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Abstracts in alphabetical order

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LARGE DEFORMATIONS OF BILAYER PLATES

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The bending of bilayer plates is a mechanism which allows for large deformations via small externally induced lattice mismatches of the underlying materials. Its mathematical modeling consists of a geometric nonlinear fourth order problem with a nonlinear pointwise isometry constraint and where the lattice mismatches act as a spontaneous curvature. A gradient flow is proposed to decrease the system energy and is coupled with finite element approximations of the plate deformations based on Kirchhoff quadrilaterals. In this talk, we focus on the convergence of the iterative algorithm towards stationary configurations and the Γ-convergence of their finite element approximations. We also explore the performances of the numerical algorithm as well as the reduced model capabilities via several insightful numerical experiments involving large (geometrically nonlinear) deformations.

ERROR ESTIMATES FOR THE NUMERICAL APPROXIMATION OF A DISTRIBUTED OPTIMAL CONTROL PROBLEM GOVERNED BY THE VON KÁRMÁN EQUATIONS

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We consider numerical approximation of a distributed optimal control problem governed by the von Kármán plate equations, defined on polygonal domains with pointwise control constraints. Conforming finite elements are employed to discretize the state and adjoint variables. The control is discretized using piece-wise constant approximations. *A priori* error estimates are derived for the state, adjoint and control variables under minimal regularity assumptions.

A C^0 METHOD FOR THE BIHARMONIC PROBLEM WITHOUT EXTRINSIC PENALIZATION.

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A symmetric C^0 finite element method for the biharmonic problem is presented and analyzed. In our approach, we introduce one-sided discrete second order derivatives and Hessian matrices to formulate our scheme. We show that the method is stable and converge with optimal order in a variety of norms. A distinctive feature of the method is that the results hold without extrinsic penalization of the gradient across inter-element boundaries. Numerical experiments are given that support the theoretical results, and the extension to Kirchhoff plates is also discussed.

NEW MIXED FEMS FOR THE BIHARMONIC EQUATION BASED ON THE HELMHOLTZ DECOMPOSITION

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The non-conforming Morley finite element method (FEM) for the biharmonic equation seems to be the simplest discretization for the Kirchhoff plate from structural mechanics. A common criticism is that this non-conforming FEM does not come in a natural hierarchy. This talk generalizes the non-conforming FEM of Morley to higher polynomial degrees. The crucial point is to reformulate the problem in a proper mixed formulation with the help of a Helmholtz decomposition which decomposes an unstructured symmetric tensor field into a Hessian and a symmetric curl. The inherent integral mean property of the non-conforming interpolation operator of the Morley FEM is preserved.

The approach can naturally be generalized to arbitrary mth-Laplace equations of the form $(-1)^m\Delta^m u = f$ for arbitrary $m = 1, 2, 3, \ldots$

Besides the a priori and a posteriori analysis, the talk presents optimal convergence rates for adaptive algorithms for the new discretizations.