MAFELAP 2016

Conference on the Mathematics of Finite Elements and Applications

14–17 June 2016

Mini-Symposium: Recent developments in isogeometric analysis

Organisers:

Carla Manni and Hendrik Speleers

Abstracts in alphabetical order

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AN ISOGEOMETRIC APPROACH TO SYMMETRIC GALERKIN BOUNDARY ELEMENT METHOD

Alessandra Aimi[1a], Mauro Diligenti[1b], Maria Lucia Sampoli[2] and Alessandra Sestini[3]

[1]Department of Mathematics and Computer Science, University of Parma, Italy
[a]alessandra.aimi@unipr.it, [b]mauro.diligenti@unipr.it

[2]Department of Information Engineering and Mathematical Sciences,
University of Siena, Italy
sampoli@unisi.it

[3]Department of Mathematics and Computer Science, University of Florence, Italy
alessandra.sestini@unifi.it

The Isogeometric Analysis (IgA) approach, introduced by Hughes and collaborators [4], establishes a strict relation between the geometry of the problem domain and the approximate solution representation, giving surprising computational advantages. It has also brought a renewed interest for Boundary Element Methods (BEMs), since one has to discretize only the boundary of the problem domain and this can be done in an accurate way by powerful geometric modeling techniques.
Among BEMs, the Symmetric Galerkin version (SGBEM) [1] is recognized as particularly suitable for mixed boundary value problems and for coupling with FEM.
In this context, we have recently introduced the IgA concept into SGBEM, using classical B-splines [2] to represent both the boundary and the approximate solution.
In this talk we will discuss about an extension including NURBS and generalized B-splines [5, 3]. The computational advantages over standard and curvilienar SGBEMs, where the numerical solution is given by means of Lagrangian basis functions, will be underlined by several numerical results.

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ON THE DESIGN OF DUAL-COMPATIBLE T-MESH REFINEMENT ALGORITHMS

Cesare Bracco and Carlotta Giannelli

Department of Mathematics and Computer Science "U. Dini",
University of Florence, Italy
cesare.bracco@unifi.it

Amongst the several types of adaptive spline spaces proposed in recent years , also in connection with related application in isogeometric analysis, the spaces of T-splines have some interesting and unique features. The functions spanning the space, the T-splines, are a natural generalization of tensor-product B-splines depending on the local topology of the T-mesh. If the T-mesh is dual-compatible (or, equivalently, analysis-suitable), the T-splines are linear independent, and therefore they form a basis (see, for instance, [L. Beirão da Veiga, A. Buffa, G. Sangalli and R. Vazquez, Analysis-suitable T-splines of arbitrary degree: definition and properties, Math. Mod. Meth. Appl. Sci. 23 (2013), pp. 1979-2003]). As a consequence, a refinement algorithm that preserves the dual-compatible structure of the T-mesh guarantees that the corresponding T-splines form a basis. In this talk we will discuss possible alternatives to existing T-mesh refinement algorithms (see [M.A. Scott, X. Li, T.W. Sederberg and T.J.R. Hughes, Local refinement of analysis-suitable T-splines, Comput. Methods Appl. Mech. Engrg. 213 (2012) pp. 206-222] and [P. Morgenstern and D. Peterseim, Analysis-suitable adaptive T-mesh refinement with linear complexity, Comput. Aided Geom. D. 34 (2015), pp. 50-66]). Our arguments are based on studying the influence of refinements on the local preservation of the dual-compatible structure, and allow us to study the complexity of the algorithm, a fundamental ingredient for the analysis of adaptive isogeometric methods.

OPTIMAL CONVERGENCE FOR ADAPTIVE IGA BOUNDARY ELEMENT METHODS

Michael Feischl[1], Gregor Gantner[2a], Alexander Haberl[2b], Dirk Praetorius[2c] and Stefan Schimanko[2d]

[1]School of Mathematics and Statistics,
University of New South Wales, Australia
m.feischl@unsw.edu.au

[2]Institute for Analysis and Scientific Computing, TU Wien, Austria
[a]gregor.gantner@tuwien.ac.at, [b]alexander.haberl@asc.tuwien.ac.at
[c]dirk.praetorius@tuwien.ac.at, [d]stefan.schimanko@tuwien.ac.at

A posteriori error estimation and optimal adaptive mesh-refinement are well-established for the Galerkin boundary element method (BEM) with piecewise polynomial ansatz functions on polygonal boundaries. In contrast to that, the mathematically reliable a posteriori error analysis for isogeometric BEM (IGABEM) is still in its infancy. In our talk, we discuss recent results on reliable a posteriori error estimators (see [1] for Galerkin IGABEM resp. [2] for collocation IGABEM in 2D) and on optimal convergence of corresponding adaptive IGABEM algorithms in 2D (see e.g. [3]).

As model example, we consider the weakly-singular as well as the hyper-singular integral equation for the 2D Laplacian and the corresponding weighted-residual error estimator which controls the (in general, non-computable and unknown) discretization error in the $\widetilde{H}^{-1/2}$ resp. $\widetilde{H}^{1/2}$ norm. Its local contributions are used for adaptive IGABEM computations to steer an adaptive algorithm of the form

$$\boxed{\texttt{Solve}} \longrightarrow \boxed{\texttt{Estimate}} \longrightarrow \boxed{\texttt{Mark}} \longrightarrow \boxed{\texttt{Refine}}$$

for which optimal convergence behaviour is proved. Unlike available results in the literature, the adaptive algorithm steers the local mesh-refinement as well as the local smoothness of the ansatz functions across nodes of the boundary partition. The algorithm automatically detects and resolves jumps and singularities of the exact solution as well as possible smooth parts. If compared to uniform mesh-refinement as well as adaptive standard BEM based on piecewise polynomials, this dramatically reduces the storage requirements and the computing time needed to achieve a certain prescribed accuracy.

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ROBUST MULTIGRID FOR ISOGEOMETRIC ANALYSIS USING SUBSPACE CORRECTION

Clemens Hofreither[1] and Stefan Takacs[2]

[1]Institute of Computational Mathematics,
Johannes Kepler University Linz, Austria
chofreither@numa.uni-linz.ac.at

[2]Johann Radon Institute for Computational and Applied Mathematics (RICAM),
Austrian Academy of Sciences
stefan.takacs@ricam.oeaw.ac.at

We present a robust and efficient multigrid method for isogeometric discretizations using tensor product B-splines of maximum smoothness. Our method is based on a stable splitting of the spline space into a large subspace of "interior" splines which satisfy a robust inverse inequality, as well as one or several smaller subspaces which capture the boundary effects responsible for the spectral outliers known to occur in Isogeometric Analysis. We then construct a multigrid smoother based on a subspace correction approach, applying a different smoother to each of the subspaces. For the interior splines, we use a mass smoother, whereas the remaining components are treated with suitably chosen Kronecker product smoothers or direct solvers.

The resulting multigrid method exhibits iteration numbers which are robust with respect to the spline degree and the mesh size. Furthermore, it can be efficiently realized both for two- and three-dimensional problems. Our numerical examples show further that the iteration numbers also scale relatively mildly with the problem dimension.

SPECTRAL ANALYSIS OF MATRICES ARISING IN GB-SPLINE ISOGEOMETRIC METHODS

Carla Manni[1], Fabio Roman[2] and Hendrik Speleers[1]

[1]Department of Mathematics, University of Roma " Tor Vergata", Italy
[a]manni@mat.uniroma2.it, [b]speleers@mat.uniroma2.it

[2]Department of Mathematics, University of Torino, Italy
fabio.roman@unito.it

Generalized splines are smooth piecewise functions with sections in spaces more general than classical algebraic polynomials. Interesting examples are spaces comprising trigonometric or hyperbolic functions. Under suitable assumptions, generalized splines enjoy all the desirable properties of polynomial splines, including a representation in terms of basis functions (the so-called GB-splines) that are a natural extension of the polynomial B-splines.

Tensor-product GB-splines are an interesting problem-dependent alternative to tensor-product polynomial B-splines and NURBS in isogeometric analysis (IgA). Like any discretization method, the IgA paradigm requires to solve large linear systems. A deep understanding of the spectral properties of the related matrices is crucial for the design of fast solvers for these linear systems.

In this talk we focus on IgA discretizations based on trigonometric or hyperbolic GB-splines. In particular, we prove that the corresponding stiffness matrices possess an asymptotic eigenvalue distribution which can be compactly described by a function, the so-called symbol, see [2]. These results extend those obtained for IgA discretization methods based on polynomial B-splines, see [1], and strengthen the structural similarity between the polynomial and the generalized setting.

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ON THE USE OF NUMERICAL TENSOR METHODS IN ISOGEOMETRIC ANALYSIS

Angelos Mantzaflaris

RICAM, Austrian Academy of Sciences, Linz, Austria
Angelos.Mantzaflaris@oeaw.ac.at

The use of tensor methods in the field of numerical simulation was explored the last decade, with the aim to overcome the curse of dimensionality, ie. the exponential complexity with respect to the spatial dimension of the computational domain. With the advent of Isogeometric Analysis (IGA) during the same period of time, the very same difficulty of dimensionality has appeared, in particular in the task of matrix assembly. Indeed, this task is more challenging than in the case of traditional finite element methods. This is due to factors such as the increased degree and the larger supports of the ansatz functions (tensor-product B-splines), that burden the sparsity pattern and bandwidth of the system matrix.

In an attempt to address this problem, we developed an interpolation-based approach that approximately transforms the integrands into piecewise polynomials and uses look-up tables to evaluate their integrals [1]. Shortly after, this led us to employ tensor methods to accelerate the assembly process further [2], focusing on the two-dimensional (bivariate) case.

In particular, we obtained a compact representation of the matrices that occur in IGA as sums of a small number of Kronecker products of auxiliary matrices, which are defined by univariate integrals. This representation, which is based on a low-rank tensor approximation of certain parts of the integrands, made it possible to achieve a significant speedup of the assembly process without compromising the overall accuracy of the simulation. The talk will describe our recent progress towards the extension of these methods to the multivariate case (i.e., to any dimension).

This is joint work with Bert Jüttler, Ulrich Langer and Boris Khoromskij.

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DESIGN AND ANALYSIS ON SURFACES WITH IRREGULARITIES

Jörg Peters[1a], Kęstutis Karčiauskas[2] and Thien Nguyen[1b]

[1]Department of Computer & Information Science & Engineering,
University of Florida, USA.
[b]tt0@cise.ufl.edu, [a]jorg@cise.ufl.edu

[2]Department of Mathematics, Vilnius University, Lithuania.

Based on the fact that 'every G^k construction yields a finite element suitable for the iso-parametric IGA framework', this talk explores issues of computing across parametric singularities, including the design of free-form surfaces and the analysis of functions on those surfaces.

TWO MATHEMATICAL ASPECTS OF ISOGEOMETRIC ANALYSIS: QUASI-OPTIMAL ADAPTIVE MESH REFINEMENT AND SUPERIOR EIGENVALUE APPROXIMATION

Daniel Peterseim

Institute for Numerical Simulation, Bonn University, Germany
peterseim@ins.uni-bonn.de

This talk presents two results in the context of Isogeometric Analysis. The first result concerns the analysis-suitable adaptive refinement of T-meshes and its quasi-optimality. The second part discusses global stability properties of the Rayleigh-Ritz approximation of Laplace eigenvalues by B-splines and the possible superiority over classical finite elements. This talk is based on joint works with Dietmar Gallistl, Pascal Huber and Philipp Morgenstern.

PARALLEL ISOGEOMETRIC TIME DOMAIN MAXWELL AND VLASOV-MAXWELL SOLVERS

Ahmed Ratnani[1,2] and Eric Sonnendrücker[1,2]

[1]Max-Planck Institut für Plasmaphysik, Garching, Germany
ahmed.ratnani@ipp.mpg.de

[2]Technische Universität München, Garching, Germany
sonnen@ipp.mpg.de

Modern finite elements techniques for Maxwell equations rely on ideas from differential geometry and more precisely on the existence of discrete spaces that provide an exact De Rham sequence. In [1] the classical theory of discrete DeRham complexes, was extended to iso-geometric analysis for the steady-state Maxwell's equations, providing a discrete exact DeRham sequence involving discrete spaces based on B-splines. In [2], we have derived a 2D B-Splines solver for the Time Domain Maxwell problem.

In this work, we present a parallel 2D/3D IsoGeometric solver for both the Time Domain Maxwell equations and the Vlasov-Maxwell problem. In the later, a hybrid Particle In Cell method is introduced, where particles *live* in the logical domain while the velocity is advanced in the physical domain.

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THE GLT CLASS AS A GENERALIZED FOURIER ANALYSIS AND APPLICATIONS

Stefano Serra-Capizzano

Department of Science and High Technology, Insubria University, Italy
stefano.serrac@uninsubria.it

Recently, the class of Generalized Locally Toeplitz (GLT) sequences has been introduced [5, 6] as a generalization both of classical Toeplitz sequences and of variable coefficient differential operators and, for every sequence of the class, it has been demonstrated that it is possible to give a rigorous description of the asymptotic spectrum [1, 7] in terms of a function (the symbol) that can be easily identified. This generalizes the notion of a symbol for differential operators (discrete and continuous) or for Toeplitz sequences for which it is identified through the Fourier coefficients and is related to the classical Fourier analysis.

The GLT class has nice algebraic properties and indeed it has been proven that it is stable under linear combinations, products, and inversion when the sequence which is inverted shows a sparsely vanishing symbol (sparsely vanishing symbol = a symbol which vanishes at most in a set of zero Lebesgue measure). Furthermore, the GLT class virtually includes any approximation of partial differential equations (PDEs) by local methods (finite difference, finite element, isogeometric analysis, etc.) and, based on this, we demonstrate that our results on GLT sequences can be used in a PDE setting in various directions, including preconditioning, multigrid, spectral detection of branches, stability issues. We will discuss specifically the spectral potential of the theory with special attention to the IgA setting [2, 3, 4].

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QUASI-INTERPOLANTS AND LOCAL APPROXIMATION ESTIMATES FOR HIERARCHICAL SPLINE SPACES

Hendrik Speleers

Department of Mathematics, University of Rome 'Tor Vergata', Italy
speleers@mat.uniroma2.it

Hierarchical spline spaces provide a flexible framework for local refinement coupled with a remarkable intrinsic simplicity. They are defined in terms of a hierarchy of locally refined meshes, reflecting different levels of refinement. The so-called *truncated hierarchical basis* is an interesting basis for the hierarchical spline space with an enhanced set of properties compared to the classical hierarchical basis: its elements form a convex partition of unity, they are locally supported and strongly stable [1, 2].

In this talk we discuss a general approach to construct quasi-interpolants in hierarchical spline spaces expressed in terms of the truncated hierarchical basis [3, 4]. The main ingredient is the property of *preservation of coefficients* of the truncated hierarchical basis representation. Thanks to this property, the construction of the hierarchical quasi-interpolant is basically effortless. It is sufficient to consider a quasi-interpolant in each space associated with a particular level in the hierarchy, which will be referred to as a one-level quasi-interpolant. Then, the coefficients of the proposed hierarchical quasi-interpolant are nothing else than a proper subset of the coefficients of the one-level quasi-interpolants. No additional manipulations are required. Important properties – like polynomial reproduction – of the one-level quasi-interpolants are preserved in the hierarchical construction. We also discuss the local approximation order of the hierarchical quasi-interpolants in different norms, and we illustrate the effectiveness of the approach with some numerical examples.

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DISCONTINUOUS GALERKIN ISOGEOMETRIC ANALYSIS OF ELLIPTIC DIFFUSION PROBLEMS ON SEGMENTATIONS WITH GAPS AND OVERLAPS

Ioannis Toulopoulos[a], Christoph Hofer[b] and Ulrich Langer[c]

[1]Johann Radon Institute for Computational and Applied Mathematics (RICAM), Austrian Academy of Sciences (ÖAW), Linz Austria.
[a]ioannis.toulopoulos@oeaw.ac.at, [b]christoph.hofer@ricam.oeaw.ac.at
[c]ulrich.langer@ricam.oeaw.ac.at

In the Isogeometric Analysis framework for treating realistic problems, it is usually necessary to decompose the domain into volumetric subdomains (patches). More precisely, we apply a segmentation technique for splitting the initial domain into simpler subdomains and then we define the coresponding control nets of the subdomains that are used for constructing the parametrizations of the subdomains. Usually, we obtain compatible parametrizations of the subdomains, meaning that using a relative coarse control mesh, the parameterizations of the adjoining subdomain interfaces are identical.

However, this is not always the case. Due to an incorrect segmentation procedure, we can lead to non-compatible parametrizations of the geometry, meaning that the parametrized interfaces of adjusting subdomains are not identical. The result of this phenomenon is the creation of overlapping subdomains or gap regions between adjacent subdomains. It is clear that, we can not apply directly the dGIgA methods which have been proposed so far in the literature and are referred to matching interface parametrizations. In this talk, we will present a discontinuous Galerkin Isogeometric Analysis method applied on decompositions, where gap and overlapping regions can appear. We apply a multi-patch approach and derive suitable numerical fluxes on the boundaries of overlapping and gap regions, using the interior subdomain solutions, (i.e., the solution on points which are not located on the overlaps and on gaps), and in that way we connect the values of the solution of the regions where we have unique representation of the solution. The ideas are illustrated on a model diffusion problem with discontinuous diffusion coefficients. We develop a rigorous theoretical framework for the proposed method clarifying the influence of the gap/overlapping region size onto the convergence rate of the method. The theoretical estimates are supported by numerical examples in two- and three-dimensional computational domains.

This talk is based on works [1, 2, 3]. We gratefully acknowledge the financial support of this research work by the Austrian Science Fund (FWF) under the grant NFN S117-03.

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ISOGEOMETRIC DIVERGENCE-CONFORMING VARIATIONAL MULTISCALE FORMULATION OF INCOMPRESSIBLE TURBULENT FLOWS

Timo M. van Opstal[1], Jinhui Yan[2], Chris Coley[3], John A. Evans[3], Trond Kvamsdal[1] and Yuri Bazilevs[2]

[1]Department of Mathematical Sciences,
Norwegian University of Science and Technology, Norway
`timo.vanopstal@math.ntnu.no`

[2]Department of Structural Engineering,
University of California, San Diego, USA

[3]Department of Aerospace Engineering,
University of Colorado, Boulder, USA

We explore the application of the Variational Multiscale Method to divergence-conforming B-splines. Residual-based VMS has established itself as a versatile turbulence model, having been successfully applied to such complex problems as parachute deployment [1] and wind turbines [2]. Within IGA, divergence-conforming B-spline spaces have established themselves as attractive discretizations for flow problems [3, 4, 5]. One important reason for this is that the discrete problem inherits much of the structure of the continuous level, i.e., many of the conservation properties are satisfied by the numerical approximation in a pointwise sense. Much of this structure is thought to be important for the accurate modeling of turbulence, and it is thus natural to explore the application of divergence-conforming discretizations to turbulence models such as RB-VMS.

However, the RB-VMS technique is not immediately transferrable to compatible B-splines, as extra terms in the continuity equation ruin the structure of these div-conforming discretizations. The crux is that the approximation of the fine-scale velocity is itself not divergence-free in general. Therefore, the fine-scale problem is revisited, and fine-scale solutions are similarly sought in the space of pointwise solenoidal functions. We suggest different strategies to arrive at such divergence-conforming VMS formulations, and present planar and 3D numerical results.

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