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Abstracts in alphabetical order

Contents

EFFICIENT QUADRATURE FOR HIGH DEGREE ISOGEOMETRIC ANALYSIS

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In this talk we present a result on the assembly of the linear system arising in the Galerkin isogeometric method. The main interest are the cases where the degree of the approximation is raised, so that the computational cost in assembling become challenging.
Key ingredients are the application of weighted quadrature and sum-factorization. These modifications demand for a change of paradigm the existing fem-based codes. The resulting method is more efficient compared to the other approaches known in literature.

Paper in preparation:
F. Calabrò, G. Sangalli, M. Tani: "Fast formation of isogeometric Galerkin matrices: beyond element-wise assembling"

Related litterature:
Antolin, P., Buffa, A., Calabrò, F., Martinelli, M., Sangalli, G. Efficient matrix computation for tensor-product isogeometric analysis: The use of sum factorization. CMAME 285 (2015), 817-828.
Auricchio, F., Calabrò, F., Hughes, T. J. R., Reali, A., Sangalli, G. A simple algorithm for obtaining nearly optimal quadrature rules for NURBS-based isogeometric analysis. CMAME, 249 (2012), 15-27.
Calabrò F., Manni C., Pitolli F.; Computation of quadrature rules for integration with respect to refinable functions on assigned nodes, App Num 90 (2015), 168–189.
Hughes, T. J., Reali, A., Sangalli, G. Efficient quadrature for NURBS-based isogeometric analysis. CMAME, 199 (5) (2010), 301-313.
Mantzaflaris A., Juttler B., Integration by interpolation and look-up for galerkin-based isogeometric analysis, CMAME 284 (2015), 373–400

SPACE-TIME ISOGEOMETRIC ANALYSIS OF PARABOLIC EVOLUTION PROBLEMS

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We present and analyze a new stable space-time Isogeometric Analysis (IgA) method for the numerical solution of parabolic evolution equations in fixed and moving spatial computational domains. The discrete bilinear form is elliptic on the IgA space with respect to a discrete energy norm. This property together with a corresponding boundedness property, consistency and approximation results for the IgA spaces yields an a priori discretization error estimate with respect to the discrete norm. The theoretical results are confirmed by several numerical experiments with low- and high-order IgA spaces including experiments on large-scale distributed memory computers with several thousand cores.

DIRECT VISUALIZATION OF IGA SIMULATION MODELS ON MODERN GPUS

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Using isogeometric analysis, design problems can be formulated and investigated through simulation and optimization in a unified workflow without relying on intermediate discretization steps. Visualization, however, a key component in understanding results, still forces tesselation if typical software tools are used. In this talk, we will discuss a visualization software prototype that leverages the massively-parallel computational power of modern GPUs to achieve accurate and efficient visualization of IGA models without requiring pre-tesselation; rather, tesselation is performed on the fly, on-GPU at just the required level of detail for pixel-accurate representation. This approach eschews the data amplification inherent in tesselation and thus provides (near) realtime visualization capabilities even for complex models. Beyond the general architecture of our system, we illustrate how typical visualization algorithms can be adapted to our system. To conclude, we present examples from several practical applications.

SOLVING COMPRESSIBLE FLOW PROBLEMS BY ISOGEOMETRIC ANALYSIS

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Isogeometric Analysis (IgA), introduced in [3], aims at bridging the gap between Finite Element Analysis (FEA) and Computer-Aided Design (CAD) by extending classical FEA towards ansatz functions such as B-splines or NURBS (non-uniform rational B-splines), which enable the more precise or even exact representation of complex geometry objects. Since its introduction IgA gained popularity in many computational mechanics and fluid dynamics applications but its use in compressible flow calculations is very limited.

In this paper, we present our implementation of a positivity-preserving isogeometric high-resolution scheme for compressible flow problems in the open-source library G+Smo [4]. It builds upon the generalization of the algebraic flux correction paradigm [5] to multi-patch IgA as universal building block for the design of positivity-preserving high-order discretizations. In particular, we analyze techniques for the positivity-preserving multi-patch coupling and boundary treatment.

Our implementation adopts Fletcher's group formulation [1] together with an efficient edge-based formation of system matrices and vectors [2] from pre-computed coefficients to overcome the high computational costs that are typically observed in quadrature-based IgA-assembly algorithms. Finally, we extend our solution algorithm to a space-time formulation that makes it possible to combine high-order approximations in space and time. The suggested approach is applied to several test problems for compressible flows.

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ON THE DESIGN AND VERSATILE IMPLEMENTATION OF ISOGEOMETRIC METHODS

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Isogeometric analysis (IGA), introduced by Hughes and collaborators in 2005, requires the seamless integration of Finite Element Analysis (FEA) and Computer-aided Design (CAD) software. Towards the realization of this revolutionary goal, we have initiated in the year 2012 the development of the open-source, collaborative software project "Geometry + Simulation Modules" (for short, gismo, see http://gs.jku.at/gismo), which aims at providing a unified development framework for IGA. In this talk we present some highlights of the on-going development of the library. G+Smo is an object-oriented, template C++ library, that implements a generic concept for IGA, based on abstract classes for geometry map, discretization basis, assemblers, solvers and so on. It makes use of object polymorphism and inheritance techniques in order to support a variety of different discretization bases, namely B-spline, Bernstein, NURBS bases, hierarchical and truncated hierarchical B-spline bases of arbitrary polynomial order, and so on. Our design allows the treatment of geometric entities such as surfaces or volumes through dimension independent code, realized by means of template meta-programming. Available features include simulations based on continuous and discontinuous Galerkin approximation of PDEs, over conforming and non-conforming multi-patch computational domains. The codebase, currently in beta stage, focuses on both the efficiency and ease of use, promotes code quality and cross-platform compatibility and encourages the exploration of new strategies potentially better suited for isogeometric analysis, before adopting known FEA practices.

ISOGEOMETRIC ANALYSIS FOR SCALED BOUNDARY PARAMETRIZATIONS

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This contribution is concerned with a new numerical method to solve the elasticity problem for solids in boundary representation. Starting with the basic idea of the scaled boundary finite element method [Song, C. and Wolf, J.P.: The scaled boundary finite-element method for elastodynamics, CMAME 147 (1997)], we derive a formulation where the geometrical description of the boundary is sufficient for defining the equations of elasticity of the complete solid. This approach fits perfectly to the boundary representation modeling technique ('b-rep') commonly employed in computer aided design. For the analysis, the weak form of the equilibrium equations is first enforced for the circumferential direction. Applying the isogeometric paradigm, the NURBS functions that describe the boundary of the geometry form also the basis for the approximation of the displacement at the boundary. The displacement field in the radial scaling direction, on the other hand, is approximated by one-dimensional NURBS, and here we have the choice of using again a weak form and Galerkin projection or, alternatively, collocation. Overall, this procedure yields a linear system of equilibrium equations whose solution gives rise to the displacement response.

In the talk, the relation of this approach to the classical concept of Isogeometric Analysis is analyzed. Moreover, computational results from the recent paper [Chen, L., Simeon, B. and Klinkel, S.: A NURBS based Galerkin approach for the analysis of solids in boundary representation. To appear in CMAME, DOI:10.1016/j.cma.2016.03.019] are reported. Finally, we discuss also issues such as the treatment of the singularity in the scaling center and the restriction of star-shaped domains.

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ROBUST APPROXIMATION ERROR AND INVERSE ESTIMATES FOR B-SPLINES AND APPLICATIONS TO ISOGEOMETRIC ANALYSIS

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In this talk, we will discuss approximation error estimates for B-splines of polynomial degree p and maximal smoothness $p-1$, which are robust in the polynomial degree p. We will see that there are large subspaces of the spline space satisfying a corresponding inverse estimate. One example is the space of splines whose odd derivatives vanish on the boundary. However, the inverse estimate does not extend to the whole spline space, i.e., there is a small subspace of outlier splines. For many numerical methods, it is important to have both, a robust approximation error estimate and a robust inverse estimate. We will discuss possibilities how a precise characterization of the outliers can be used for the construction of fast linear solvers for problems in isogeometric analysis.

ISOGEOMETRIC PRECONDITIONERS BASED ON FAST SOLVERS FOR THE SYLVESTER EQUATION

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We address the problem of numerically solving linear systems that arise in Isogeometric Analysis (IGA). It is known that many standard methods, when applied to IGA systems, have a computational cost which significantly increase with the degree p of the splines employed as basis functions. This fact contributes in making high degree splines prohibitive for real world applications. As a consequence, a number of recent papers have attempted to reduce the dependence of computational cost from p.

Let $\mathcal{K}$ be the stiffness matrix for the Poisson problem on an arbitrary physical domain. A simple but crucial observation is that $\mathcal{K}$ can be preconditioned by the stiffness matrix for the unit hypercube, which has the form

$$\mathcal{P} = \sum_{i=1}^{d} M_1 \otimes \ldots \otimes M_{i-1} \otimes K_i \otimes M_{i+1} \otimes \ldots \otimes M_d,$$

where M_i and K_i, $i = 1, \ldots, d$, represent one-dimensional mass and stiffness matrices, and d is the problem dimension.

Our approach is based on the fact that the application of $\mathcal{P}^{-1}$ is equivalent to the solution of a tensor equation. For example, when $d = 2$ the linear system $\mathcal{P}s = r$ is equivalent to the solution of the matrix equation

$$M_2 S K_1 + K_2 S M_1 = R$$

where S and R are matrices obtained by a proper reshape of vectors s and r.

The literature on the numerical solution of such problems is vast, and we select among the available methods the ones which seem the most suited for the particular features of IGA problems. Application to 2D and 3D problems is shown, and robustness with respect to the problems parameters (such as the spline degree) is discussed. We also discuss how the geometry of the PDE domain affects the spectral properties of $\mathcal{P}^{-1}\mathcal{K}$, and propose simple strategies to partially include information on the geometry in the preconditioner.