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Abstracts in alphabetical order

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A NITSCHE-TYPE METHOD FOR HELMHOLTZ EQUATION WITH AN EMBEDDED, ACOUSTICALLY PERMEABLE INTERFACE

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We consider the Helmholtz equation of acoustic wave propagation in the situation where a permeable interface is embedded in the computational domain. The presence of the interface is represented by a complex-valued impedance function Z that relates the jump in the solution over the interface to the flux through the interface. Thus, the flux is assumed to be continuous over the interface whereas the solution may contain jump discontinuities. Such an interface condition constitutes, for instance, a macro model of a perforated plate through which sound is leaking. The real part of Z, assumed to be nonnegative, represents losses in the interface, whereas the imaginary part, which can be of either sign, corresponds to reactive effects. For low-loss interfaces with negative imaginary part of Z, so-called surface waves can appear in a layer around the interface.

The straight-forward, standard finite-element discretization of this problem leads to a variational form in which the impedance function appears in the *denominator* of a surface integral along the interface, which means that partly or fully vanishing impedance functions cannot be handled without this term blowing up. We propose another formulation, based on a variant of Nitsche's method, which seamlessly handles a complex-valued impedance function Z that is allowed to vanish. The method can be seen as an interpolation between the standard method and a classic Nitsche method that weakly enforces continuity over the interface.

We show stability of the method, in terms of a discrete Gårding inequality, for a quite general class of surface impedance functions, provided that possible surface waves are sufficiently resolved by the mesh. Moreover, we prove an a priori error estimate under the assumption that the absolute value of the impedance is bounded away from zero almost everywhere. Numerical experiments illustrate the performance of the method for a number of test cases in 2D and 3D with different interface conditions, with and without surface waves.

FINITE ELEMENT-DISCONTINUOUS GALERKIN METHOD FOR THE NUMERICAL SIMULATION OF TWO-PHASE FLOW

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The subject of the contribution is the numerical simulation of two-phase flow of immiscible fluids. Their motion is described by the incompressible Navier-Stokes equations with piecewise constant density and viscosity. The interface between the fluids is defined with the aid of the level-set method using a transport first-order hyperbolic equation. The Navier-Stokes system equipped with initial and boundary conditions and transmission conditions on the interface between the fluids is discretized by the Taylor-Hood $P2/P1$ conforming finite elements in space and the second-order BDF method in time. The transport level-set problem is solved with the aid of the space-time discontinuous Galerkin method (DGM). Numerical experiments demonstrate the applicability, accuracy and robustness of the developed method.

The results were obtained in cooperation with E. Bezchlebová, V. Dolejší and P. Sváček.

ACCURATE SPATIAL AND TEMPORAL DISCRETISATION TECHNIQUES FOR INTERFACE PROBLEMS AND FLUID-STRUCTURE INTERACTIONS IN EULERIAN COORDINATES

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Interface problems pose several challenges for discretisation, especially in the case of moving interfaces. If the interface is not resolved by the discretisation, one obtains a reduced order of convergence and possibly stability issues.

In this talk, we present discretisation schemes in both space and time in order to avoid these issues. The proposed finite element discretisation in space corresponds to a fitted finite element method that uses a fixed patch mesh that is independent of the interface location in combination with an interiour refinement that resolves the interface. For time discretisation, we use a modified time-stepping scheme that is based on a space-time continuous Galerkin approach (cG(1)). Instead of using polynomials in direction of time that cross the interface, we define Galerkin spaces on trajectories that stay within each subdomain. Similar techniques have been used within the fixed-mesh ALE method by Codina et al. We show second-order convergence for both discretisation in space and time and give a bound on the condition of the system matrix. Finally, we illustrate the capability of our approach in the context of fluid-structure interaction problems.

A LOCALLY MODIFIED FITTED FINITE ELEMENT METHOD FOR INTERFACE PROBLEMS IN SHAPE AND TOPOLOGY OPTIMIZATION

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We consider the design optimization of an electric motor by means of PDE-constrained topology and shape optimization. The goal is to find the optimal distribution of ferromagnetic material within a design subregion of the computational domain. In the course of the optimization procedure, the interface between ferromagnetic material and air regions evolves.

In every iteration of the optimization procedure, the interface between different subdomains is updated. On the updated geometry, which is in general not resolved by the finite element discretization, the state and adjoint equations have to be solved. We present an easy to implement numerical method that allows us to resolve a piecewise linear interface exactly in every iteration by only locally modifying the underlying triangular mesh. Moreover, the chosen mesh adaptation strategy ensures a maximum angle condition which yields optimal order of convergence independent of the location of the interface relative to the mesh. The presented method is based on [1].

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A STUDY ON THE ACCURACY OF IMMERSED FINITE ELEMENT METHODS

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Immersed Finite Element Methods (IFEM) are an evolution of the original Immersed Boundary Element Method (IBM) developed by Peskin [6] in the early seventies for the simulation of complex Fluid Structure Interaction (FSI) problems. In the IBM, the coupled FSI problem is discretised using a single (uniformly discretised) background fluid solver, where the presence of the solid is taken into account by adding appropriate forcing terms in the fluid equation. Dirac delta distributions are used to interpolate between the Lagrangian and the Eulerian framework in the original formulation by Peskin, while a variational formulation was introduced by Boffi et al. [1], and later generalised in Heltai and Costanzo [4] that does not require any Dirac delta approximation.

One of the key issues that kept people from adopting IBM or IFEM techniques is related to the loss in accuracy attributed to the non-matching nature of the discretisation between the fluid and the solid domains, leading to only formally optimal solvers (see, for example, Lai and Peskin [5]). In this work we exploit some techniques introduced by D'Angelo and Quarteroni [2, 3], to show that, for variational formulations, the loss in accuracy is only restricted to a thin layer of elements around the solid-fluid interface, and optimal error estimates in all norms are recovered if one uses appropriate weighted norms, or by removing the layer of non-matching cells from the error estimates.

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CONVERGENCE RESULTS WITH NATURAL NORMS: STABILIZED LAGRANGE MULTIPLIER METHOD FOR ELLIPTIC INTERFACE PROBLEMS

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A stabilized Lagrange multiplier method for second order elliptic interface problems is presented in the framework of mortar method. The requirement of LBB (Ladyzhenskaya-Babuška-Brezzi) condition for mortar method is alleviated by introducing penalty terms in the formulation. Optimal convergence results are established in natural norm which is independent of mesh. Error estimates are obtained with an assumption that: the multiplier space satisfies the strong regularity property in the sense of Babuška (see, [1]). Numerical experiments are conducted in support of the theoretical derivations.

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