MAFELAP 2016

Conference on the Mathematics
of Finite Elements and Applications

14–17 June 2016

Mini-Symposium: Numerical methods for viscous flow in porous media

Organisers:

Ricardo Oyarzua and Ricardo Ruiz Baier

Abstracts in alphabetical order

Contents

A POSTERIORI ERROR ANALYSIS FOR A VISCOUS FLOW–TRANSPORT PROBLEM

Mario Alvarez[1a], Gabriel N. Gatica[1b] and Ricardo Ruiz-Baier[2]

[1]CI^{2}MA and Departamento de Ingeniería Matemática,
Universidad de Concepción, Casilla 160-C, Concepción, Chile,
[a]mguadamuz@ci2ma.udec.cl, [b]ggatica@ci2ma.udec.cl

[2]Mathematical Institute, Oxford University,
Andrew Wiles Building, Woodstock Road, Oxford, UK,
ruizbaier@maths.ox.ac.uk

In this talk we present an *a posteriori* error analysis for an augmented mixed–primal finite element approximation of a stationary viscous flow and transport problem. The governing system corresponds to a scalar, nonlinear convection-diffusion equation coupled with a Stokes problem with variable viscosity, and it serves as a prototype model for sedimentation-consolidation processes and other phenomena where the transport of species concentration within a viscous fluid is of interest. The solvability of the continuous mixed–primal formulation along with *a priori* error estimates for a finite element scheme using Raviart-Thomas spaces of order k for the stress approximation, and continuous piecewise polynomials of degree $\leq k+1$ for both velocity and concentration, have been recently established in [M. Alvarez et al., ESAIM: Math. Model. Numer. Anal. 49 (5) (2015) 1399–1427]. Here we derive two efficient and reliable residual-based a posteriori error estimators for that scheme: For the first estimator, and under suitable assumptions on the domain, we apply a Helmholtz decomposition and exploit local approximation properties of the Clément interpolant and Raviart-Thomas operator to show its reliability. On the other hand, its efficiency follows from inverse inequalities and the localization arguments based on triangle-bubble and edge-bubble functions. Secondly, an alternative error estimator is proposed, whose reliability can be proved without resorting to Helmholtz decompositions. Our theoretical results are then illustrated via some numerical examples, highlighting also the performance of the scheme and properties of the proposed error indicators.

References

[1] M. Alvarez, G.N. Gatica and R. Ruiz-Baier, *An augmented mixed–primal finite element method for a coupled flow–transport problem.* ESAIM: Math. Model. Numer. Anal., 49 (2015), no. 5, 1399–1427.

[2] M. Alvarez, G.N. Gatica and R. Ruiz-Baier, *A mixed-primal finite element approximation of a steady sedimentation-consolidation system.* M3AS: Math. Models Methods Appl. Sci., 26 (2016), no. 5, 897–900.

[3] I. Babuška and G.N. Gatica, *A residual-based a posteriori error estimator for the Stoke-Darcy coupled problem.* SIAM J. Numer. Anal. 48 (2010), no. 2, 498–523.

[4] A.E. Boycott, *Sedimentation of blood corpuscules.* Nature 104 (1920), 532.

[5] G.N. Gatica, *A note on stable Helmholtz decompositions in 3D*. Preprint 2016-03, Centro de Investigación en Ingeniería Matemática (CI^{2}MA). Universidad de Concepción, Chile, (2016). [available from `http://www.ci2ma.udec.cl`]

[6] R. Verfürth , *A Posteriori Error Estimation Techniques for Finite Element Methods*. Numerical Mathematics and Scientific Computation, Oxford University Press, 2013.

A POSTERIORI ERROR ANALYSIS FOR A FULLY-MIXED FORMULATION OF THE NAVIER–STOKES/DARCY COUPLED PROBLEM WITH NONLINEAR VISCOSITY

Sergio Caucao[1a], Gabriel N. Gatica[1b] and Ricardo Oyarzúa[2]

[1]CI^{2}MA and Departamento de Ingeniería Matemática,
Universidad de Concepción, Chile
[a]`scaucao@ci2ma.udec.cl`, [b]`ggatica@ci2ma.udec.cl`

[2]GIMNAP-Departamento de Matemática,
Universidad del Bío-Bío, Chile,
and CI^{2}MA, Universidad de Concepción, Chile,
`royarzua@ubiobio.cl`

We present an *a posteriori* error analysis for an augmented fully-mixed finite element method for the coupling of fluid flow with porous media flow in 3D. The flows are governed by a class of nonlinear Navier–Stokes and linear Darcy equations, respectively, and the corresponding transmission conditions are given by mass conservation, balance of normal forces, and the Beavers–Joseph–Saffman law. We consider dual-mixed formulations in both domains, and the nonlinearity involved in the Navier–Stokes region is handled by setting the strain and vorticity tensors as auxiliary unknowns. In turn, since the transmission conditions become essential, they are imposed weakly, which yields the introduction of the traces of the porous media pressure and the fluid velocity as associated Lagrange multipliers. A feasible choice of finite element subspaces includes piecewise constants, Raviart–Thomas elements of lowest order, continuous piecewise linear elements, and piecewise constants for the strain tensor, stress, velocity, and vorticity in the fluid, Raviart–Thomas elements of lowest order and piecewise constants for the velocity and pressure in the porous medium, together with continuous piecewise linear elements for the traces. We derive a reliable and efficient residual-based *a posteriori* error estimator for the coupled problem. By making use of the global inf-sup condition, Helmholtz decompositions in both media, and local approximation properties of the Clément interpolant and Raviart–Thomas operator, the reliability of the estimator is obtained under a suitable smallness data assumption. On the other hand, inverse inequalities, the localization technique based on tetrahedron-bubble and face-bubble functions and known results from previous works, are the main tools for proving the efficiency of the estimator. Finally, some numerical results confirming the properties of the estimator and illustrating the performance of the associated adaptive algorithm are also reported.

THE INTERFACE CONTROL DOMAIN DECOMPOSITION (ICDD) METHOD FOR THE STOKES-DARCY COUPLING

Marco Discacciati[1], Paola Gervasio[2], Alessandro Giacomini[2] and Alfio Quarteroni[3]

[1]Department of Mathematical Sciences, Loughborough University, UK
m.discacciati@lboro.ac.uk

[2]DICATAM, Università degli Studi di Brescia, Italy
paola.gervasio@unibs.it, alessandro.giacomini@unibs.it

[3]MATHICSE, Chair of Modelling and Scientific Computing,
École Polytechnique Fédérale de Lausanne, Switzerland
alfio.quarteroni@epfl.ch

In this talk we present a new technique called Interface Control Domain Decomposition (ICDD) method to couple the Stokes and the Darcy equation to model the filtration of a free fluid through a porous medium. According to this approach the coupled problem is reformulated as an optimal control problem with control variables corresponding to the traces of the velocity and of the pressure on the boundary of an overlapping region between the Stokes and the Darcy subdomains. The coupling between the two models is practically achieved by imposing the continuity of velocity and pressure across the boundary of the overlapping region in a least-squares sense. No additional modelling is required. We will show that the optimal control problem is well-posed, we will present some numerical tests to illustrate the behaviour of the method, and we will compare it with the more classical approach based on the Beavers-Joseph-Saffman condition.

A CONSERVATIVE DISCRETIZATION OF BIOT'S MODEL FOR SOIL CONSOLIDATION

Guido Kanschat

IWR, Universität Heidelberg, Germany
kanschat@uni-heidelberg.de

We present a finite element discretization of Biot's linear consolidation model. The method couples the volumetric part of displacement with the fluid in a projection free and thus strongly conservative way. We discuss a priori error estimates and present numerical results.

PARAMETER-ROBUST DISCRETIZATION AND PRECONDITIONING OF BIOT'S CONSOLIDATION MODEL

Jeonghun J. Lee[a], Kent-Andre Mardal[b] and Ragnar Winther[c]

Department of Mathematics, University of Oslo, Norway
[a]jeonghul@math.uio.no, [b]kand-and@math.uio.no,
[c]rwinther@math.uio.no

Biot's consolidation model describes behaviors of a poroelastic solid saturated by a Newtonial fluid. The model has wide applications from geophysics to computational biomechanics, so there is a strong need of good numerical methods for it.

For the studies with numerical simulations, it is important to develop efficient numerical methods for problems with realistic parameter ranges. In this talk, we discuss finite element discretization and preconditioners for the problem, which are robust for realistic ranges of the elastic moduli and the permeability.

A MIXED FEM FOR A VORTICITY FORMULATION OF THE BRINKMAN EQUATIONS

Verónica Anaya[1], David Mora[1,2a], Ricardo Oyarzúa[1,2b] and Ricardo Ruiz-Baier[3]

[1]GIMNAP, Departamento de Matemática, Universidad del Bío-Bío,
Casilla 5-C, Concepción, Chile.
vanaya@ubiobio.cl

[2]Centro de Investigación en Ingeniería Matemática (CI2MA),
Universidad de Concepción, Concepción, Chile.
[a]dmora@ubiobio.cl, [b]royarzua@ubiobio.cl

[3]Mathematical Institute, Oxford University,
Andrew Wiles Building, Woodstock Road, Oxford, UK.
ruizbaier@maths.ox.ac.uk

In this talk, we develop a mixed finite element method for the Brinkman equations formulated in terms of velocity, vorticity and pressure. Employing the Babuška–Brezzi theory, it is proved that the resulting continuous and discrete variational formulations are well-posed. In particular, we show that Raviart-Thomas elements of order $k \geq 0$ for the approximation of the velocity field, piecewise continuous polynomials of degree $k+1$ for the vorticity, and piecewise polynomials of degree k for the pressure, yield unique solvability of the discrete problem. We establish a priori error estimates in the natural norms. Finally, we report several numerical experiments illustrating the behavior of the proposed scheme and confirming our theoretical results.

DIVERGENCE-FREE-PRESERVING DISCRETIZATIONS OF INCOMPRESSIBLE FLOW

Michael Neilan

Department of Mathematics, University of Pittsburgh, United States
neilan@pitt.edu

We construct conforming finite element spaces for the Stokes and Navier–Stokes problem in two and three dimensions that yield divergence–free velocity approximations. The derivation of the finite element pairs is motivated by a smooth de Rham complex that is well–suited for the Stokes problem. We discuss the stability and convergence properties of the new elements and outline the construction of reduced elements that have fewer unknowns.

DISCONTINUOUS APPROXIMATION OF VISCOUS TWO-PHASE FLOW IN HETEROGENEOUS POROUS MEDIA

Raimund Bürger[1], Sarvesh Kumar[2], Sudarshan Kumar K.[3] and Ricardo Ruiz-Baier[4]

[1]CI^{2}MA and Departamento de Ingeniería Matemática, Universidad de Concepción, Casilla 160-C, Concepción, Chile.
rburger@ing-mat.udec.cl

[2]Department of Mathematics, Indian Institute of Space Science and Technology, Thiruvananthapuram 695 547, Kerala, India.
sarvesh@iist.ac.in

[3]CI^{2}MA, Universidad de Concepción, Casilla 160-C, Concepción, Chile.
skumar@ci2ma.udec.cl

[4]Mathematical Institute, Oxford University, Andrew Wiles Building, Woodstock Road, Oxford, UK.
ruizbaier@maths.ox.ac.uk

Runge-Kutta Discontinuous Galerkin (RKDG) and Discontinuous Finite Volume Element (DFVE) methods are applied to a coupled flow-transport problem describing the immiscible displacement of a viscous incompressible fluid in a non-homogeneous porous medium. The model problem consists of a nonlinear pressure-velocity equation assuming Brinkman flow, coupled to a nonlinear hyperbolic equation governing the mass balance (saturation equation). The mass conservation properties inherent to finite volume-based methods motivate a DFVE scheme for the approximation of the Brinkman flow in combination with a RKDG method for the spatio-temporal discretization of the saturation equation. The stability of the uncoupled schemes for the flow and for the saturation equation are analyzed, and several numerical experiments illustrate the robustness of the numerical method.

MULTI-TIME STEPPING SCHEMES FOR COUPLED POROUS MEDIUM AND FREE FLOW PROBLEMS

Iryna Rybak

Institute of Applied Analysis and Numerical Simulation,
University of Stuttgart, Germany
rybak@ians.uni-stuttgart.de

Physical systems, where a porous medium is in contact with a free fluid, arise in a variety of environmental and industrial problems. Striking examples originate from terrestrial-atmospheric contact zones, surface water–groundwater interaction, filters and fuel cells. Flow and transport processes in these systems evolve on different space and time scales yielding different sets of equations in the flow domains and the necessity of interface conditions to complete the model formulation. The multiscale nature of the processes in the coupled system also contributes to the complexity of the problem from the numerical point of view. These multiple length and time scales should be taken into account for accurate and efficient numerical implementation of transitions between the flow domains. The talk is focused on the development and analysis of mass conservative multi-time stepping algorithms to simulate such coupled systems in environmental applications.