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Abstracts in alphabetical order

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HIGH-ORDER DISCONTINUOUS GALERKIN APPROXIMATIONS TO SECOND-ORDER ORDINARY DIFFERENTIAL EQUATIONS WITH APPLICATIONS TO ELASTODYNAMICS

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In this talk we propose and analyze a high-order discontinuous Galerkin finite element method for the time integration of linear second-order ordinary differential equations. These equations typically arise after space semi-discretization of second order hyperbolic-type differential problems, e.g., the elastodynamics equation. After introducing the new method, we analyze its well-posedness and present a-priori error estimates in a suitable (mesh-dependent) norm. Numerical results are also presented to verify the theoretical estimates.

FINITE ELEMENT METHODS FOR DEGENERATE HAMILTON-JACOBI-BELLMAN EQUATIONS

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Hamilton-Jacobi-Bellman (HJB) equations characterise the value functions of optimal control problems. For a wide range of control problems one can compute optimal control policies from the partial derivatives of the value function.

An important tool in the analysis of HJB equations and their numerical approximations is the concept of viscosity solutions. Its definition is based on sign information on function values of candidate solutions, leading typically to proofs of uniform convergence of numerical methods. It is more difficult to prove convergence in other norms if solely viscosity solutions are used.

The use of weak solutions, familiar from semilinear differential equations, in the context of Hamilton-Jacobi-Bellman equations is delicate because often uniqueness cannot be ensured. However, we believe that combining the notions of viscosity and weak solution is attractive for numerical analysis: the former to deal with uniqueness and the later to study convergence of partial derivatives.

In a previous work the uniform convergence of P1 finite element approximations to the viscosity solutions of isotropic, degenerate parabolic HJB equations was shown. In addition $L^2(H^1)$ convergence was demonstrated, under the assumption that the HJB equation is uniformly parabolic.

Yet in a wide range of applications, in particular from financial mathematics, the resulting Bellman equations are only degenerately parabolic. In this talk I will explain how to remove the assumption of uniform parabolicity and verify that strong convergence in weighted $L^2(H^1_\gamma)$ spaces can be maintained. This provides the basis to recover optimal control policies for degenerate optimal control problems.

TWO LEVEL NON-OVERLAPPING AND OVERLAPPING SCHWARZ METHODS FOR DISCONTINUOUS GALERKIN APPROXIMATIONS OF SECOND AND FOURTH ORDER ELLIPTIC PROBLEMS

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We present some two-level non-overlapping and overlapping additive Schwarz domain decomposition methods for the solution of the linear systems resulting from symmetric interior penalty discontinuous Galerkin discretizations of second and fourth order elliptic problems. In particular we investigate the influence of the penalty terms as well as the choice of the coarse mesh spaces on the condition numbers of the preconditioned linear systems. We identify significant differences between the two methods as far as such dependences are concerned. The numerical experiments conducted are largely in agreement with the theoretical results.

A POSTERIORI ERROR CONTROL & ADAPTIVITY FOR EVOLUTION SCHRÖDINGER EQUATIONS (PART A)

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We provide a posteriori error estimates in the $L^\infty(L^2)-$norm for fully discrete approximations for a class of evolution Schrödinger equations, including nonlinear Schrödinger equations up to the critical exponent. For the discretisation in time we use the relaxation Crank-Nicolson scheme, introduced by Besse in [Ch. Besse, *A relaxation scheme for the nonlinear Schrödinger equation*, SIAM J. Numer. Anal. 42 (2004) 934– 952]. For the spatial discretisation we use finite element spaces that are allowed to change from one time-step to another.

For the derivation of estimates we use the reconstruction technique and nonlinear stability arguments as in the continuous problem. More precisely, key ingredients for our analysis include the time-space reconstruction for the relaxation Crank-Nicolson finite element scheme; the conservation laws available for the continuous problem; and appropriate bounds of the $L^\infty(L^2)-$norm of the gradient of the exact solution of the continuous problem.

Various numerical experiments verify and complement our theoretical results. The numerical implementations are performed using uniform partitions in time and space, and verify that the a posteriori estimator converges with the same rate as the exact error.

Based on the a posteriori estimator, we further design and analyse a time-space adaptive algorithm. The adaptive algorithm is shown to perform satisfactorily. More precisely, it drastically reduces the computational cost for Schrödinger equations in the semiclassical regime and nonlinear Schrödinger equations on the critical exponent.

This work will be presented in two linked talks (as Part A and Part B), with this talk (Part A) being concentrated more on the theoretical aspects.

A POSTERIORI ERROR CONTROL & ADAPTIVITY FOR EVOLUTION SCHRÖDINGER EQUATIONS (PART B)

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We provide a posteriori error estimates in the $L^\infty(L^2)-$norm for fully discrete approximations for a class of evolution Schrödinger equations, including nonlinear Schrödinger equations up to the critical exponent. For the discretisation in time we use the relaxation Crank-Nicolson scheme, introduced by Besse in [Ch. Besse, *A relaxation scheme for the nonlinear Schrödinger equation*, SIAM J. Numer. Anal. 42 (2004) 934– 952]. For the spatial discretisation we use finite element spaces that are allowed to change from one time-step to another.

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WELL-BALANCED KINETIC SCHEMES FOR THE SHALLOW WATER EQUATION WITH BULK RECHARGE AND DISCHARGE

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Kinetic schemes developed in the late nineties and throughout the noughties [3,2] to include source terms coming from gravitational pull. These have proved crucial in modeling river flows over long (physical) times.

In flood risk assessment models Saint-Venant's shallow water equation must be coupled with other equations describing interacting meteorological and hydrogeological phenomena such as rain and groundwater flows. The SWE must therefore be appropriately modified to accommodate source and sink terms, so "classical kinetic schemes" as described above are no longer valid. While modifications of SWE in this direction have been recently proposed, e.g., [1, eq.(1.3–4)], in our approach we depart from the extant literature by proposing a model that is, to the best of our knowledge, novel in that it is both "entropy-consistent" and "naturally extends" the SWE by respecting its kinetic formulation connections. This allows the derivation of fluxes that make finite volume, and possibly discontinuous Galerkin schemes able to tackle long time integration.

We call our extension "natural" as we approached it via matched asymptotic expansions from the Navier–Stokes model with infiltration–recharge boundary conditions similar to the Beavers–Joseph–Saffmann conditions appearing in fluid-solid interaction problems. This puts our model on very solid "first-principle" bases.

Of course, only validation against physical data will decide whether our model is more accurate than previous ones; we are working in this direction with our industrial partners, in view of this, we have identified many numerical examples where our model exhibits features such as shocks and waves (caputured thanks to the kinetic formulation) that are quite different from competing models. I will conclude by showing these modes.

1. O. Delestre. *Simulation du ruissellement d'eau de pluie sur des surfaces agricoles.* Docteur, Université D'Orléans, École Doctorale sciences et technologie laboratoire : MAPMO, juillet 2010.

2. B. Perthame and C. Simeoni. A kinetic scheme for the Saint-Venant system with a source term. *Calcolo*, 38(4):201–231, 2001.

3. J.-F. Gerbeau and B. Perthame. Derivation of viscous Saint-Venant system for laminar shallow water; numerical validation. *Discrete Contin. Dyn. Syst. Ser. B*, 1(1):89–102, 2001.

APPROXIMATIONS TO TRANSPORT, CONVECTION-DIFFUSION AND NONLINEAR HYPERBOLIC PROBLEMS

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Results on a posteriori error control of transport, convection-diffusion and nonlinear hyperbolic problems are discussed. We consider classical discretisation methods combined with self adapted meshes. The methods are compared to certain recently proposed "idealised" discretisations.

hp-ADAPTIVE GALERKIN TIME STEPPING METHODS FOR NONLINEAR IVPS

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In this talk, we will discuss the derivation of conditional a posteriori error estimates for continuous Galerkin approximations to nonlinear IVPs with an emphasis on those with solutions which exhibit finite-time existence. We then discuss the design of adaptive algorithms based on this error estimator with the goal of approximating the blow-up time. Numerical experiments complement the theoretical results.

APPROXIMATION OF LIQUID CURTAINS

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Liquid curtains arise in various industrial applications. Curtain coating is where an object is passed through a smooth curtain of liquid. Examples of this range from painting of materials to coating of photographic films but all are essentially coating procedures allowing for uniform coating of various substrates with some film. The stability of curtains in these applications is crucial as a stable procedure for this allows for a uniform coating with minimal materials and time.

In this work we propose a numerical method to approximate a thin film model of liquid curtains. The stability of the numerical approximation of the curtain is examined at a variety of length scales. At small length scales we compare our simulations to experimental work already conducted demonstrating the numerical method correctly predicts the stability of curtains and then test our numerical method at larger length scales, illustrating the applicability of large scale curtains for commercial use.