemes, and allows to use very general polygonal/polyhedral meshes.

This talk is concerned with a new method introduced in \cite{Paula2020}, based on inserting plane wave basis functions within the VEM framework in order to construct an $H^1$-conforming, high-order method for the discretisation of the Helmholtz problem, in the spirit of the partition of unity method (PUM, see e.g., \cite{Hiptmair2019}).

Plane wave functions are a particular case of Trefftz functions for the Helmholtz problem, i.e., functions belonging to the kernel of the Helmholtz operator. Finite element methods based on inserting Trefftz basis functions within the approximating spaces \cite{Hiptmair2020} allow to obtain, compared to standard polynomial finite element methods, similar accuracy with less degrees of freedom, mitigating the strong requirements in terms of number of degrees of freedom per wavelength due to the pollution effect.

The main ingredients of the plane wave VEM scheme (PW-VEM) are: \textit{i}) a low order VEM space whose basis functions, which form a partition of unity and are associated to the mesh vertices, are not explicitly computed in the element interiors; \textit{ii}) a proper local projection operator onto the plane wave space, which has to provides good approximation properties for Helmholtz solutions; \textit{iii}) an approximate stabilization term. Convergence of the $h$-version of the PW-VEM was proved, and numerical results testing its performance on general polygonal meshes were presented.

References


NON-UNIFORM FILTERS VIA SHIFT AND SCALE FOR DISCONTINUOUS GALERKIN OUTPUT

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Convolving the output of Discontinuous Galerkin computations with symmetric Smoothness-Increasing Accuracy-Conserving (SIAC) filters can improve both smoothness and accuracy. To extend convolution to the boundaries, several one-sided spline filters have recently been developed. This paper interprets these filters as instances of a general class of position-dependent (PSIAC) spline filters that can have non-uniform knot sequences and skip B-splines of the sequence.

PSIAC filters with rational knot sequences have rational coefficients. For prototype knot sequences, such as integer sequences that may have repeated entries, PSIAC filters can be expressed in symbolic form. Based on the insight that filters for shifted or scaled knot sequences are easily derived by non-uniform scaling of one prototype filter, a single filter can be re-used in different locations and at different scales. Computing a value of the convolution then simplifies to forming a scalar product of a short vector with the local output data. Restating one-sided filters in this form improves both stability and efficiency compared to their original formulation via numerical integration. PSIAC filtering is demonstrated for several established and one new boundary filter.
DESIGN AND ANALYSIS ON SURFACES WITH IRREGULARITIES

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Based on the fact that ‘every $G^k$ construction yields a finite element suitable for the iso-parametric IGA framework’, this talk explores issues of computing across parametric singularities, including the design of free-form surfaces and the analysis of functions on those surfaces.

RELAXING THE CFL CONDITION FOR THE WAVE EQUATION ON ADAPTIVE MESHES

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The Courant-Friedrichs-Lewy (CFL) condition limits the choice of the time-step size for the popular explicit leapfrog method for the wave equation to be bounded by the minimal mesh-size in the spatial finite element mesh. This makes the scheme expensive for locally refined meshes. On the other hand, locally refined meshes are necessary to reveal the optimal convergence rate on domains with re-entrant corners. This talk introduces a reduced ansatz space based on a uniform mesh that allows to balance the CFL condition and adaptive spatial approximation in an optimal way, even in the presence of spatial singularities.
TWO MATHEMATICAL ASPECTS OF ISOGEOEMTRIC ANALYSIS:
QUASI-OPTIMAL ADAPTIVE MESH REFINEMENT
AND SUPERIOR EIGENVALUE APPROXIMATION

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This talk presents two results in the context of Isogeometric Analysis. The first result concerns the analysis-suitable adaptive refinement of $T$-meshes and its quasi-optimality. The second part discusses global stability properties of the Rayleigh-Ritz approximation of Laplace eigenvalues by $B$-splines and the possible superiority over classical finite elements. This talk is based on joint works with Dietmar Gallistl, Pascal Huber and Philipp Morgenstern.

ADAPTED NUMERICAL METHODS FOR THE
POISSON EQUATION WITH $L^2$ BOUNDARY
DATA IN NON-CONVEX DOMAINS

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This talk is concerned with adapted numerical methods for the Poisson equation with $L^2$ boundary data and emphasis on non-convex domains. Due to the rough boundary data, the equation needs to be understood in the very weak sense. For a standard finite element discretization with regularized boundary data, a convergence order of $1/2$ in the $L^2(\Omega)$-norm can be proved provided that the domain is convex. However, in non-convex domains the convergence rate is reduced although the solution remains to be contained in $H^{1/2}(\Omega)$. The reason is a singularity in the solution of the dual problem. In this talk, as a remedy, both a standard finite element method with mesh grading and a dual variant of the singular complement method are proposed and analyzed in order to retain a convergence rate of $1/2$ also in non-convex domains. Finally, numerical experiments are presented in order to illustrate the theoretical results.
Enhanced and controlled delivery of molecules (e.g. drug, proteins, DNA) into cells with the aid of electric fields is a hot topic in molecular medicine. In this context, the role of the electric field is two-fold: one, to permeabilize the cellular membrane allowing the introduction of the molecules into the cell, a process known as electroporation; two, to advance and control the migration of the charged molecules into the cell. This last technique, known as electrophoresis, is particularly crucial when the cells have a short resealing time or when large molecules need to be loaded into cells.

The mathematical modeling of these biological and physical processes involves two main equations, a parabolic convection-diffusion equation that describes the evolution of the concentration of molecules, in and outside the cell, and Maxwell’s equations for the electromagnetic waves. Ignoring the magnetic field, the Maxwell system can be reduced to a wave equation for the electric field.

In this work we study a finite difference method for the wave-diffusion coupled problem that is based on piecewise linear finite element approximations. Second order error estimates with respect to a discrete $H^1$-norm are established provided that, for each time level, the solution of the coupled problem is in $H^3(\Omega) \times H^3(\Omega)$, where \( \Omega \) is the spatial domain.
I will introduce an adaptive framework developed to solve nonlinear elliptic partial differential equations (PDE) starting from a coarse mesh. The target problem class includes quasi-linear problems with steep gradients and thin internal layers in the solution-dependent diffusion coefficients, for which standard methods such as Newton or Picard iterations are known to fail. The method is designed to start with a discretization that does not resolve the problem coefficients. The discrete problem on the initial sequence of meshes is not assumed to inherit the stability, coercivity, monotonicity or solvability properties of the continuous system; essentially, the initial sequence of discrete problems is assumed ill-posed.

A sequence of partial solves of regularized problems is used to adaptively refine the discretization to uncover the layers and resolve the problem coefficients and data. Automatic control of the regularization parameters is developed to ultimately solve the discrete problem without regularization. I will discuss improving stability of the method by choice of regularization and pseudo-time integrators. The method will be demonstrated with numerical examples using an underlying linear finite element discretization.
BOUNDARY DOMAIN INTEGRAL EQUATIONS FOR THE MIXED COMPRESSIBLE STOKES SYSTEM WITH VARIABLE VISCOSITY IN BOUNDED DOMAINS

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The mixed boundary value problem related to the Stokes system is reduced to two different BDIES which are equivalent to the original boundary value problem, see \cite{1, 2, 3}. These Boundary Integral Equation Systems can be expressed in terms of surface and volume parametrix-based potential type operators whose properties are also analysed in appropriate Sobolev spaces. The invertibility and Fredholm properties related to the matrix operator that defines the BDIES are also presented.

References


EFFICIENT ERROR ESTIMATION AND FAST SOLVERS FOR
STOCHASTIC GALERKIN FINITE ELEMENT APPROXIMATION

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We discuss two issues related to the efficient implementation of stochastic Galerkin finite element methods (SGFEMs) for elliptic PDEs with random coefficients: a posteriori error estimation and fast iterative solvers.

An a posteriori error estimator was recently proposed in [A. Bespalov, C.E. Powell, D. Silvester, Energy norm a posteriori error estimation for parametric operator equations, SIAM Journal Sci. Comp. 36(2), A339–A363, 2014]. A strengthened Cauchy Schwarz (or CBS) constant associated with a deterministic problem related to the mean diffusion coefficient determines both the efficiency of the error estimate, and the estimate of the error reduction that would be achieved by enriching the SGFEM approximation space. We present new analysis of CBS constants for use in developing adaptive SGFEM algorithms.

A novel reduced-basis solver for the associated discrete linear systems of equations was also recently introduced in [C.E. Powell, V. Simoncini, D. Silvester, An efficient reduced basis solver for stochastic Galerkin matrix equations, submitted (2015)]. When we re-cast the linear systems as matrix equations, the solution matrix often has low rank and can be well approximated in a low-dimensional space. We describe a novel strategy for adaptively building such a space, leading to an algorithm with lower memory requirements than standard Krylov solvers.
We present an axiomatic proof of optimal convergence rates for adaptive FEM as well as BEM in the spirit of [5]. For this purpose, an overall set of four axioms on the error estimator is sufficient and (partially even) necessary [2]: These four axioms are stability on non-refined element domains (A1), reduction on refined element domains (A2), discrete reliability (A3), and general quasi-orthogonality (A4). The presentation shall discuss those properties and motivate the different arguments which guarantee convergence with optimal rate in terms of certain nonlinear approximation classes which coincide from the literature, e.g., [3], if the error estimator is efficient.

The contributions of [2] can be summarized as follows: First, a general framework is presented which covers the existing literature on rate optimality of adaptive schemes for both, linear as well as nonlinear problems. Second, efficiency of the error estimator is exclusively needed to characterize the approximation classes. Third, some general quasi-Galerkin orthogonality is not only sufficient, but also necessary for the R-linear convergence of the error estimator, which is a fundamental ingredient in the current quasi-optimality analysis [5, 3]. Finally, the general analysis allows for various generalizations like equivalent error estimators and inexact solvers as well as different non-homogeneous and mixed boundary conditions.

One particular focus of the presentation will be on nonlinear model problems like strongly-monotone operators [4] or the p-Laplacian [1].

References

AN OPTIMAL SOLVER FOR LINEAR SYSTEMS ARISING FROM STOCHASTIC FEM APPROXIMATION OF DIFFUSION EQUATIONS WITH RANDOM COEFFICIENTS

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This paper discusses the design and implementation of efficient solution algorithms for symmetric linear systems associated with stochastic Galerkin approximation of elliptic PDE problems with correlated random data. The novel feature of our preconditioned MINRES solver is the incorporation of error control in the natural “energy” norm in combination with a reliable and efficient a posteriori estimator for the PDE approximation error. This leads to a robust and optimally efficient inbuilt stopping criterion: the iteration is terminated as soon as the algebraic error is insignificant compared to the approximation error.

ADAPTIVE REGULARISATION

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The design of numerical schemes for nonlinear PDEs is delicate. In many important cases, for example when tackling conservation laws, there are infinitely many weak solutions and it is paramount that the underlying scheme respects certain physically motivated selection criteria. In the design of numerical methods for linear problems, high order perturbations tend to be neglected. The main difference in treating nonlinear problems over their linear counterparts is that high order perturbations cannot just be dropped, especially in the case when infinitely many weak solutions may exist.

We propose a methodology of introducing regularisation in an a posteriori fashion. This will allow us to construct numerical approximations of a particularly challenging set of solution concepts, namely entropy and viscosity solutions. These are appropriate “weak” solutions of conservation laws and Hamilton-Jacobi equations. In this talk we illustrate the ideas and application to some simple problems.
APPROXIMATION OF LIQUID CURTAINS

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Liquid curtains arise in various industrial applications. Curtain coating is where an object is passed through a smooth curtain of liquid. Examples of this range from painting of materials to coating of photographic films but all are essentially coating procedures allowing for uniform coating of various substrates with some film. The stability of curtains in these applications is crucial as a stable procedure for this allows for a uniform coating with minimal materials and time.

In this work we propose a numerical method to approximate a thin film model of liquid curtains. The stability of the numerical approximation of the curtain is examined at a variety of length scales. At small length scales we compare our simulations to experimental work already conducted demonstrating the numerical method correctly predicts the stability of curtains and then test our numerical method at larger length scales, illustrating the applicability of large scale curtains for commercial use.

HIGHER ORDER FINITE ELEMENTS IN OPTIMAL CONTROL

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In this talk we propose a new method for solving control constrained optimal control problems. We use a non-conform discretization with higher order finite elements. A mass lumping approach is proposed to obtain a simple and very accurate numerical scheme. Under certain assumptions we are able to show convergence order up to \(h^4\).
This work is devoted to a higher order scheme for the non-stationary diffusion equation. The scheme is based on continuous Galerkin in time and mixed finite element method (MFEM) in space. Precisely, Raviart-Thomas elements of arbitrary order are involved. Continuous, semi-discrete and fully-discrete variational formulations are set up. Existence and uniqueness of solutions for the all formulations is rigorously proved. A priori error estimates are derived to show the convergence of the scheme. This is done for arbitrary orders in time and space. To obtain optimal order estimates a duality argument is involved. Numerical experiments are shown to confirm the theoretical results. We refer to [1] for the details of the analysis.

References

COMPUTABLE A POSTERIORI ERROR ESTIMATORS FOR FINITE ELEMENT APPROXIMATIONS OF AN OPTIMAL CONTROL PROBLEM

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We consider an optimal control problem with control constraints, where the state is governed by a convection–reaction–diffusion equation. We will discuss how computable a posteriori error estimators are obtained for the case when piecewise affine stabilized finite element methods are used to approximate the solutions to the state and adjoint equations and piecewise constants are used to approximate the control. The estimators provide guaranteed upper bounds on the norms of the errors and, up to a constant and oscillation terms, local lower bounds on the norms of the errors. Numerical examples, in two and three dimensions, will be presented to illustrate the theory.

PARALLEL ISOGEOMETRIC TIME DOMAIN MAXWELL AND VLASOV-MAXWELL SOLVERS

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Modern finite elements techniques for Maxwell equations rely on ideas from differential geometry and more precisely on the existence of discrete spaces that provide an exact De Rham sequence. In [1] the classical theory of discrete DeRham complexes, was extended to iso-geometric analysis for the steady-state Maxwell’s equations, providing a discrete exact DeRham sequence involving discrete spaces based on B-splines. In [2], we have derived a 2D B-Splines solver for the Time Domain Maxwell problem.

In this work, we present a parallel 2D/3D IsoGeometric solver for both the Time Domain Maxwell equations and the Vlasov-Maxwell problem. In the later, a hybrid Particle In Cell method is introduced, where particles live in the logical domain while the velocity is advanced in the physical domain.

References

Problems with very large deformation, motion of the solid and even contact raise problems for the ALE formulation of fluid-structure interactions. A domain map between a reference system and the current system cannot deal with changes of topology. If a strictly monolithic system is desirable due to reasons of stability of efficiency, a Fully Eulerian formulation, where both subproblems are cast onto the current coordinate system is a promising alternative.

In this contribution we report on recent advances and applications of the Fully Eulerian Formulation for fluid-structure interactions. In particular, we focus on the question of interface accuracy, which is critical, as the Fully Eulerian method is of interface capturing type. Furthermore, we present extensions of this model to incorporate active material growth and pre-stressing.
We consider a discretization of the hypersingular integral operator for the Poisson problem in $\mathbb{R}^3$ using the $hp$-version of the Galerkin boundary element method on a mixed mesh consisting of triangles and quadrilaterals. We propose and analyze a preconditioner based on the overlapping additive Schwarz framework. The underlying decomposition consists of a global block of piecewise linears/bilinears and blocks of higher order polynomials supported on the vertex, edge and element patches. The resulting preconditioned system has a condition number that is uniformly bounded with respect to the mesh size $h$ and the polynomial degree $p$. We also briefly discuss some options to improve the computational complexity of this preconditioner by replacing the piecewise linears/bilinears with a decomposition of multilevel type and by reducing the higher order block associated with the patches to a finite set of reference configurations.

References

A POSTERIORI ERROR ESTIMATES FOR THE BIOT PROBLEM BASED ON EQUILIBRATED $H(\text{div})$-CONFORMING FLUX RECONSTRUCTIONS

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Over the last few years, adaptive algorithms based on a posteriori error estimates have been put forward, comprising the adaptive stopping of the iterative solvers and the dynamic adaptation of the mesh and the time step (see, e.g. [3]). These two applications are the major motivations for EDF to include a posteriori error estimates in the hydro-mechanical part of their finite element code Code_Aster. We present here an approach allowing us to obtain a posteriori error estimations for a poro-elastic problem, where we handle the hydraulic part as proposed in [3] and develop equivalent techniques for the elasticity. The estimators are obtained by introducing equilibrated reconstructions of the velocity and the mechanical stress tensor, obtained as mixed finite element solutions of local Neumann problems posed over patches of elements (cf., e.g., [4]). With this approach it is possible to distinguish the different error sources: spatial and temporal discretization, and algebraic resolution. In the spirit of [2], the velocity reconstruction is sought in the Raviart-Thomas finite element space, while the difficulty of reconstructing a symmetric $H(\text{div})$-conforming stress tensor is overcome by choosing the mixed finite element space proposed by Arnold and Winther in [1].

References

A new variant of the Adaptive Cross Approximation (ACA) for approximation of dense block matrices is presented. This algorithm can be applied to matrices arising from the Boundary Element Methods (BEM) for elliptic or Maxwell systems of partial differential equations. The usual interpolation property of the ACA is generalised for the matrix valued case \[1\]. Some numerical examples demonstrate the efficiency of the new method. The main example will be the electromagnetic scattering problem, i.e. the exterior boundary value problem for the Maxwell system. Here, we will show that the matrix valued ACA method works well for high order BEM \[2\] and the corresponding high rate of convergence is preserved. Another example shows the efficiency of the new method in comparison with the standard technique while approximating the smoothed version of the matrix valued fundamental solution of the time harmonic Maxwell system.

References


The discontinuous Petrov-Galerkin finite element methodology of Demkowicz and Gopalakrishnan (DPG) [1, 2] offers a host of appealing features, including automatic stability and minimization of the residual in a user-controllable energy norm. DPG is, moreover, well-suited for high-performance computing, in that the extra work required by the method is embarrassingly parallel; the use of a discontinuous test space allows the computation of optimal test functions to be done element-wise. Additionally, the approach gives almost total freedom in the choice of basis functions, so that high-order discretizations can be employed to increase computational intensity (the number of floating point operations per unit of communication). Finally, since the method is stable even on a coarse mesh and comes with a built-in error measurement, it enables robust adaptivity which in turn means less human involvement in the solution process, a desirable feature when running large-scale computations.

Camellia [3] is a software framework for DPG with the aim of enabling rapid development of DPG solvers both for running on a laptop and at scale. Camellia supports spatial meshes in 1D through 3D; initial support for space-time elements is also available. Camellia supports \( h \)- and \( p \)-adaptivity, and offers distributed computation of essentially all the algorithmic components of a DPG solve. (One exception, which we plan to address, is the generation and storage of the mesh geometry; at present, this happens redundantly on each MPI rank.) Camellia supports static condensation for reduction of the global problem, and has a robust, flexible interface for using third-party direct and iterative solvers for the global solve.

Until recently, we have almost always solved the global DPG system matrix using parallel direct solvers such as SuperLU Dist. This is not a scalable strategy, particularly for 3D and space-time meshes. Both memory and time costs therefore motivate our recent work, developing and studying iterative solvers in the context of a range of example problems. Since Camellia’s adaptive mesh hierarchy provides us with rich geometric information, we focus on \( hp \)-geometric multigrid preconditioners with additive Schwarz smoothers of minimal or small overlap. Preconditioning a conjugate gradient solve using such preconditioners, we are able to solve much larger problems within the same memory footprint.

References


NUMERICAL MODELLING OF EVANESCENT AND PROPAGATING MODES IN PHONONIC STRUCTURES

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The phononic plates are periodic structures made of elastic components with large differences in their elastic coefficients, the soft phase being distributed in a form of inclusions embedded in a stiff matrix. The homogenization approach of such elastic structures occupying domain \(\Omega\) with the “dual porosity” type of the scaling ansatz applied in the inclusions \(\Pi\) leads to the following problem describing the acoustic wave propagation in a homogenized medium: Find polarization \(q \in Q(\Omega), q(x) \in \mathbb{R}^d\) for \(x \in \Omega\) (\(Q(\Omega)\) is the admissibility set reflecting boundary conditions), such that

\[
-\omega^2 \text{IM}(\omega^2) q + \text{IK} q = f(\omega^2), \quad \text{in} \ \Omega, \tag{1}
\]

where \(\omega \in \mathbb{R}\) is a fixed frequency, \(\text{IM} : \mathbb{R}^d \mapsto \mathbb{R}^d\) is the mass tensor (real symmetric, but possibly indefinite, depending on \(\omega \in \mathbb{R}_+\)) and \(\text{IK}\) is the 2nd order (elliptic) differential operator, the stiffness. For the 3D elasticity problem (with \(q = u = (u_i), i = 1, 2, 3\)) \(\text{IK}\) attains the form \((\text{IK})_{ij} = -\partial_k D_{ikjl} \partial_j\) with \(D_{ikjl}\) being the usual symmetric positive definite elasticity tensor. The problem for a phononic Reissner-Mindlin plate which is issued in the paper attains the same form, although \(q\) involves plate deflections and rotations and the \(\text{IK}\) and \(\text{IM}\) have a more complex structure.

Using the spectral decomposition of \(\text{IM}\), see \([2]\), the wave equation (1) can be transformed to a “diagonalized” form,

\[
-\omega^2 \langle \Lambda \xi, \zeta \rangle_{\Omega} + a_{\Omega}(\xi, \zeta) = \langle b(\omega^2), \zeta \rangle_{\Omega}, \quad \text{for all} \ \zeta \in W_0(\Omega), \tag{2}
\]

where \(a_{\Omega}(\cdot, \cdot)\) is an elliptic bilinear form and \(\Lambda = \Lambda^+ + \Lambda^-\) is the spectral matrix associated with \(\text{IM}\), decomposed into the positive and the negative parts. This is the basis for introducing two subspaces by solving eigenvalue problems which depend on the imposed frequency. Projections of (2) into these bases yield a system which allows us to resolve the propagating and evanescent modes (when \(\Lambda^- \neq 0\)).

In the conference paper, this approach to the wave dispersion analysis in the phononic media based on the outlined spectral decomposition will be compared with...
other methods of modelling the wave propagation in homogenized periodic structures. In particular, solving the dynamic problem in the time domain, thus, involving time convolutions, due to the presence of \( \text{IM}(\omega^2) \), will be discussed. The research was supported by the Czech Scientific Foundation project GACR P101/12/2315.

References


A LOCAL PROJECTION STABILIZATION METHOD FOR NATURAL CONVECTION PROBLEMS

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In this talk, we propose a Local Projection Stabilization (LPS) finite element method applied to the numerical solution of natural convection problems.

Firstly, after recalling the mathematical model for which the Boussinesq approximation is employed to treat thermal coupling, a particular LPS scheme, the high-order term-by-term stabilization method, is introduced and analyzed (cf. [1]). This method replaces the projection-stabilized structure of standard LPS methods by an interpolation-stabilized structure, which only acts on the high frequencies components of the flow. This approach gives rise to a method which may be cast in the Variational Multi-Scale (VMS) framework (cf. [2]), and constitutes a low-cost, accurate solver for incompressible flows, despite being only weakly consistent. In [1], this method has been applied to the simulation of a high Reynolds number ($Re = 10^4$) plane mixing layer flow, with accurate results for relatively coarse grids.

Here, numerical results for the 2D problem of a buoyancy-driven airflow in a square cavity with differentially heated side walls at high Rayleigh numbers (up to $Ra = 10^7$) are given and compared with benchmark solutions. Again, a good accuracy is obtained with relatively coarse grids.

References


DISCONTINUOUS APPROXIMATION OF VISCOUS TWO-PHASE FLOW IN HETEROGENEOUS POROUS MEDIA

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Runge-Kutta Discontinuous Galerkin (RKDG) and Discontinuous Finite Volume Element (DFVE) methods are applied to a coupled flow-transport problem describing the immiscible displacement of a viscous incompressible fluid in a non-homogeneous porous medium. The model problem consists of a nonlinear pressure-velocity equation assuming Brinkman flow, coupled to a nonlinear hyperbolic equation governing the mass balance (saturation equation). The mass conservation properties inherent to finite volume-based methods motivate a DFVE scheme for the approximation of the Brinkman flow in combination with a RKDG method for the spatio-temporal discretization of the saturation equation. The stability of the uncoupled schemes for the flow and for the saturation equation are analyzed, and several numerical experiments illustrate the robustness of the numerical method.
MULTI-TIME STEPPING SCHEMES FOR COUPLED POROUS MEDIUM AND FREE FLOW PROBLEMS

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Physical systems, where a porous medium is in contact with a free fluid, arise in a variety of environmental and industrial problems. Striking examples originate from terrestrial-atmospheric contact zones, surface water-groundwater interaction, filters and fuel cells. Flow and transport processes in these systems evolve on different space and time scales yielding different sets of equations in the flow domains and the necessity of interface conditions to complete the model formulation. The multiscale nature of the processes in the coupled system also contributes to the complexity of the problem from the numerical point of view. These multiple length and time scales should be taken into account for accurate and efficient numerical implementation of transitions between the flow domains. The talk is focused on the development and analysis of mass conservative multi-time stepping algorithms to simulate such coupled systems in environmental applications.
The purpose of this work is the design and analysis of a reliable and efficient a posteriori error estimator for the so-called pointwise tracking optimal control problem. This linear-quadratic optimal control problem entails the minimization of a cost functional that involves point evaluations of the state, thus leading to an adjoint problem with Dirac measures on the right hand side; control constraints are also considered. The proposed error estimator relies on a posteriori error estimates in the maximum norm for the state and in Muckenhoupt weighted Sobolev spaces for the adjoint state. We present an analysis that is valid for two and three-dimensional domains. We conclude by presenting several numerical experiments which reveal the competitive performance of adaptive methods based on the devised error estimator.
A PDE APPROACH TO THE FRACTIONAL OBSTACLE PROBLEM

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We study solution techniques for the elliptic and parabolic obstacle problem with fractional diffusion. The fractional diffusion operator is realized as the Dirichlet-to-Neumann map of a nonuniformly elliptic problem posed on a semi-infinite cylinder. This allows us to localize the problem and consider instead a thin obstacle problem. We present, for the elliptic case, optimal error estimates based on recent regularity results. For the parabolic case we present an error analysis with minimal smoothness and one using the best regularity results available to date.
We present a partitioned algorithm for fluid-shell interaction analysis using the finite element method (FEM) considering large structural displacements. The shell structure is modeled according to the Reissner-Mindlin kinematics, allowing thick shells modeling, and its FEM formulation is written with nodal positions and components of an unconstrained vector as degrees of freedom instead of displacements and rotations, avoiding problems related to large rotations approximations. Newmark time integrator is used for the structure and reveals to be stable and to present momentum conserving properties and enough energy conservation for most of the problems. The fluid governing equations are written in the arbitrary Lagrangian-Eulerian (ALE) description and solved by an implicit time integrator algorithm with mixed FEM approach for the incompressible cases and by one explicit characteristic based time integrator and standard finite elements for the compressible cases. The fluid-shell coupling is performed by a partitioned explicit Dirichlet-Neumann algorithm and the fluid mesh is updated by using a linear Laplacian smoothing. In order to save computing time and avoid element inversion in the Laplacian smoothing scheme, we introduce a coarse higher order auxiliary mesh which we call space mesh and use it only to capture the structural deformation and extend it to the fluid domain. Finally, the methodology is tested by numerical examples.
Materials such as ferromagnets, liquid crystals, and granular media involve orientation degrees of freedom. Mathematical descriptions of such materials involve fields of nonlinear objects such as unit vectors, rotations matrices, or unitary matrices. Classical numerical methods like the finite element method cannot be applied in such situations, because linear and polynomial interpolation is not defined for such nonlinear objects. Instead, a variety of heuristic approaches is used in the literature, which are difficult to analyze rigorously. We present nonlinear generalizations of the finite element method that allow to treat problems with orientation degrees of freedom in a mathematically sound way. This allows to show solvability of the discrete problems, makes the construction of efficient solvers easier, and allows to obtain reliable bounds for the finite element approximation error. We use the technique to calculate stable configurations of chiral magnetic skyrmions, and wrinkling patterns of a thin elastic polyimide film.
In this paper we discuss a discontinuous finite volume method for the approximation of distributed optimal control problem governed by the Brinkman equations written in terms of velocity and pressure. An additional force field is sought such that it produces a velocity matching a desired, known value. The discretization of state and co-state velocity and pressure fields follows a lowest order discontinuous finite volume scheme, whereas three different approaches are used for the control approximation: variational discretization, element-wise constant, and element-wise linear functions. We employ the optimize-then-discretize approach to approximate the control problem, and the resulting discretized formulation is non-symmetric. We derive a priori error estimates for velocity, pressure, and control in natural norms. A set of numerical examples is finally presented to illustrate the performance of the method and to confirm the predicted accuracy of the state, co-state and control approximations under various scenarios including 2D and 3D cases.
We present a Balancing Domain Decomposition by Constraints (BDDC) preconditioner for the model of cardiac mechanics. The contraction-relaxation process of the cardiac muscle, induced by the spread of the electrical excitation, is quantitatively described by a mathematical model called electro-mechanical coupling. The electric model consists of a non-linear degenerate parabolic system of two partial differential equations (PDEs), the so-called Bidomain model, which describes the spread of the electric impulse in the heart muscle. The PDE is coupled with the non-linear elasticity system, where the myocardium is considered as a nearly-incompressible transversely isotropic hyperelastic material. The discretization of the whole electro-mechanical model is performed by Q1 finite elements in space and a semi-implicit finite difference scheme in time. This approximation strategy yields at each time step the solution of a large scale linear system deriving from the discretization of the Bidomain model and a non-linear system deriving from the discretization of the finite elasticity equations. The parallel mechanical solver consists of solving the non-linear system with a Newton-Krylov-BDDC method, with different choices of coarse spaces. Three-dimensional parallel numerical tests on a Linux cluster show that the parallel solver proposed is scalable and quasi-optimal. Simulations based on the solver developed are performed to study the reliability of extracellular markers of repolarization in presence of domain deformations.
NGS-PY: A NATURAL LANGUAGE FOR HP-FEM IN MULTIPHYSICS

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Netgen/NGSolve is open source high order finite element code which provides a variety of scalar, vector-valued and tensor-valued hp-finite element spaces as needed for simulation in multiphysics. The design is object oriented, where grid-functions, bilinear- and linear forms, preconditions are C++ objects. NGSolve has a variety of built-in integrators for the classical equations and different discretisation concepts.

We present the recent redesign NGS-Py, where the C++ objects are accessible from the Python scripting language. Furthermore, equations can be provided symbolically in variational formulation, similar to the popular FEniCS system. This renders the variety of hand-written integrators obsolete. We explain in detail how element matrix calculation is now implemented, and how performance compares to the hand-written C++ code.

We show several examples demonstrating the flexibility of the interface, and the obtained performance for system assembly and solver parts.

We think this tool is in particular useful for algorithm development for multiphysics problems. The software and documentation is available from https://gitlab.asc.tuwien.ac.at/jschoeberl/ngsolve-docu/wikis/ngspy
The numerical solution of wave propagation problems requires discretizations in space and time. Latest since the great success of Discontinuous Galerkin methods it is accepted that adaptive space-time methods are preferable against time stepping techniques. In the context of Boundary Element Methods (BEM) space-time methods are used from the beginning on [Mansur(1983)]. Using a constant time step size results in a lower triangular Toeplitz system for the discretized retarded potentials. Hence, the complexity in time is linear. Also the convolution quadrature method (CQM) in its initial form requires a constant time step size [Lubich(1988)], which results as well in a linear complexity in time.

A variable time step size for BEM has been proposed by [Sauter and Veit(2013)] using a global shape function in time and by [Lopez-Fernandez and Sauter(2013)] with a generalized convolution quadrature method. The latter approach shares all benefits of the original CQM but allows a variable time step size. The complexity in time is $\mathcal{O}(N \log N)$. This approach is used in this presentation to formulate a BE formulation for acoustics and elastodynamics. Numerical studies will show the behaviour of this formulation with respect to temporal discretization. The formulation will be based on a collocation approach in space.

References


NEW MIXED FEMS FOR THE BIHARMONIC EQUATION
BASED ON THE HELMHOLTZ DECOMPOSITION

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The non-conforming Morley finite element method (FEM) for the biharmonic equation seems to be the simplest discretization for the Kirchhoff plate from structural mechanics. A common criticism is that this non-conforming FEM does not come in a natural hierarchy. This talk generalizes the non-conforming FEM of Morley to higher polynomial degrees. The crucial point is to reformulate the problem in a proper mixed formulation with the help of a Helmholtz decomposition which decomposes an unstructured symmetric tensor field into a Hessian and a symmetric curl. The inherent integral mean property of the non-conforming interpolation operator of the Morley FEM is preserved.

The approach can naturally be generalized to arbitrary $m$th-Laplace equations of the form $(-1)^m \Delta^m u = f$ for arbitrary $m = 1, 2, 3, \ldots$

Besides the a priori and a posteriori analysis, the talk presents optimal convergence rates for adaptive algorithms for the new discretizations.
We study the discontinuous Galerkin time discretization (dG(k)-method) for the transient Stokes problem which is discretized in space by means of an inf-sup stable pair of finite element spaces \((V_h, Q_h)\) for velocity and pressure, respectively. Here, the fully discrete solution \((u_h(t), p_h(t))\) on each time interval is a polynomial in time of order \(k\) with values in the finite element product space \(V_h \times Q_h\). By means of a simple post-processing step we can compute in a very inexpensive way a lifted solution \((\tilde{u}_h(t), \tilde{p}_h(t))\) which is globally continuous in time and a polynomial of order \(k + 1\) on each time interval. For this approximation \((\tilde{u}_h(t), \tilde{p}_h(t))\), we prove an optimal estimate for the velocity error in \(L^2(L^2)\) of the higher order in time \(\tau^{k+2} + h^{r+1}\), where \(\tau\) denotes the time step size, \(h\) the mesh size and \(r\) the polynomial degree for the velocity approximation in \(V_h\). Moreover, we prove an optimal \(L^2(L^2)\) estimate for the pressure error of the order \(\tau^{k+2} + h^r\), where the polynomial degree for the pressure approximation in \(Q_h\) is \(r - 1\) due to the inf-sup condition. Key ingredients of the analysis are a special higher order interpolate in time of the exact solution and a special stability estimate for the lifted velocity error (for both see \([1]\)) applied in the discretely divergence free subspace of \(V_h\) as well as the proof of superconvergence of the error in the time derivative for the velocity. We present some numerical results which confirm the theoretical error bounds.

References


ANALYSIS OF THE ENSEMBLE KALMAN FILTER FOR INVERSE PROBLEMS

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The ideas from the Ensemble Kalman Filter introduced by Evensen in 1994 can be adapted to inverse problems by introducing artificial dynamics. In this talk, we will discuss an analysis of the EnKF based on the continuous time scaling limits, which allows to derive estimates on the long-time behavior of the EnKF and, hence, provides insights into the convergence properties of the algorithm. In particular, we are interested in the properties of the EnKF for a fixed ensemble size, in order to better understand current practice, and to suggest future directions for development of the algorithm. Results from various numerical experiments supporting the theoretical findings will be presented.

ASYMPTOTIC EXPANSION TECHNIQUES FOR SINGULARLY PERTURBED BOUNDARY INTEGRAL EQUATIONS

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We will consider singularly perturbed elliptic transmission problems in the framework of boundary integral equations and boundary element Galerkin discretisations [1]. For this we demonstrate the use of asymptotic expansion techniques both for establishing regularity results for the solution and for deriving a priori error estimates for boundary element discretisation. The dependence of the corresponding bounds on the singular perturbation parameter is studied in detail. This dependence clearly manifests itself in numerical experiments.

References

Mortar element methods use a decomposition of the computational domain and couple different discretization spaces in the subdomains weakly by a mortar condition. We use for example a high-order mortar element method for full-potential electronic structure calculations [1]. For this we use a spherical discretization in spherical elements around each nucleus, which is adapted to resolve the core singularity due to an unbounded potential term, is coupled to a finite element discretization in between the nuclei. We discuss the error of the mortar element method with uniform refinement as well as the reliability of a residual error estimator. With a series of numerical experiments we illustrate the theoretical convergence results for uniform refinement also in comparison with a conforming $hp$-adaptive finite element method and a $p$-adaptive refinement strategy based on the residual error estimator.

References

Mixed methods based on the introduction of gradient or stress fields as additional unknowns in $H(\text{div})$-spaces are well-established. They are available for variational equations [1] as well as variational inequalities [2]. In these methods, the discretization of the $H(\text{div})$-space necessitates continuity in the normal direction of the edges of the underlying mesh. Usually, Raviart-Thomas finite elements are used in order to guarantee this continuity condition. Alternatively, one can also apply mixed-hybrid methods where additional Lagrange multipliers on the edges are introduced to enforce the desired continuity.

In this talk, we discuss reliable error estimates and adaptivity of $hp$-adaptive finite elements for mixed and mixed-hybrid methods. In particular, we consider the Poisson problem and the obstacle problem leading to a variational equation and a variational inequality, respectively. The mixed-hybrid approach enables the use of tensor product shape functions based on Lagrange polynomials for all fields and, thus, an effective implementation of assembling routines (numerical integration, static condensation, parallelization) for quadrilateral or hexahedral mesh elements with varying polynomial degree distribution and (multilevel) hanging nodes. The basic idea of the a posteriori error control is to reconstruct the solution of the primal variable in the $H^1$-space so that error controls for $H^1$-conforming finite elements can be applied [3]. The reconstruction can be done globally, but also locally in many cases. Several numerical examples confirm the applicability of the proposed techniques within $hp$-adaptive refinements.

References


WE CONSIDER THE GALERKIN-FEM FOR THE INCOMPRESSIBLE AND NON-ISOTHERMAL NAVIER-
STOKES EQUATIONS:

\[
\begin{align*}
\partial_t \mathbf{u} - \nu \Delta \mathbf{u} + (\mathbf{u} \cdot \nabla) \mathbf{u} + \nabla p &= f_u(\theta), \\
\nabla \cdot \mathbf{u} &= 0, \\
\partial_t \theta - \lambda \Delta \theta + \mathbf{u} \cdot \nabla \theta &= f_\theta
\end{align*}
\]

with the Oberbeck-Boussinesq assumption \(f_u(\theta) = \beta (\theta - \theta_{ref}) g\). More precisely, for the discrete velocity and pressure spaces \(V_h \times Q_h\), we apply Scott-Vogelius element pairs \([P_{k+1}^d] \times P^{-k}\) with \(k \geq d\) on barycentrically refined simplicial meshes which are known to satisfy the constraint \([2]\) even pointwise. The discrete temperature space \(W_h\) consists of \(P_{k+1}^d\)-elements.

In case of \(\mathbf{u} \in L^\infty(0,T;\mathcal{W}^{1,\infty}(\Omega)^d)\) and \(\theta \in L^\infty(0,T;\mathcal{W}^{1,\infty}(\Omega))\), we extend the semi-robust semidiscrete error estimates of \([1]\) to the given case. In particular, we point out the gain of pressure-robust schemes in the sense of \([2]\). Finally, we briefly discuss extensions to finite-element pairs \(V_h \times Q_h\) with \(\nabla \cdot V_h \subseteq Q_h\).

References


NON-STATIONARY ADVECTION-DIFFUSION PROBLEMS IN NETWORKS OF FRACTURES WITH AN OPTIMIZATION APPROACH

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Time-dependent advection-diffusion problems in large discrete fracture networks (DFN) are addressed via an optimization-based approach. In DFN models, underground fractures are modelled individually and are represented by planar polygons intersecting each other in the three dimensional space and forming an intricate network resembling the fracture-system in the underground. Fracture geometrical data and hydraulic properties are only known through probability distribution functions, tuned through sampling and testing on specific geological sites. DFN models are particularly well suited for the simulation of transport phenomena in which the directionality of the flow is of paramount importance but some difficulties are addressed to perform effective simulations. We have: geometrical complexities, as the generation of a mesh suitable for finite elements and conforming to interfaces (i.e. fracture intersections) on intricate networks of fractures often results infeasible or leads to poor quality elements; the multiscale nature of the problem, due to the simultaneous presence of large geological entities (as, e.g., faults) and very small fractures; domain size with networks for practical applications counting up to millions of fractures; and uncertainty in input data.

A solution to the above mentioned issues is proposed, based on a PDE constrained optimization method \cite{2, 4, 3}. The method allows for an independent mesh generation on each fracture of the network, resorting to the minimization of a cost functional to enforce conditions at the interfaces on the non-conforming meshes. In such a way the mesh can be adjusted locally to fit with the scale of each fracture. The method can also be readily implemented in parallel computers, thus effectively handling problem dimensions. Thanks to the robustness of the method stochastic analyses considering randomness in DFN data are performed, also exploiting modern uncertainty quantification techniques \cite{1}.

References


THE GLT CLASS AS A GENERALIZED FOURIER ANALYSIS AND APPLICATIONS

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Recently, the class of Generalized Locally Toeplitz (GLT) sequences has been introduced \cite{1,2} as a generalization both of classical Toeplitz sequences and of variable coefficient differential operators and, for every sequence of the class, it has been demonstrated that it is possible to give a rigorous description of the asymptotic spectrum \cite{1,7} in terms of a function (the symbol) that can be easily identified. This generalizes the notion of a symbol for differential operators (discrete and continuous) or for Toeplitz sequences for which it is identified through the Fourier coefficients and is related to the classical Fourier analysis.

The GLT class has nice algebraic properties and indeed it has been proven that it is stable under linear combinations, products, and inversion when the sequence which is inverted shows a sparsely vanishing symbol (sparsely vanishing symbol = a symbol which vanishes at most in a set of zero Lebesgue measure). Furthermore, the GLT class virtually includes any approximation of partial differential equations (PDEs) by local methods (finite difference, finite element, isogeometric analysis, etc.) and, based on this, we demonstrate that our results on GLT sequences can be used in a PDE setting in various directions, including preconditioning, multigrid, spectral detection of branches, stability issues. We will discuss specifically the spectral potential of the theory with special attention to the IgA setting \cite{2,3,4}.

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\end{enumerate}


Viscoelastic media such as polymers and biotissue are dispersive and are usually described by a hereditary constitutive law. The physically reasonable assumption of fading memory in these problems makes it possible to derive stability and error bounds which are ‘sharp’ in so much as they can be derived without recourse to Gronwall’s inequality. This means that they do not contain an exponential growth in time, and this provides some confidence in the quality of long-time simulations.

An example of this type of result will be given for a high order space-time Galerkin finite element method (continuous in space; discontinuous in time) for a dynamic linear solid viscoelasticity problem. This problem is of interest to us because, in a proof-of-concept project, we as a multidisciplinary group are aiming to model the passage of shear waves from the wall of a diseased coronary artery to the chest surface. Our long term aim is a relatively cheap and non-invasive screening or diagnostic device, based on solving the inverse problem, for coronary artery disease.

Within the context of that project we have followed the heat equation formulations in [Werder et al., Comput. Methods Appl. Mech. Engrg., 190:6685—6708, 2001] and developed a time diagonalised space-time finite element solver for the viscodynamic wave equation. This approach allows for both coarse and fine grained parallelism, and high degree polynomial approximation in both space and time. This formulation will be illustrated for the simpler case of the acoustic wave equation in order to describe the main points.

Surprisingly, perhaps, Maxwell’s equations for a Debye media have at a high enough level of abstraction essentially the same structure as those for viscodynamics. The same type of sharp estimates will be illustrated, for finite difference time discretization, for this application along with some further results for Lorentz media. Difficulties in extending the space-time Galerkin formulation (as above) for these materials, as well as for the Drude model for metamaterials, will be touched upon.

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Various aspects of this material are joint work with any or all of the following: SE Greenwald (QMUL); MJ Birch, MP Brewin (Barts and the London NHS Trust); HT Banks, ZR Kenz, S Hu (NC State); J Li (UNLV); C Kruse and JR Whiteman (Brunel).
A HYBRIDIZABLE DISCONTINUOUS GALERKIN METHOD FOR THE $p$-LAPLACIAN

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We propose the first hybridizable discontinuous Galerkin (HDG) method for the $p$-Laplacian equation. When using polynomials of degree $k \geq 0$ for the approximation spaces of $u$, $\nabla u$, and $|\nabla u|^{p-2}\nabla u$, the method exhibits optimal $k+1$ order of convergence for all variables in $L^1$- and $L^p$-norms in our numerical experiments. For $k \geq 1$, an element-wise computation allows us to obtain a new approximation $u_h^*$ that converges to $u$ with order $k+2$. We rewrite the scheme as discrete minimization problems in order to solve them with nonlinear minimization algorithms. The unknown of the first problem is the approximation of $u$ on the skeleton of the mesh but requires solving nonlinear local problems. The second problem has the approximation on the elements as an additional unknown but it only requires solving linear local problems. We present numerical results displaying the convergence properties of the methods, demonstrating the utility of using frozen-coefficient preconditioners, and indicating that the second method is superior to the first one even though it has more unknowns.
The use of computational tools in industrial flow simulations is well established. As engineering design continues to evolve and become ever more complex there is an increasing demand for more accurate transient flow simulations. It can, using existing methods, be extremely costly in computational terms to achieve sufficient accuracy in these simulations. Accordingly, advanced engineering industries, such as the Formula One (F1) industry, are looking to academia to develop the next generation of techniques which may provide a mechanism for more accurate simulations without excessive increases in cost.

This demand for modelling of accurate flow physics around complex geometries are therefore making high order methods such as spectral/hp type discretisations more attractive to industry. Nevertheless a number of challenges still exist in translating academic tools into engineering practice. As the start of the pipeline, meshing techniques for high order methods are required to handle highly complex geometries. Next many engineering problems require high Reynolds numbers leading to turbulent flow that typically are only marginally resolved. Therefore, there is a need for greater robustness in marginally resolved conditions where aliasing errors and high frequency damping are typically required. Finally maintaining computational efficiency is also obviously important.

In this presentation we will outline the demands imposed on computational aerodynamics within the highly competitive F1 sector and discuss the numerical challenges which have to be overcome to translate academic tools into this environment.
Recently in a series of papers, we developed a class of reduced order HDG methods for various linear and nonlinear problems. A main feature of this approach is to apply different polynomial spaces for the unknowns. It was first discovered in 2009 by Lehrenfeld in his thesis for diffusion problem. Under the standard HDG framework, if we apply $P_{k+1}$ polynomial space for the pressure while we still use $P_k$ spaces for the other two unknowns, by a simple modification of the numerical flux we can obtain optimal order of convergence for all unknowns. The analysis is valid for general polygonal meshes. In this talk, we will present this general framework for linear elasticity, convection-diffusion and steady Navier-Stokes equations.
ADAPTIVE ALGORITHMS DRIVEN BY A POSTERIORI
ESTIMATES OF ERROR REDUCTION
FOR PDES WITH RANDOM DATA

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An efficient adaptive algorithm for computing stochastic Galerkin finite element approximations of elliptic PDE problems with random data will be outlined in this talk. The underlying differential operator will be assumed to have affine dependence on a large, possibly infinite, number of random parameters. Stochastic Galerkin approximations are sought in a tensor-product space comprising a standard $h$-finite element space associated with the physical domain, together with a set of multivariate polynomials characterising a $p$-finite-dimensional manifold of the (stochastic) parameter space.

Our adaptive strategy is based on computing distinct error estimators associated with the two sources of discretisation error. These estimators, at the same time, will be shown to provide effective estimates of the error reduction for enhanced approximations. Our algorithm adaptively ‘builds’ a polynomial space over a low-dimensional manifold of the infinite-dimensional parameter space by reducing the energy of the combined discretisation error in an optimal manner. Convergence of the adaptive algorithm will be demonstrated numerically.
This contribution is concerned with a new numerical method to solve the elasticity problem for solids in boundary representation. Starting with the basic idea of the scaled boundary finite element method [Song, C. and Wolf, J.P.: The scaled boundary finite-element method for elastodynamics, CMAME 147 (1997)], we derive a formulation where the geometrical description of the boundary is sufficient for defining the equations of elasticity of the complete solid. This approach fits perfectly to the boundary representation modeling technique ('b-rep') commonly employed in computer aided design. For the analysis, the weak form of the equilibrium equations is first enforced for the circumferential direction. Applying the isogeometric paradigm, the NURBS functions that describe the boundary of the geometry form also the basis for the approximation of the displacement at the boundary. The displacement field in the radial scaling direction, on the other hand, is approximated by one-dimensional NURBS, and here we have the choice of using again a weak form and Galerkin projection or, alternatively, collocation. Overall, this procedure yields a linear system of equilibrium equations whose solution gives rise to the displacement response.

In the talk, the relation of this approach to the classical concept of Isogeometric Analysis is analyzed. Moreover, computational results from the recent paper [Chen, L., Simeon, B. and Klinkel, S.: A NURBS based Galerkin approach for the analysis of solids in boundary representation. To appear in CMAME, DOI:10.1016/j.cma.2016.03.019] are reported. Finally, we discuss also issues such as the treatment of the singularity in the scaling center and the restriction of star-shaped domains.

This work is supported by the European Union within the Horizon 2020 project MOTOR – Multi-ObjecTive design Optimization of fluid eneRgy machines.
A WAY TO IMPROVE THE SOLUTION OF LOCAL PROJECTION STABILIZATION

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Composite finite elements and novel postprocessing based on the local $L_2$ projection are proposed in order to improve the solution of standard one-level Local Projection Stabilization (LPS) on quadrilateral meshes, cf. [1,2,3]. Theoretical results are justified by several tests for convection-dominated problems in two dimensions. Numerical results show that the discrete solution is oscillation-free and of optimal accuracy in the regions away from the boundary layer whereas the spurious oscillations are significantly reduced near the boundary layers when the postprocessing is applied.

References


From earlier investigations it is well known that some composite materials can provide superior properties compared to their virgin monolithic constituent materials (Ryu et al., 2002). Remarkably larger magnetoelectric (ME) effect is observed in composites as compared to those in either composite constituents (Nan, 1994). The ME effect is intensively studied to utilize it for energy conversion between the magnetic and electric fields and the ME memory elements, smart sensors and transducers (Wang et al., 2005). Pan and Wang (2009) showed larger ME effect in layered composites than in monoliths. Applied magnetic field induces strain in the magnetostrictive constituent of the bilayer multiferroic composite; this is passed on to the piezoelectric constituent, where it induces an electric polarization. They observed a strong ME effect in artificially fabricated multiferroic composites. The thickness ratio of piezomagnetic and piezoelectric layers has influence on the ME effect (Laletin et al., 2008).

Coating of a base material plays an important role in various engineering applications. Enhancement of ME coupling for fibrous magnetoelectroelastic composites seems to be a great challenge for research. The subject of piezoelectric/piezomagnetic fibrous composites with multicoated circular/elliptic fibres is seldomly analyzed in literature. These composites are analyzed only under anti-plane shear deformation (Kuo and Pan 2011). In the present paper influence of coating layer on the ME coefficient in fibrous composites with piezoelectric and piezomagnetic phases is investigated. The effective material parameters are computed on the base of homogenization techniques performed on the RVE. The solution of general boundary value problems for coupled multi-field problems requires advanced numerical methods due to the high mathematical complexity. Such a multi-field problem is described by a system of partial differential equations because of the interactions among the magnetic, electric and mechanical fields involved in the constitutive equations. In this paper the finite element method (FEM) is implemented for investigation of coating layer on effective material properties, particularly the so-called ME effect which is important to optimal design of multiferroics composites.

References


A POSTERIORI ERROR ESTIMATES FOR HIGHER-ORDER TIME DISCRETIZATIONS

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We present equilibrated flux guaranteed a posteriori error estimates with respect to the $L^2(H^1) \cap H^1(H^{-1})$ and $L^2(H^1)$ parabolic energy norms for fully discrete schemes for the heat equation based on high-order conforming FEM in space and high-order discontinuous Galerkin methods in time. Extending the ideas in \cite{ern2010} to high-order methods, the equilibration is obtained by solving, for each timestep, local mixed FEM problems posed on the patches of the current mesh. We further show that the error estimates are locally efficient with respect to the space-time local $L^2(H^1) \cap H^1(H^{-1})$-error and temporal jumps, and, building on \cite{braess2009, ern2013}, we establish full robustness with respect to both the temporal and spatial polynomial degrees, thus making the estimates well-suited for high-order schemes. In the practically relevant situation where the time-step size $\tau \gtrsim h^2$ the mesh-size, the spatial estimators are in addition locally efficient with respect to the space-time local $L^2(H^1)$-error and temporal jumps.

References


Devices containing a periodically corrugated metallic backreflector have become of interest since surface gratings are able to enhance the electromagnetic field due to the excitation of multiple surface plasmon polariton waves. Design of this type of structure requires a rapid and reliable way to simulate the optical characteristics for wide ranges of wavelength and angle of incidence.

Recently, several simulations of wave-guide concentrators and solar cells ([1, 2, 3]) have been performed using two different numerical methods: the rigorous coupled-wave approach (RCWA) and the finite element method (FEM). In this work we compare the performance of these methods. RCWA is fast and flexible, but FEM has predictable convergence even for discontinuous constitutive properties.

On the other hand, for devices involving shallow-surface relief gratings, we numerically test the accuracy of an asymptotic model which replaces the shallow grating by a planar interface with suitable transmission conditions ([5]).

References


ANOMALOUS DIFFUSION WITH RESETTING

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We consider a fractional partial differential equation that describes the diffusive motion of a particle, performing a random walk with Lévy distributed jump lengths, on one dimension with an initial position $x_0$. The particle is additionally subject to a resetting dynamics, whereby its diffusive motion is interrupted at random times and is reset to $x_0$. A numerical method is presented for this diffusive problem with resetting. The influence of resetting on the solutions is analysed and physical quantities such as pseudo second order moments and pseudo fractional order moments will be discussed. Some comments about what happens in the presence of boundaries will be also included. This talk is based on joint work with Amal K. Das from Dalhousie University (Canada).

QUASI-INTERPOLANTS AND LOCAL APPROXIMATION ESTIMATES FOR HIERARCHICAL SPLINE SPACES

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Hierarchical spline spaces provide a flexible framework for local refinement coupled with a remarkable intrinsic simplicity. They are defined in terms of a hierarchy of locally refined meshes, reflecting different levels of refinement. The so-called truncated hierarchical basis is an interesting basis for the hierarchical spline space with an enhanced set of properties compared to the classical hierarchical basis: its elements form a convex partition of unity, they are locally supported and strongly stable [1, 2].

In this talk we discuss a general approach to construct quasi-interpolants in hierarchical spline spaces expressed in terms of the truncated hierarchical basis [3, 4]. The main ingredient is the property of preservation of coefficients of the truncated hierarchical basis representation. Thanks to this property, the construction of the hierarchical quasi-interpolant is basically effortless. It is sufficient to consider a quasi-interpolant in each space associated with a particular level in the hierarchy, which will be referred to as a one-level quasi-interpolant. Then, the coefficients of the proposed hierarchical quasi-interpolant are nothing else than a proper subset of the coefficients of the one-level quasi-interpolants. No additional manipulations are required. Important properties – like polynomial reproduction – of the one-level quasi-interpolants are preserved in the hierarchical construction. We also discuss the local approximation order of the hierarchical quasi-interpolants in different norms, and we illustrate the effectiveness of the approach with some numerical examples.
We consider the Helmholtz equation with variable wavenumber, i.e.

\[ \Delta u + \kappa^2 nu = f \]

where \( \kappa > 0 \) is a constant and \( n \) (the refractive index) is a function of position. Under a condition on \( n \) (which has a natural interpretation as a non-trapping condition), we prove bounds that are explicit in \( \kappa \), \( n_{\min} \), and \( n_{\max} \) on the solution of the following Helmholtz boundary value problems:

1. the interior impedance problem when the 2- or 3-d domain is Lipschitz and star-shaped with respect to a ball,
2. the exterior Dirichlet problem when the 2- or 3-d obstacle is Lipschitz and star-shaped,
3. the exterior Neumann problem when the 2-d obstacle is \( C^2 \) and has strictly positive curvature.

The bounds in 1 and 2 are sharp in their \( \kappa \) dependence, whereas the bound in 3 is \( \kappa^{2/3} \) away from being sharp.
I will show that for realistic simulations (with heterogeneous materials for instance) convergence of domain decomposition methods becomes very slow. Then I will explain how this can be fixed by injecting more information into the solver. In particular, robustness can be achieved by using multiple search directions within the conjugate gradient algorithm. Efficiency is also taken into account since our solvers are adaptive.

This work is a particular application of the adaptive multipreconditioned conjugate gradient algorithm [2, 1].

References


LOCAL PROJECTION STABILIZATION WITH DISCONTINUOUS
GALERKIN METHOD IN TIME APPLIED TO TRANSIENT
SCALAR EQUATION IN TIME DEPENDENT DOMAINS

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In this work, a stabilized finite element scheme combined with the discontinuous
Galerkin (dG) discretization in time for the solution of a transient convection diffusion
reaction equation in a time-dependent domain is analyzed. In particular, the local
projection stabilization (LPS) and the higher order dG in time discretization is con-
sidered. Further, arbitrary Lagrangian Eulerian (ALE) is used to handle the domain
deformation. The stability and error estimates are presented for the proposed numeri-
cal scheme.

The stabilization term in local projection method is based on a projection \(\pi_h : V_h \rightarrow D_h\)
of finite element approximation space \(V_h\) into a discontinuous space \(D_h\). LPS was orig-
inally given as a two level method in which projection space \(D_h\) lies on a coarser grid,
but this approach increases the discretization stencil \[1\]. In this work, we use the one
level approach, in which the approximation space \(Y_h\) and projection space \(D_h\) are de-
defined on the same mesh, with enrichment of the approximation space \(Y_h\) \[3\].

The analysis is mainly based on a quadrature formula for approximating the integrals
in time. We consider exact integration in time which is impractical to implement,
and the Radau quadrature in time, which can be used in practical \[2\]. The stability
and error estimates are given for both the time integration methods. Analysis reveals
that the numerical scheme for exact in time integration is unconditionally stable, while
Radau quadrature in time is conditionally stable only with time step restriction de-
pending on ALE map. The validation of the proposed local projection stabilization
scheme with higher order discontinuous Galerkin time discretization is demonstrated
with appropriate numerical examples.

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GUARANTEED AND ROBUST A POSTERIORI BOUNDS FOR LAPLACE EIGENVALUES AND EIGENVECTORS

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In this talk we present a posteriori error estimates for conforming numerical approximations of the Laplace eigenvalue problem with a homogeneous Dirichlet boundary condition. In particular, upper and lower bounds for the first eigenvalue are given. These bounds are guaranteed, fully computable, and converge with the optimal speed to the exact eigenvalue. They are valid under an explicit, a posteriori, minimal resolution condition on the computational mesh and the approximate solution; we also need to assume that the approximate eigenvalue is smaller than a computable lower bound on the second smallest eigenvalue, which can be satisfied in most cases of practical interest by including the computational domain into a rectangular parallelepiped or a d-sphere. Guaranteed, fully computable, and polynomial-degree robust bounds for the energy error in the approximation of the first eigenvector are derived as well, under the same conditions. Remarkably, there appears no unknown (solution-, regularity-, or polynomial-degree-dependent) constant in our theory, and no convexity/regularity assumption on the computational domain/exact eigenvector(s) is needed.
A PARTITION-OF-UNITY BOUNDARY ELEMENT METHOD WITH SPACE-TIME ENRICHMENT FOR THE WAVE EQUATION

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This talk considers a time-domain partition-of-unity boundary element method for wave propagation problems at high frequency. Travelling plane waves are included as enrichment functions into a space-time boundary element Galerkin scheme. We present some first numerical experiments with this method for high-frequency scattering problems in $\mathbb{R}^3$, discuss algorithmic aspects and comment on relevant engineering applications.

FIRST-ORDER SYSTEM $LL^*$ USING NONCONFORMING TEST FUNCTIONS

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The first-order system $LL^*$ formulation is based on the ultra weak formulation

$$\langle U, L^*V \rangle = F(V) \quad \forall V$$

of some first-order system of differential equations $LU = F$ and closely related to the DPG methodology. It is obtained by setting $U = L^*W$ with $W$ being in the test space, therefore leading to a self-adjoint coercive variational problem. We consider the $H(\text{div}) \times H^1$ first-order system $LL^*$ formulation studied in [Z. Cai, R. Falgout and S. Zhang, SIAM J. Numer. Anal. 53 (2015), 405–420] for Poisson-type equations. The local conservation properties of the method using next-to-lowest-order Raviart-Thomas spaces for $H(\text{div})$ combined with quadratic nonconforming elements for $H^1$ are investigated in this contribution. This will also be discussed in the context of conservation of momentum in a stress-velocity formulation of the Stokes system.
We present an adaptive hp-finite element algorithm. It consists of iterating two routines: hp-NERBEST finds a near-best hp-approximation of the current discrete solution and data to a desired accuracy, and REDUCE improves the discrete solution to a finer but comparable accuracy. The former hinges on a recent algorithm by P. Binev for adaptive hp-approximation, and acts as a coarsening step. We prove convergence and instance optimality. For controlling the computational cost, we present results on saturation that are uniform in $p$. 
A STABLE DPG FORMULATION OF TRANSPORT EQUATIONS

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We formulate and analyze a Discontinuous Petrov Galerkin formulation of linear transport equations with variable convection fields. We show that a corresponding infinite dimensional mesh-dependent variational formulation, in which besides the principal field also its trace on the mesh skeleton is an unknown, is uniformly stable with respect to the mesh, where the test space is a certain product space over the underlying domain partition.

Our main result states then the following. For piecewise polynomial trial spaces of degree $m$, we show under mild assumptions on the convection field that piecewise polynomial test spaces of degree $m + 1$ over a refinement of the primal partition with uniformly bounded refinement depth give rise to uniformly (with respect to the mesh size) stable Petrov-Galerkin discretizations.

Finally we show how rigorously computable a posteriori error bounds can drive a convergent adaptive algorithm.
Space-time discretization methods require a well-posed space-time variational formulation. Such formulations are well-known for parabolic problems. The (Navier)-Stokes equations can be viewed as a parabolic problem for the divergence-free velocities. Yet to avoid the cumbersome construction of divergence-free trial spaces, we present well-posed variational formulations for the saddle-point problem involving the pair of velocities and pressure. We discuss adaptive wavelet methods for the optimal adaptive solution of simultaneous space-time variational formulations of evolutionary PDEs. Thanks to use of tensor products of temporal and spatial wavelets, the whole time evolution problem can be solved at a complexity of solving one instance of the corresponding stationary problem.

Modelling surface active agents (surfactants) in multi-phase flow leads to coupled bulk and interface advection-diffusion equations. Aiming for a phase field description we present a Cahn-Hilliard-Navier-Stokes system which is coupled to suitable PDEs for surfactant(s). The challenge is to correctly recover the conditions in the triple junctions if the fluid interfaces are given by thin layers rather than by hypersurfaces as in classical approaches. We will therefore focus on a diffuse interface approach to partial differential equations on evolving bubble clusters and network-like structures. Thanks to smoothing the problem by replacing the interfaces with thin layers we can use standard bulk finite elements though mesh adaptation is mandatory to make simulations feasible. Simulation results will be presented which support the theory behind the modelling approach.
CURVE SHORTENING FLOW COUPLED TO LATERAL DIFFUSION

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A semi-discrete finite element scheme for a system consisting of a geometric evolution equation for a curve and a parabolic equation on that evolving curve is presented. More precisely, curve shortening flow with a forcing term that depends on a conserved field is coupled with a diffusion equation for that field. Such a system can be considered as a prototype for more complicated problems as they may arise in applications. Our scheme is based on ideas of Dziuk for the curve shortening flow and Dziuk/Elliott for the parabolic equation on the moving curve. However, additional estimates particularly with respect to the time derivative of the length element are required. Numerical simulation results support the theoretical findings.
AN ADAPTIVE MIXED FINITE ELEMENT METHOD
FOR DARCY FLOW IN FRACTURED POROUS MEDIA

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In this work, we propose an adaptive mixed finite element method for simulating the single-phase Darcy flow in fractured porous media. The reduced model that we use for simulation is a discrete fracture model coupling Darcy flows in the matrix and the fractures, and the fractures are modeled by lower-dimensional fractures. The Raviart-Thomas mixed finite element methods are utilized for the solution of the coupled Darcy flows in the matrix and the fractures. In order to improve the efficiency of the simulation, we use adaptive mixed finite element method based on the residual-based \textit{a posteriori} error estimators. Several examples of Darcy flow in the fractured porous media are provided to demonstrate the robustness of the algorithm.
A POSTERIORI ERROR ESTIMATES
FOR THE VIRTUAL ELEMENT METHOD

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The virtual element method is a recently introduced method for approximating solutions to partial differential equations on meshes consisting of arbitrary (convex or non-convex) polygonal elements. We present a reliable and efficient residual-type a posteriori error estimator for the virtual element method applied to linear second-order elliptic problems, and demonstrate its effectiveness when used to drive an adaptive algorithm. In keeping with the usual virtual element methodology, the estimator is fully computable since it may be evaluated using just the degrees of freedom of the discrete space and element-wise projections onto the polynomial subspace. The flexibility of polygonal meshes offers many attractive features for mesh adaptation, such as the fact that hanging nodes can be handled with no mesh post-processing since elements with coplanar edges are allowed.

References


ENERGY-CORRECTION METHOD FOR
DIRICHLET BOUNDARY CONTROL PROBLEM

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Let Ω ⊂ ℜ² be a bounded polygonal domain with a re-entrant corner, i.e. corner with an angle Θ > π, with disjoint boundary parts Γ_D and Γ_C, satisfying ∂Ω = Γ_D ∪ Γ_C.

In this talk we consider the optimal Dirichlet control problem in the energy space [2]. This problem is defined as a minimization of the following tracking-type functional

\[ J(u, z) = \frac{1}{2} \| u - \overline{u} \|^2_{L^2(\Omega)} + \frac{\rho}{2} \| z \|^2_{H^\frac{1}{2}(\Gamma_C)}, \]

subject to the constraint

\[ -\Delta u = f \quad \text{in } \Omega, \]
\[ u = 0 \quad \text{on } \Gamma_D, \]
\[ u = z \quad \text{on } \Gamma_C, \]

and the control constraints

\[ z_a \leq z \leq z_b \quad \text{a.e. on } \Gamma_C. \]

We present the saddle-point structure of the problem and investigate the behaviour of the piecewise linear finite element approximation. Its convergence order is lower due to the reduced regularity in the presence of re-entrant corner. Recently, an effective method of recovering the full second-order convergence for elliptic equations on domains with re-entrant corners, when measured in locally modified L_2 and H^1 norms, known as energy-correction, has been proposed [1]. This method is based on a modification of a fixed number of entries in the system’s stiffness matrix. We show how energy-correction method can be successfully applied to regain optimal convergence in weighted norms for optimal control problems. All theoretical results are confirmed by numerical test.

References


A FINITE ELEMENT FORMULATION FOR MAXWELL EIGENVALUE PROBLEM USING CONTINUOUS LAGRANGIAN INTERPOLATIONS

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In this work, we consider the stabilized finite element formulation based on the subgrid scale concept for solving the Maxwell eigenvalue problem. The application of a stabilization technique based on a projection of the residual to an eigenproblem, leads to a system resulting in a quadratic eigenvalue problem. As a consequence, eigenpairs which are not solutions of the original problem are introduced, and a considerable increase in complexity of the problem is involved. In this study, the unresolved subscales are taken to be orthogonal to the finite element space. Thus, the components leading to a quadratic structure vanish, the residual is simplified, and the implementation of term by term stabilization is allowed. Moreover, the method allows the use of continuous Lagrangian interpolations. Apart from its novelty, we show that the approach is essential to establish the original structure of the eigenproblem. We present the problem formulation, and provide some numerical results from the solution of the Maxwell eigenvalue problem on two-dimensional regions. The numerical results we have obtained from the formulation described above, demonstrate a very good agreement with the previously published results.
In this talk, we will discuss approximation error estimates for B-splines of polynomial degree \( p \) and maximal smoothness \( p - 1 \), which are robust in the polynomial degree \( p \). We will see that there are large subspaces of the spline space satisfying a corresponding inverse estimate. One example is the space of splines whose odd derivatives vanish on the boundary. However, the inverse estimate does not extend to the whole spline space, i.e., there is a small subspace of outlier splines. For many numerical methods, it is important to have both, a robust approximation error estimate and a robust inverse estimate. We will discuss possibilities how a precise characterization of the outliers can be used for the construction of fast linear solvers for problems in isogeometric analysis.
We address the problem of numerically solving linear systems that arise in Isogeometric Analysis (IGA). It is known that many standard methods, when applied to IGA systems, have a computational cost which significantly increase with the degree $p$ of the splines employed as basis functions. This fact contributes in making high degree splines prohibitive for real world applications. As a consequence, a number of recent papers have attempted to reduce the dependence of computational cost from $p$.

Let $K$ be the stiffness matrix for the Poisson problem on an arbitrary physical domain. A simple but crucial observation is that $K$ can be preconditioned by the stiffness matrix for the unit hypercube, which has the form

$$P = \sum_{i=1}^{d} M_1 \otimes \ldots \otimes M_{i-1} \otimes K_i \otimes M_{i+1} \otimes \ldots \otimes M_d,$$

where $M_i$ and $K_i$, $i = 1, \ldots, d$, represent one-dimensional mass and stiffness matrices, and $d$ is the problem dimension.

Our approach is based on the fact that the application of $P^{-1}$ is equivalent to the solution of a tensor equation. For example, when $d = 2$ the linear system $Ps = r$ is equivalent to the solution of the matrix equation

$$M_2 SK_1 + K_2 SM_1 = R$$

where $S$ and $R$ are matrices obtained by a proper reshape of vectors $s$ and $r$.

The literature on the numerical solution of such problems is vast, and we select among the available methods the ones which seem the most suited for the particular features of IGA problems. Application to 2D and 3D problems is shown, and robustness with respect to the problems parameters (such as the spline degree) is discussed. We also discuss how the geometry of the PDE domain affects the spectral properties of $P^{-1}K$, and propose simple strategies to partially include information on the geometry in the preconditioner.
A major challenge in the application of sampling methods to large scale inverse problems, is the high computational cost associated with solving the forward model for a given set of input parameters. To overcome this difficulty, we consider using a surrogate model that approximates the solution of the forward model at a much lower computational cost. We focus in particular on Gaussian process emulators, and analyse the error in the posterior distribution resulting from this approximation.
A GUARANTEED EQUILIBRATED ERROR ESTIMATOR
FOR THE A − φ AND T − Ω
MAGNETODYNAMIC HARMONIC FORMULATIONS
OF THE MAXWELL SYSTEM

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Key Words: Maxwell equations, potential formulation, a posteriori estimators, finite element method.

This communication is devoted to the development and analysis of an equilibrated a posteriori error estimator for the harmonic eddy current problems. Therefore the system of interest is given by the quasi-static approximation of Maxwell’s equations in the magnetoharmonic regime, completed by the constitutive laws: \( B = \mu H \) in the whole domain \( D \) and \( J_e = \sigma E \) in the conductor domain \( D_c \). Here \( B \), \( H \), \( J_e \) and \( E \) represent respectively the magnetic flux density, the magnetic field, the eddy current density and the electric field, while \( \mu \) stands for the magnetic permeability and \( \sigma \) for the electrical conductivity.

In order to obtain numerical solutions, we solve the two classical potential formulations. The first one is a recast of the original system through a magnetic vector potential \( A \), defined in \( D \), as well as an electrical scalar potential \( \phi \), defined only in \( D_c \). The finite element method applied to the \( A - \phi \) formulation provides the numerical solutions:

\[ B_h = \text{curl } A_h \text{ in } D \]
\[ E_h = -i\omega A_h - \nabla \phi_h \text{ in } D_c. \]

Similarly, a recast of the original system through an electric vector potential \( T \), defined in \( D_c \), as well as a magnetic scalar potential \( \Omega \), defined in \( D \), gives the so-called \( T - \Omega \) formulation. The finite element method provides the numerical solutions:

\[ H_h = H_s + T_h - \nabla \Omega_h \text{ in } D \]
\[ J_h = \text{curl } T_h \text{ in } D_c, \]

where \( J_s = \text{curl } H_s \) denotes the source term.

The aim is to estimate the energy norm of the error \( \epsilon \):

\[ \epsilon = \left( \| \mu^{1/2} (B - B_h) \|^2_{L^2(D)} + \| \mu^{1/2} (H - H_h) \|^2_{L^2(D)} \right) + \| (\omega \sigma)^{-1/2} (J - J_h) \|^2_{L^2(D_c)} + \| \omega^{-1/2} \sigma^{1/2} (E - E_h) \|^2_{L^2(D_c)} \right)^{1/2}. \]

To do that, we derive an error estimator based on the non-verification property of the constitutive laws for the numerical fields [Creusé, S. Nicaise and R. Tittarelli, A guaranteed equilibrated error estimator for the \( A - \varphi \) and \( T - \Omega \) magnetodynamic harmonic formulations of the Maxwell system, IMA Journal of Numerical Analysis, submitted for publication]. Let us denote by \( \mathcal{T}_h \) a tetrahedral regular mesh. The estimator \( \eta \) is defined as

\[ \eta^2 = \sum_{T \in \mathcal{T}_h} \eta^2_{m,T} + \sum_{T \in \mathcal{T}_h, T \subset D_c} \eta^2_{e,T}, \text{ where} \]
\[ \eta_{m,T} = \| \mu^{1/2} (H_h - \mu^{-1} B_h) \|_T \text{ and } \eta_{e,T} = \| (\omega \sigma)^{-1/2} (J_h - \sigma E_h) \|_T. \]
First of all, the global equivalence between the error $\epsilon$ and the estimator $\eta$ up to higher order terms (h.o.t.) without unknown constants is proved, that is:

$$\eta^2 = \epsilon^2 + \text{h.o.t.}.$$ 

Secondly, the local efficiency property is proved i.e. $\eta_T = (\eta_{n,T}^2 + \eta_{e,T}^2)^{1/2} \leq \sqrt{2}\epsilon_T$ with $T \in \mathcal{T}_h$. This latter inequality gives the key ingredient for driving an adaptive remeshing process. Finally, these theoretical results are validated through an analytical benchmark test.
NUMERICAL METHODS FOR P-LAPLACE TYPE PROBLEMS

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In this presentation, we consider numerical methods for solving power-law diffusion problems, e.g. p-Laplace type problems. For the space discretization we use continuous Galerkin finite element methods (FE) with high order polynomial spaces. For the solution of the resulting nonlinear system we employ different Newton methods, such as residual-based and error-oriented globalization techniques. In addition, we also transform the original problem into a saddle point problem using an augmented Lagrangian (ALG) decomposition technique.

Assuming sufficient regularity for the solution, we derive high order interpolation and error estimates in relevant quasi-norms. We mainly focus on a systematic comparison of first and second order finite element approximations in order to confirm our theoretical findings. Our second goal is a very detailed comparison of two different Newton methods: a residual-based procedure and an error-oriented procedure. Lastly, we discuss the solution of the produced ALG saddle point problem. We discretize it using a FE methodology and then we present two iterative methods for solving the resulting nonlinear algebraic system. The first iterative method is the classical ALG1 iterative method, which is usually used in the literature. It can be interpreted as a variant of the Uzawa algorithm, where the Lagrange multiplier is separately updated. The second proposed iterative method can be characterized as a monolithic approach where all the unknown variable are simultaneously computed in one step. All, proposed methods are compared with respect to computational cost and to the convergence rates in several examples.

This talk is based on a joint work with Thomas Wick, \cite{1}. We gratefully acknowledge the financial support of this research work by the Austrian Science Fund (FWF) under the grant NFN S117-03.

References

In the Isogeometric Analysis framework for treating realistic problems, it is usually necessary to decompose the domain into volumetric subdomains (patches). More precisely, we apply a segmentation technique for splitting the initial domain into simpler subdomains and then we define the corresponding control nets of the subdomains that are used for constructing the parametrizations of the subdomains. Usually, we obtain compatible parametrizations of the subdomains, meaning that using a relative coarse control mesh, the parameterizations of the adjoining subdomain interfaces are identical.

However, this is not always the case. Due to an incorrect segmentation procedure, we can lead to non-compatible parametrizations of the geometry, meaning that the parametrized interfaces of adjusting subdomains are not identical. The result of this phenomenon is the creation of overlapping subdomains or gap regions between adjacent subdomains. It is clear that we cannot apply directly the dGIgA methods which have been proposed so far in the literature and are referred to matching interface parametrizations. In this talk, we will present a discontinuous Galerkin Isogeometric Analysis method applied on decompositions, where gap and overlapping regions can appear. We apply a multi-patch approach and derive suitable numerical fluxes on the boundaries of overlapping and gap regions, using the interior subdomain solutions, (i.e., the solution on points which are not located on the overlaps and on gaps), and in that way we connect the values of the solution of the regions where we have unique representation of the solution. The ideas are illustrated on a model diffusion problem with discontinuous diffusion coefficients. We develop a rigorous theoretical framework for the proposed method clarifying the influence of the gap/overlapping region size onto the convergence rate of the method. The theoretical estimates are supported by numerical examples in two- and three-dimensional computational domains.

This talk is based on works [1, 2, 3]. We gratefully acknowledge the financial support of this research work by the Austrian Science Fund (FWF) under the grant NFN S117-03.

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The computation of the stress intensity factors governing the behaviour of cracked bodies is complicated by the presence of stress singularities at crack tips. One method of overcoming this difficulty is to use an enriched form of a discrete numerical method. The eXtended Finite Element Method (XFEM) has become a popular research topic, allowing accurate results from coarse finite element discretisations, and freeing the meshing from the constraint to follow the geometry of the crack. The similar type of enrichment can also be applied to the Boundary Element Method, as shown in [Alatawi and Trevelyan (2015), Engineering Analysis with Boundary Elements, 52:56-63], allowing accurate evaluation of the stress intensity factors directly in the solution vector and without the requirement for postprocessing such as the J-integral. This approach has come to be known as the eXtended Boundary Element Method (XBEM).

In the current work we extend the XBEM to consider anisotropic media. The enrichment functions based on the Williams expansions for isotropic media are replaced by the corresponding anisotropic expressions found from the Stroh formalism approach. We present results that, with very small numbers of degrees of freedom, correspond well with XFEM solutions. Finally we show how the matrices governing these enriched systems are amenable to low rank approximation using Adaptive Cross Approximation, accelerating the matrix vector product embedded in each iteration of an iterative solver.
We present a simple and highly efficient algorithm for evaluation of quasi-periodic Green functions that is seamlessly incorporated into a boundary integral equation numerical method for the solution of wave scattering problems by bi-periodic arrays of scatterers in three-dimensional space. Except at certain “Wood frequencies” at which the quasi-periodic Green function ceases to exist, the proposed approach, which is based on use of smooth windowing functions, gives rise to lattice sums which converge to the Green function superalgebraically fast—that is, faster than any power of the number of terms used—in sharp contrast with the extremely slow convergence exhibited by the corresponding sums in absence of smooth windowing. A variety of numerical results, in turn, demonstrate the practical efficiency of the proposed approach.
MULTILEVEL MONTE CARLO ANALYSIS FOR OPTIMAL CONTROL OF ELLIPTIC PDES WITH RANDOM COEFFICIENTS

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This work is motivated by the need to study the impact of data uncertainties and material imperfections on the solution to optimal control problems constrained by partial differential equations. We consider a pathwise optimal control problem constrained by a diffusion equation with random coefficient together with box constraints for the control. For each realization of the diffusion coefficient we solve an optimal control problem using the variational discretization [M. Hinze, Comput. Optim. Appl., 30 (2005), pp. 45-61]. Our framework allows for lognormal coefficients whose realizations are not uniformly bounded away from zero and infinity. We establish finite element error bounds for the pathwise optimal controls. This analysis is nontrivial due to the limited spatial regularity and the lack of uniform ellipticity and boundedness of the diffusion operator. We apply the error bounds to prove convergence of a multilevel Monte Carlo estimator for the expected value of the pathwise optimal controls. In addition we analyze the computational complexity of the multilevel estimator. We perform numerical experiments in 2D space to confirm the convergence result and the complexity bound.
Complex processes in fractured media lead to multiscale problems because of a hierarchy of fracture sizes. To represent the microscale interaction between the fractures and the matrix, various coarse-grid models have been developed. These include dual-continua like approaches, coarse-scale continuum model, upscaling methods, Multiscale Finite Volume, and so on. In this talk, I will describe an approach, which is based on Generalized Multiscale Finite Element Method. The main idea of the approach is to extract important local information from local snapshot spaces via local spectral problems. This computational approach leads to extracting important flow patterns in fractured media and results to accurate predictions when using a few basis functions. In my talk, I will describe the algorithm. I will show numerical results for two applications. One is for a model problem describing the transport of shale gas and the other is wave propagation.
We present a scalable parallel solver for H(div) problems discretized by arbitrary order finite elements on general unstructured meshes. The solver is based on hybridization and algebraic multigrid (AMG). Unlike some previously developed H(div) solvers, the hybridization solver does not require discrete curl and gradient operators as additional input from the user. Instead, only fine-grid element information is needed in the construction of the solver. The hybridization results in a $H^1$-equivalent symmetric positive definite system, which is then rescaled and solved by AMG solvers designed for $H^1$ problems. Weak and strong scaling of the method are examined through several numerical tests. Our numerical results show that the proposed solver provides a competitive alternative to ADS, a state-of-the-art solver for H(div) problems from the LLNL parallel solvers library HYPRE. In fact, it outperforms ADS for high order elements.

The presentation is based on joint works with C. S. Lee (Texas A & M University), V. Dobrev (LLNL), and Tz. Kolev (LLNL).
Preserving positivity in approximation is a useful property. This is illustrated, e.g., in [Z. Chen, R. H. Nochetto, Residual type a posteriori error estimates for elliptic obstacle problems, Numer. Math. 84 (2000), 527–548], where a linear interpolation operator with this property is constructed and used in a posteriori error estimation. However, preserving positivity entails also obstructions: the impossibility results in [R. H. Nochetto, L. B. Wahlbin, Positivity preserving finite element approximation, Math. Comp. 71 (2001), 1405–1419] show in particular that second order cannot be reached with optimal local approximation properties.


Recently, Christian Kreuzer and the author developed an approach to a posteriori error estimation that clarifies the role of oscillation; see also the talk of Christian Kreuzer in this mini-symposium. For Poisson’s problem, this approach provides an $H^{-1}$-oscillation that is bounded in terms of the error.

This talk revisits previous approaches in the a posteriori error analysis with obstacles, assessing their compatibility with this new $H^{-1}$-oscillation.
Coupled systems of semilinear parabolic equations arise in a number of applications in fields such as biology, chemistry and material science. Often the applications are such that the equations are posed on complex or evolving geometries. In this talk we address the design and analysis of finite element approximations of such systems with implicit-explicit time discretisation. The theoretical results will be supported by examples of application driven numerical simulations.
Topology optimization is a fertile area of research that is mainly concerned with the automatic generation of optimal layouts to solve design problems in Engineering. The classical formulation addresses the problem of finding the best distribution of an isotropic material that minimizes the work of the external loads at equilibrium, while respecting a constraint on the assigned amount of volume. This is the so-called minimum compliance formulation that can be conveniently employed to achieve stiff truss-like layout within a two-dimensional domain. A classical implementation resorts to the adoption of four node displacement-based finite elements that are coupled with an elementwise discretization of the (unknown) density field. When regular meshes made of square elements are used, well-known numerical instabilities arise, see in particular the so-called checkerboard patterns. On the other hand, when unstructured meshes are needed to cope with geometry of any shape, additional instabilities can steer the optimizer towards local minima instead of the expected global one. Unstructured meshes approximate the strain energy of truss-like members with an accuracy that is strictly related to the geometrical features of the discretization, thus remarkably affecting the achieved layouts. In this talk we will consider several benchmarks of truss design and explore the performance of the Virtual Element Method (VEM) in driving the topology optimization procedure. In particular, we will show how the capability of VEM of efficiently approximating elasticity equations on very general polygonal meshes can contribute to overcome the aforementioned mesh-dependent instabilities exhibited by classical finite element based discretization techniques.
PARTITIONED ALGORITHMS FOR FLUID-STRUCTURE INTERACTION ARISING IN HEMODYNAMICS

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We propose a unified convergence analysis of the generalized Schwarz method applied to a linear elliptic problem for a general interface (flat, cylindrical or spherical) in any dimension. In particular, we provide the exact convergence set of the interface symbols related to the operators involved in the transmission conditions. We also provide a general procedure to obtain estimates of the optimized interface symbols within the constants. We apply such general results to the fluid-structure interaction problem arising in haemodynamics, obtaining partitioned algorithms based on Robin interface conditions. A proper choice of the interface parameters involved in these conditions allows us to obtain efficient algorithms which do not suffer from the high added mass effect which characterizes haemodynamic applications. The numerical results both in ideal and real geometries highlighted the suitability of our proposals.

DISCRETE MAXIMAL PARABOLIC REGULARITY AND BEST APPROXIMATION RESULTS FOR GALERKIN FINITE ELEMENT SOLUTIONS OF PARABOLIC PROBLEMS

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In this talk we present discrete maximal parabolic regularity results [1] for linear parabolic equations discretized by discontinuous Galerkin methods in time and Lagrange finite elements in space. These results provide a novel flexible technique for establishing optimal error estimates in various non-Hilbertian norms without any coupling conditions between the spatial mesh size and time steps. Especially we present global and interior best approximation type estimates in the $L^\infty(\Omega)$ norm [2].

References


In this talk we discuss optimal control problems subject to parabolic equations, where the support of the control is potentially of measure zero. This includes sparse optimal control problems \cite{KunischPieperVexler} and problems with pointwise controls \cite{LeykekhmanVexlerParabolicOptimal, LeykekhmanVexlerParabolicOptimal3D}. For this type of problems we consider finite element discretizations in space and time and derive a priori error estimates. The main technical tools are recently established discrete maximal parabolic regularity \cite{LeykekhmanVexlerDiscreteMaximal} and pointwise best approximation results \cite{LeykekhmanVexlerLocal}. 

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SPH AS A NONLOCAL REGULARISATION METHOD FOR INSTABILITIES DUE TO STRAIN-SOFTENING

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Meshless methods, such as Smoothed Particle Hydrodynamics (SPH), are of particular interest for the accurate prediction of failure including fragmentation and fracture. Within the framework of continuum damage mechanics (CDM) material mechanical properties are degraded locally with evolution of damage. These local models evaluate state and internal variables at points or particles with a limited ability to take into account the length scales (characteristic lengths) of the effects taking place at a sub continuum scale \([1]\).

The presented work investigated the strain-softening effects in the SPH spatial discretisation combined with local and nonlocal CDM damage models. The simple uniaxial wave propagation in presence of damage induced material softening for which Bazant et al. \([2]\) derived an exact solution was used in this investigation. The simulations were performed with the in-house SPH code MCM (“Meshless Continuum Mechanics”).

The strain softening related problems observed by Bazant et. al. were not present in the total Lagrangean formulation of SPH \([3]\) due to the nonlocal character of the SPH method. It was established that stress wave propagation continues in the presence of strain-softening and the waves continue to propagate within the damage localisation zone unlike in the FE simulation. Furthermore, it was demonstrated that the smoothing length represents characteristic length for the material considered and has to be defined with caution when modelling damage and failure.

References


Carbon fibre reinforced composites (CFRP) are often exposed to the impact loading with high strain rates in the range from to (e.g. debris, hail stone, bird strike and ballistic impacts). These extreme impact loadings almost always involve generation and propagation of shock waves within the material. The material behaviour under such a complex loading needs to be accurately modelled, in order to minimise the risk of the catastrophic impact related failure. The presented research is related to development and validation of a thermodynamically consistent constitutive model for CFRP materials under high velocity impact loading. The model is capable of modelling damage, failure and formation and propagation of shock waves in non-homogeneous anisotropic material. The model has two main parts: the strength part which defines the material response to shear deformation and an equation of state (EOS) which defines the material response to isotropic volumetric deformation [1]. The constitutive model was implemented into the transient nonlinear finite element code DYNA3D [2] and our in house SPH code. Limited model validation was performed by simulating a number of high velocity material characterisation and validation impact tests.

The new damage model was developed in the framework of configurational continuum mechanics and irreversible thermodynamics with internal state variables. It is applicable to large deformations.

The damage was represented as a second order tensor, which was divided into the volume change related damage (e.g. voids, cavities) and damage related to shear deformation. The damage evolution equations were based on the modified Tuler Bucher “time to failure” [3] approach which was coupled with a thermo elastic model and the shock EOS. The failure initiation was based on a critical value of a specific dissipation function. Validated model was used for modelling of composite aircraft engine blade impacts.
Figure: a) front and b) rear side of the impacted blade, c) released blade.

References


POLYNOMIAL-DEGREE-ROBUST ESTIMATES
IN THREE SPACE DIMENSIONS

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Braess \textit{et al.} \cite{1} proved for the first time that equilibrated flux a posteriori error estimates for conforming finite elements do not suffer from increased overestimation for higher polynomial degrees, i.e., that they are robust with respect to the polynomial degree. This result has been extended in \cite{2} to a unified framework covering all conforming, nonconforming, discontinuous Galerkin, and mixed finite element discretizations of the Poisson problem, still in two space dimensions. On each patch of elements sharing the given interior vertex, one solves here a homogeneous local Neumann problem by the mixed finite element method to obtain an equilibrated flux reconstruction in $H(\text{div}, \Omega)$, as well as a homogeneous local Dirichlet problem by the conforming finite element method to obtain a potential reconstruction in $H^1_0(\Omega)$. We extend here this methodology to three space dimensions. Details are given in \cite{3}.

References


We show that dual norms of bounded linear functionals on the Sobolev space \( W^{1,p}_0(\Omega) \) are localizable provided that the functional in question vanishes over locally supported test functions which form a partition of unity. This allows, a fortiori, to establish local efficiency and robustness for a posteriori analysis of nonlinear partial differential equations in divergence form. This result holds true even in presence of linearization and algebraic errors from inexact solvers, provided that these are relatively small. Along these lines, we propose a fully adaptive inexact Newton method. Here, at each step of the nonlinear and linear solvers, the discretization, linearization, and algebraic error components are balanced via local stopping criteria based on guaranteed a posteriori estimates. Numerical experiments are presented in confirmation of the theory. The details can be found in [1, 2].

References


EXPONENTIAL CONVERGENCE OF $hp$-FINITE ELEMENT DISCRETIZATION OF OPTIMAL BOUNDARY CONTROL PROBLEMS WITH ELLIPTIC PARTIAL DIFFERENTIAL EQUATIONS

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We investigate the numerical solution of a boundary control problem with elliptic partial differential equation by the $hp$-finite element method. We prove exponential convergence with respect to the number of unknowns for an a-priori chosen discretization. Here, we have to prove that derivatives of arbitrary order of the solution belong to suitably chosen weighted Sobolev spaces. This result relies on the assumption that the number of switching points of the optimal control is finite. Numerical experiments confirm the theoretical findings.

OPTIMAL CONVERGENCE ORDER FOR CONTROL CONSTRAINED OPTIMAL CONTROL PROBLEMS

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In this talk we consider the numerical solution of control constrained optimal control problems. We are interested in obtaining the optimal convergence rate for the $L^2(\Omega)$-error w.r.t. the number of degrees of freedom. Due to the control constraint, the optimal control possesses a kink at the interface between the active and inactive set w.r.t. the control constraint. This kink limits the convergence order of a uniform discretization to $h^{3/2}$.

We compare some approaches from the literature. Moreover, we provide a new, efficient and robust error estimator which is used for an adaptive refinement of the mesh.

We also present a new method for solving control constrained problems. In this method, we move the nodes of the mesh at the interface between the active and inactive set. This yields optimal order of convergence.
Fractional partial differential equations (FPDEs) provide a powerful tool for modeling challenging phenomena including anomalous transport, and long-range time memory or spatial interactions in nature, science, social science, and engineering. However, FPDEs present mathematical and numerical difficulties that have not been encountered in the context of integer-order PDEs.

Computationally, because of the nonlocal property of fractional differential operators, the numerical methods for space-fractional FPDEs often generate dense stiffness matrices for which widely used direct solvers have a computational complexity of $O(N^3)$ (per time step for a time-dependent problem) and memory requirement of $O(N^2)$ where $N$ is the number of unknowns (per time step for a time-dependent problem). This makes numerical simulation of three-dimensional FPDE modeling computationally very expensive.

What further complicates the scenario results from the fact that the solutions to fractional elliptic PDEs with smooth data and domain may have boundary layers and poor regularity. Consequently, a fast numerical scheme discretized on a uniform mesh cannot be effective. Hence, finite-difference methods, which are obtained via a discretization of Grünwald-Letnikov fractional derivatives, are out of the question. On the other hand, a numerical scheme discretized on an adaptively refined unstructured mesh offers great flexibility in resolving the boundary layers and other singularities, it destroys the structure of the dense stiffness matrix and so the efficiency of the numerical scheme.

We derive an accurate and fast numerical scheme by balancing the flexibility and efficiency: (i) This would use a composite mesh that consists of gridded mesh near the interface regions and a structured mesh in most of the domain. (ii) This would utilize the structure of the stiffness matrices on respective subdomains. (iii) This would use low-rank approximations to the “off-diagonal” dense matrix blocks in the stiffness matrix. (iv) The resulting fast method has approximately linear computational complexity (per time step) and optimal memory requirement.

This work was supported in part by the National Science Foundation under Grant DMS-1216923 and by the OSD/ARO MURI Grant W911NF-15-1-0562.
In the talk, the speaker shall first introduce the weak Galerkin (WG) finite element method for partial differential equations. Weak Galerkin is a finite element method for PDEs where the differential operators (e.g., gradient, divergence, curl, Laplacian etc.) in the weak forms are approximated by discrete generalized distributions. The WG discretization procedure often involves the solution of inexpensive problems defined locally on each element. The solution from the local problems can be regarded as a reconstruction of the corresponding differential operators. The fundamental difference between the weak Galerkin finite element method and other existing methods is the use of weak functions and weak derivatives (i.e., locally reconstructed differential operators) in the design of numerical schemes based on existing weak forms for the underlying PDEs. Weak Galerkin is a natural extension of the classical Galerkin finite element method with advantages in many aspects. Due to its great structural flexibility, the weak Galerkin finite element method is well suited to most partial differential equations by providing the needed stability and accuracy in approximation.

The talk will start with the second order elliptic equation, for which WG shall be applied and explained in detail. In particular, the concept of weak gradient will be introduced and discussed for its role in the design of weak Galerkin finite element schemes. The speaker will then introduce a general notion of weak differential operators, such as weak Hessian, weak divergence, and weak curl etc. These weak differential operators shall serve as building blocks for WG finite element methods for other class of partial differential equations, such as the Stokes equation, the biharmonic equation, the Maxwell equations in electron magnetics theory, div-curl systems, and PDEs in non-divergence form (such as the Fokker-Planck equation). In particular, the speaker will introduce a primal-dual formulation for second order elliptic PDEs in non-divergence form. Numerical results and error estimates shall be discussed. The talk should be accessible to graduate students with adequate training in computational methods.
A new discretization method for homogeneous convection-diffusion-reaction boundary value problems in 3D is presented that is a non-standard finite element method with PDE-harmonic shape functions on polyhedral elements, see [1]. The element stiffness matrices are constructed by means of local boundary element techniques. The method, which is referred to as a BEM-based FEM, can therefore be considered a local Trefftz method with element-wise (locally) PDE-harmonic shape functions.

The current research combines the results of [2] with the hierarchical construction of shape functions presented in [3]. The Dirichlet boundary data for these shape functions is chosen according to a convection-adapted procedure which solves projections of the PDE onto the edges and faces of tetrahedral and polyhedral elements, respectively. This improves the stability of the discretization method for convection-dominated problems both when compared to a standard FEM and to previous BEM-based FEM approaches, as we demonstrated in several numerical experiments. Our experiments also show an improved resolution of the exponential layer at the outflow boundary for our proposed method when compared to the SUPG method.

References


Radial basis functions (RBFs) are a popular meshfree discretisation method. They are used in various areas comprising, for example, scattered data approximation, computer graphics, machine learning, aeroelasticity and the geosciences.

The approximation space is usually formed using the shifts of a fixed basis function. This simple approach makes it easy to construct approximation spaces of arbitrary smoothness and in arbitrary dimensions. It is also possible to incorporate physical features like incompressibility into the approximation space.

Multiscale RBFs employ radial basis functions with compact support. In contrast to classical RBFs they do not only use the shifts of a fixed basis function but also vary the support radius in an orderly fashion. If done correctly, this leads to an extremely versatile and efficient approximation method.

In this talk, I will introduce various ways of solving PDEs numerically using (multiscale) RBFs. I will address collocation and Galerkin techniques for elliptic and parabolic problems. I will discuss error and stability estimates and give several examples.
We present and analyze an enriched Galerkin finite element method (EG) to solve coupled flow and transport system with jump coefficients referred to miscible displacement problems. The EG is formulated by enriching the conforming continuous Galerkin finite element method (CG) with piecewise constant functions. This method is shown to be locally and globally conservative, while keeping fewer degrees of freedom in comparison with discontinuous Galerkin finite element methods (DG). In addition, we present and analyze a fast and effective EG solver simpler than DG and whose cost is roughly that of CG and can handle an arbitrary order of approximations for the flow problem.

Moreover, to avoid any spurious oscillations for the higher order transport system, we employ an entropy residual stabilization technique. Dynamic mesh adaptivity using hanging node is applied to save computational cost for large-scale physical problems. Number of numerical tests in two and three dimensions are presented to confirm our theoretical results as well as to demonstrate the advantages of the EG.
Currently, fracture propagation is a major topic in applied mathematics and engineering. It seems to turn out that one of the most promising methods is based on a variational setting and more specifically on a thermodynamically consistent phase-field model. Here a smoothed indicator function determines the crack location and is characterized through a model regularization parameter. In addition, modeling assumes that the fracture can never heal, which is imposed through a temporal constraint, leading to a variational inequality system. The basic fracture model problem is augmented with several hints and discussions of serious challenges in developing numerical methods for fracture propagation. Key aspects are robust and efficient algorithms for imposing the previously mentioned crack irreversibility constraint, treatment of the indefinite Jacobian matrix, computational analysis of the interplay of model and discretization parameters, goal-functional evaluations, coupling to other multiphysics problems such as pressurized fractures, fluid-filled fractures, proppant-filled fractures in porous media, fluid-structure interaction, and aspects of high performance computing for tackling practical field problems.
We introduce a novel hybrid discontinuous Galerkin method for elliptic problems with a discontinuous ansatz space in the cells and adaptively chosen constraints on the faces. This corresponds to a weakly conforming finite element space defined by primal and dual face degrees of freedom. In the solution process the interior degrees of freedom can be eliminated. We provide local criteria for the well-posedness and stability of this elimination process, and we derive global spectral bounds for the resulting skeleton reduction. The a priori finite element error and a residual based error estimator measuring also the primal and dual consistency error are analyzed.

The face contributions of the primal and dual consistency error are used to derive a flexible strategy to increase the number of face degrees of freedom locally. The new adaptive scheme is evaluated numerically for nearly incompressible 3D linear elasticity, and the results are compared with the symmetric interior penalty discontinuous Galerkin method. Finally, we show that the method extends to nonlinear applications such as contact problems or large strain elasticity.
CONTINUOUS AND DISCONTINUOUS GALERKIN TIME STEPPING METHODS FOR NONLINEAR INITIAL VALUE PROBLEMS WITH APPLICATION TO FINITE TIME BLOW-UP

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We consider cG and dG time stepping methods of arbitrary order as applied to nonlinear initial value problems in real Hilbert spaces. We develop new techniques to prove general Peano-type existence results for discrete solutions; in particular, our results show that the existence of solutions is independent of the local approximation order, and only requires the local time steps to be sufficiently small (independent of the polynomial degree). In addition, our theory is applied to finite time blow-up problems with nonlinearities of algebraic growth. For such problems we develop a time step selection algorithm for the purpose of numerically computing the blow-up time, and provide a convergence result.
Tent pitching algorithms construct space-time meshes by vertically erecting canopies over vertex patches. The main advantage is the ability to advance in time by different amounts at different spatial locations. These tent pitched meshes are usually combined with a space-time discretization, which leads to a rather large local problem on each tent. This talk considers a novel discretization technique, that exploits the structure of tent pitched meshes to reduce the local problem size. The reduction is obtained by transforming the tents to a reference domain with a space-time tensor product structure, which then allows to discretize space and time independently. These Mapped Tent Pitching (MTP) schemes can be applied to both, linear and non-linear systems. For linear systems a fully implicit MTP scheme is presented in [1] and this talk will focus on non-linear systems (see [1, 2]). Numerical results for the Euler equations in 2+1 dimensions and the linear wave equation in 3+1 dimensions will be shown.

References


We consider model reduction techniques for the numerical simulation of vibro-acoustics. The mathematical model is based on an eigenvalue problem for the possibly orthotropic linear elasticity equation. In addition to nine material parameters, geometrical parameters and insulation by thin elastomeric layers can be taken into account. A fine scale finite element simulation is typically expansive due to complex geometries. In this talk we cover several aspects. Firstly, weakly coupled patch-wise tensorial structured isogeometric elements are considered. These are of special interest for complex geometries with piecewise smooth but curvilinear boundaries. We discuss the well-posedness of the isogeometric Lagrange multiplier based mortar formulation. Secondly, we consider a dimension reduction technique which allows us to reformulate a layered geometry as interface equation coupling the 3D blocks by a spring. Thirdly, we provide upper bounds for the approximation of eigenvalues in a reduced basis setting. To obtain locality in the detailed system, we use the saddle point approach and do not apply static condensation techniques. However within the reduced basis context, it is natural to eliminate the Lagrange multiplier and formulate a reduced eigenvalue problem for a symmetric positive definite matrix. The selection of the snapshots is controlled by a multi-query greedy strategy taking into account an error indicator allowing for multiple eigenvalues.

As example for isogeometrical mortar methods, we consider the vibration of a violin bridge in a multi-query context and as example for dimension reduced interface couplings, we use a timber building block having thin elastomeric layers as insulation. Our numerical results illustrate several aspects such as accuracy of mortar couplings for splines, the influence of the orthotropic material and geometrical parameters on the eigenvalues and the component based decomposition for a multi-storey timber building.

References


DISCRETIZATION OF PARABOLIC OPTIMIZATION PROBLEMS WITH CONSTRAINTS ON THE SPATIAL GRADIENT OF THE STATE

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In this talk, optimization problems subject to a possibly semilinear parabolic partial differential equation (PDE) are considered. Moreover, additional pointwise constraints are imposed on the gradient of the state, i.e., the solution to the PDE. The optimization problems are discretized using a Galerkin-type approach and the convergence rates for the discretization error are discussed.

FINITE ELEMENT APPROXIMATION OF GRADIENT CONSTRAINT ELLIPTIC OPTIMIZATION PROBLEMS ON NON-SMOOTH DOMAINS

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In this talk, we are concerned with the discretization of PDE constrained optimization problems with pointwise constraints on the gradient of the state. Particular emphasis will be given to the case of non smooth domains, where the control to state mapping does not assert the gradient of the PDE solution to be Lipschitz. Nonetheless, convergence of the finite element approximation can be shown.
RECURSIVE INTEGRAL METHOD FOR A NON-LINEAR NON-SELFADJOINT TRANSMISSION EIGENVALUE PROBLEM

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We propose a robust numerical method to compute complex and real eigenvalues of a nonlinear non-selfadjoint transmission eigenvalue problem. Based on a fourth order formulation, we obtain a quadratic eigenvalue problem. The non-conforming Morley element is used for discretization, leading to a quadratic matrix eigenvalue problem. Then we propose to use a recursive integral method to compute the eigenvalues in prescribed regions on the complex plane. The effectiveness of the proposed method can be validated by numerical examples.

FULLY COMPUTABLE ERROR ESTIMATES FOR EIGENVALUE PROBLEMS

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In the talk, we will present a fully computable error estimate for the eigenvalue problem which is solved by the general conforming finite element methods on the general meshes. Based on the computable error estimate, we also give a guaranteed upper bound of the error estimate for the eigenfunction approximation. Furthermore, we also propose a simple process to compute the guaranteed lower bound of the first eigenvalue based on the upper bound of the eigenfunction error estimate. Some numerical examples are presented to validate the theoretical results.
In this work we discuss an adaptive finite element method for efficiently solving electrical impedance tomography – a severely ill-posed nonlinear inverse problem to recover the conductivity from boundary voltage measurements. The reconstruction technique is based on Tikhonov regularization with a Sobolev smoothness penalty and approximation of the forward model using continuous piecewise linear finite elements. We propose an adaptive finite element algorithm with an a posteriori error estimator involving the concerned state and adjoint variables and the recovered conductivity. The convergence of the algorithm is established, in the sense that the sequence of discrete solutions contains a convergent subsequence to a solution of the optimality system for the continuous formulation. Numerical results are presented to verify the convergence and efficiency of the algorithm.
AN ANALYSIS OF THE MODIFIED L1 SCHEME FOR THE TIME-FRACTIONAL PARTIAL DIFFERENTIAL EQUATIONS WITH NONSMOOTH DATA

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We consider the error estimates of the modified L1 scheme for solving time fractional partial differential equation. Jin et al. (2016, An analysis of the L1 scheme for the subdiffusion equation with nonsmooth data, IMA J. of Numer. Anal., 36, 197-221) established an $O(k)$ convergence rate for L1 scheme for both smooth and nonsmooth initial data. We introduce a modified L1 scheme and prove that the convergence rate is $O(k^{2-\alpha})$, $0 < \alpha < 1$ for both smooth and nonsmooth initial data. We first write the time fractional partial differential equation as a Volterra integral equation which is then approximated by using two convolution quadratures, respectively. The numerical schemes obtained are equivalent to the L1 scheme and the modified L1 scheme respectively. Laplace transform method is used to prove the error estimates for the homogeneous time fractional partial differential equation for both smooth and nonsmooth data. Numerical examples are given to show that the numerical results are consistent with the theoretical results.

WEAK GALERKIN METHODS AND APPLICATIONS

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The Weak Galerkin method is an extension of the standard Galerkin finite element method where classical derivatives were substituted by weakly defined derivatives on functions with discontinuity. The WG methods have the flexibility in handling complex geometry and the simplicity in analyzing real-world physical problems. Recent development of weak Galerkin methods will be discussed in the presentation.
A BLOCK-DIAGONAL PRECONDITIONER FOR A FOUR-FIELD MIXED FINITE ELEMENT METHOD FOR BIOT’S EQUATIONS

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In this talk, we explore an efficient preconditioning method for the saddle point system resulting from a four-field mixed finite element method applied to Biot’s consolidation model. The proposed preconditioner is a block diagonal preconditioner based on the Schur complement. We obtain bounds on the eigenvalues of the preconditioned matrix that are clustered away from 0. To reduce the computational expense, this preconditioner is inverted approximately. Some numerical results are provided to show the efficiency of our preconditioning strategy when applied to a poroelasticity problem in a layered medium.

ANISOTROPIC MESHES AND STABILIZED PARAMETERS FOR THE STABILIZED FINITE ELEMENT METHODS

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In this talk, we demonstrate a numerical strategy to generate anisotropic meshes and select the appropriate stabilization parameter simultaneously for two dimensional convection-dominated convection-diffusion equations by the stabilized continuous linear finite elements. Since the discretization error in a suitable norm can be bounded by the sum of interpolation error and its variants in different norms, we replace them by some terms which contain the Hessian matrix of the true solution, convective fields, and the geometric properties such as directed edges and the area of triangles. Based on this observation, the shape, size and equidistribution requirements are used to derive the corresponding metric tensor and the stabilization parameter. The process of the derivation reveals that the optimal stabilization parameter is coupled with the metric tensor for each element. Numerical results are also provided to validate the stability and efficiency of the proposed numerical strategy.
A MULTIPOINT STRESS MIXED FINITE ELEMENT METHOD FOR LINEAR ELASTICITY

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We discuss a new multipoint stress mixed finite element method for elasticity, based on the lowest order mixed finite element spaces with weak symmetry. A trapezoidal-type quadrature rule allows for local stress and rotation elimination and reduces the method to a cell-centered scheme for displacements. Stability and error analysis is performed on simplicial and quadrilateral grids. Numerical experiments are presented to illustrate the convergence of the method and its ability to handle heterogeneous problems.

This is joint work with Ilona Ambartsumyan and Eldar Khattatov from University of Pittsburgh, and Jan Nordbotten from University of Bergen.

A LAGRANGE MULTIPLIER METHOD FOR A BIOT-STOKES MODEL OF FLOW IN FRACTURED POROELASTIC MEDIA

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We study a mathematical model and its finite element approximation for solving the coupled problem arising in the interaction between fluid in a poroelastic material and fluid in a fracture. The fluid flow in the fracture is governed by the Stokes equations, while the poroelastic material is modeled using the Biot system. The continuity of normal velocity on the interface is imposed via a Lagrange multiplier. A stability and error analysis is performed for the semidiscrete continuous-in-time formulation. We present a series of numerical experiments to illustrate the convergence of the method and its applicability to modeling physical phenomena, as well as the sensitivity of the model with respect to its parameters.

This is joint work with Ilona Ambartsumyan, Eldar Khattatov, and Paolo Zunino from University of Pittsburgh.
AN ENERGY APPROACH TO TIME-DOMAIN BOUNDARY INTEGRAL EQUATIONS FOR THE WAVE EQUATION

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For the discretisation of the wave equation by boundary element methods the starting point is the so-called Kirchhoff’s formula, which is a representation formula by means of boundary potentials. In this talk different approaches to derive weak formulations of related boundary integral equations are considered. First, weak formulations based on the Laplace transform and second, time-space energetic formulations are introduced. In both cases coercivity is shown in appropriate Sobolev spaces. Finally, some numerical examples are presented and discussed.

FULLY STABLE AND FULLY CONSISTENT NONCONFORMING GALERKIN METHODS

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We consider symmetric and elliptic linear variational problems in a Hilbert space $V$. Generalizing conforming Galerkin methods, we propose a nonconforming discretization, which is fully stable and fully consistent. The first property guarantees that, for any load term in $V'$, the approximating function is well-defined and bounded in the energy norm. The second one ensures that the consistency error vanishes and motivates the definition of nonconforming Galerkin methods. Combining full stability and full consistency, we prove that our approximation is near-best and determine the quasi-optimality constant. We apply this framework to the discretization of second- and fourth-order model problems with the Crouzeix-Raviart and Morley element respectively.
MIXED ELEMENT METHOD FOR EIGENVALUE PROBLEM
OF THE BIHARMONIC EQUATION

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In this talk, a new kind of mixed element method for the eigenvalue problem of the biharmonic equation will be presented. Under the framework of a new mixed formulation of the biharmonic equation, finite element methods are designed so that, firstly, low-degree finite element spaces can be sufficient for the discretization schemes, secondly, an efficient multilevel method can be designed and implemented associated with the schemes, and thirdly, guaranteed upper and lower bounds of the eigenvalues can be computed with the schemes. Numerical experiments are also given for confirmation. This is a joint work with Xia Ji and Yingxia Xi.

AUXILIARY SPACE PRECONDITIONER FOR LINEAR
ELASTICITY EQUATIONS WITH WEAKLY IMPOSED SYMMETRY

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In recent years, there are considerable works on developing stable mixed finite element approximation for the linear elasticity equations with weakly imposed symmetry. However, it is still open how to efficiently solve the resulting (large-scaled) saddle point system. In this talk, we present an auxiliary space preconditioner for the mixed finite element approximation of the linear elasticity equations with weakly imposed symmetry. We apply the augmented Lagrangian Uzawa iteration for the saddle point system, which reduces to solving a nearly singular system. We then design an efficient preconditioner for solving this nearly singular equation. The preconditioner consists of a fast Poisson solver, and \(d\) copies of (vector) \(H(div)\) solvers (such as HX-preconditioner) where \(d\) is the space dimension. We show that the preconditioner is uniform with respect to the mesh size and parameters in the equation. This preconditioner also provides an efficient solver for the pseudo-stress formulation of the Stokes equation.
APPLICATION OF FINITE ELEMENTS IN NANO-OPTICS

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In this presentation we give an overview of the application of finite elements for the design of nano-optical devices. This ranges from single photon emitters, lightning (LEDs), scatterometry, solar cells, photomasks to silicon photonics. The physical modelling involves a basic understanding of quantum field theory and a deep insight in wave propagation and coherence theory. Numerically, we need to combine various concepts such as high order hp-Finite Elements, transparent boundary conditions, shape optimizer and the Reduced Basis method for fast parameter scans.
A NEW APPROACH TO MIXED METHODS FOR
BIHARMONIC PROBLEMS IN 2D AND 3D AND
EFFICIENT SOLVERS FOR THE DISCRETIZED PROBLEMS

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A new variant of a mixed variational formulation for a biharmonic problem is presented, which involves a non-standard Sobolev space for the Hessian of the original unknown. This allows to rewrite the fourth-order problem as a sequence of three (consecutively to solve) second-order problems. In 2D this decomposition relies on the Hilbert complex

\[ \hat{H}^1(\Omega)^2 \xrightarrow{\text{sym curl}} \mathbf{H}(\text{div div}; \Omega, \mathbb{S}) \xrightarrow{\text{div div}} L^2(\Omega), \]

in 3D on the Hilbert complex

\[ \hat{H}^1(\Omega)^3 \xrightarrow{\text{dev } \nabla} \mathbf{H}(\text{sym curl}; \Omega, \mathbb{T}) \xrightarrow{\text{sym curl}} \mathbf{H}(\text{div div}; \Omega, \mathbb{S}) \xrightarrow{\text{div div}} L^2(\Omega), \]

which both are exact for bounded and topologically simple domains, and on a Helmholtz-like decomposition, which is different from the Helmholtz decomposition associated to the Hilbert complexes from above.

On the discrete level this approach can be exploited in 2D either to reformulate the well-known Hellan-Herrmann-Johnson method or to construct a new class of mixed finite element methods for biharmonic problems in such a way that, in both cases, the assembling of the discretized equations involves only standard Lagrangian elements. Similar to the continuous level a decomposition of the discretized problem into three discretized second-order problems is available, which substantially simplifies the construction of efficient solution techniques on the discrete level. Possible extensions to 3D on the discrete level as well as extensions to more general classes of fourth-order problems will also be shortly discussed.
NUMERICAL SOLUTION OF NONLOCAL PROBLEMS

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Classical differential equations are formulated using derivatives of various orders which are local operators, i.e. defined using only local properties of the function. The solution process is then nonlocal, where e.g. a local change of boundary conditions affects the solution in the entire domain or an open subset thereof. However there are equations, where even the problem formulation is nonlocal. A classical example are fractional differential equations. More recently, a nonlocal differential calculus was devised by Gunzburger which gives a description of various nonlocal phenomena such as nonlocal diffusion or convection-diffusion with interesting applications. Efficient solution of such problems is very challenging. Our interest in this subject originally comes from the solution of a model of flocking dynamics using the discontinuous Galerkin method.
ISOGEOMETRIC DIVERGENCE-CONFORMING
VARIATIONAL MULTISCALE FORMULATION
OF INCOMPRESSIBLE TURBULENT FLOWS

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We explore the application of the Variational Multiscale Method to divergence-conforming B-splines. Residual-based VMS has established itself as a versatile turbulence model, having been successfully applied to such complex problems as parachute deployment \cite{1} and wind turbines \cite{2}. Within IGA, divergence-conforming B-spline spaces have established themselves as attractive discretizations for flow problems \cite{3, 4, 5}. One important reason for this is that the discrete problem inherits much of the structure of the continuous level, i.e., many of the conservation properties are satisfied by the numerical approximation in a pointwise sense. Much of this structure is thought to be important for the accurate modeling of turbulence, and it is thus natural to explore the application of divergence-conforming discretizations to turbulence models such as RB-VMS.

However, the RB-VMS technique is not immediately transferrable to compatible B-splines, as extra terms in the continuity equation ruin the structure of these divergence-conforming discretizations. The crux is that the approximation of the fine-scale velocity is itself not divergence-free in general. Therefore, the fine-scale problem is revisited, and fine-scale solutions are similarly sought in the space of pointwise solenoidal functions. We suggest different strategies to arrive at such divergence-conforming VMS formulations, and present planar and 3D numerical results.

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Goal-oriented adaptive methods aim to adaptively approximate output quantities of interest of solutions to PDEs, with the least amount of computational effort. In typical adaptive (FEM or BEM) computations, a “double” rate of convergence (the sum of the primal energy-norm rate and the dual energy-norm rate) is observed with respect to the number of degrees of freedom in the approximation space.

In this contribution we will present an analysis of the convergence of goal-oriented adaptivity in abstract settings (the work of which can be found in [1]), which extends all existing prior results for goal-oriented adaptive FEM and goal-oriented adaptive BEM. The setting allows for any linear problem that complies with the Lax–Milgram Lemma, includes axiomatic adaptive components as in [2], and uses an extension of the marking strategy in the seminal work [3] or the one from [4].

References


THE NONLINEAR PETROV–GALERKIN METHOD IN
BANACH SPACES: ELIMINATING THE GIBBS PHENOMENA

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Is it possible to obtain near-best approximations to solutions of linear operator equations in a general Banach-space setting? Can this be done with guaranteed stability?

In this talk we address these questions by considering nonstandard, nonlinear Petrov–Galerkin discretisations, proposed in [1], which aim to guarantee stability in general Banach-space settings, and builds on ideas of residual minimisation [2] and the recent Hilbert-space theory of optimal Petrov-Galerkin methods [3].

We demonstrate that the inexact (implementable) version is naturally related to a mixed method with a monotone nonlinearity. For this method, optimal a priori error estimates hold (a la Céa / Babuška), with constants depending on the geometry of the involved Banach spaces.

As an elementary, but important, application of the nonlinear Petrov–Galerkin method, we consider the advection equation in dual Sobolev spaces (of integrability $p$). It is demonstrated that in the approximation of solutions with discontinuities, the Gibbs phenomena, which is inherently present in the Hilbert case ($p = 2$), is eliminated as $p \to 1$.

References


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