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Abstracts in alphabetical order

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A-POSTERIORI ERROR ESTIMATES FOR PRESSURE-PROJECTION SCHEMES

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We give a short introduction and the historical development of pressure-correction methods for time discretization of the incompressible Stokes equations and discuss advantages and disadvantages of the different schemes. Further we present a-posteriori error estimates for the two-step backward differential formula method (BDF2) for the pressure-correction scheme in rotational form.

BEST APPROXIMATION ERROR ESTIMATES FOR THE ALLEN-CAHN EQUATION

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Fully-discrete approximations of the Allen-Cahn equation are discussed. In particular, we consider schemes of arbitrary order based on a discontinuous Galerkin (in time) approach combined with standard conforming finite elements (in space). We prove best approximation a-priori error estimates, with constants depending polynomially ypon $(1/\epsilon)$. We also prove that these schemes are unconditionally stable under minimal regularity assumptions on the given data. The key feature of our approach is an appropriate duality argument, combined with a boot-strap technique.

TIME AND SPACE ADAPTIVITY FOR THE WAVE EQUATION DESCRETIZED IN TIME BY A SECOND ORDER SCHEME

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We develop a posteriori error estimates of optimal order in time for the wave equation in the fully discrete situation discretized with the Newmark scheme in time and with finite elements in space. We look for a posteriori upper bounds in the L^∞-in-time-energy-in-space norm of the error. We adopt a particular choice for the parameters in the Newmark method, namely $\beta = 1/2$, $\gamma = 1/4$. This is a popular choice since it provides a conservative method with respect to the energy norm. Another interesting feature of this variant of the method, which is in fact essential for analysis, is the fact that the method can be reinterpreted as the Crank-Nicolson discretization of a reformulation of the governing equation as a first-order in time system of equations as in [C. Bernardi, E. Süli, Time and space adaptivity for the second-order wave equation, Math. Models Methods Appl. Sci. 15, 2 (2005), pp. 199–225]. We are thus able to use the techniques from [A. Lozinski, M. Picasso, V. Prachittham, An anisotropic error estimator for the Crank-Nicolson method: application to a parabolic problem, SIAM J. Sci. Comput. 31, 4 (2009), pp. 2757–2783], i.e. a piecewise quadratic polynomial in time reconstruction of the numerical solution, which leads to optimal a posteriori error estimates in time and also allows us to recover the estimates in space easily as well. We shall present the technical proofs and illustrate them by numerical results.

MAXIMUM-NORM A POSTERIORI ERROR ESTIMATION FOR CLASSICAL AND SINGULARLY PERTURBED PARABOLIC PROBLEMS

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Consider a semilinear parabolic equation in the form

$$\mathcal{M}u := \partial_t u + \mathcal{L}u + f(x,t,u) = 0 \qquad \text{for} \;\; (x,t) \in Q := \Omega \times (0,T],$$

with a second-order linear elliptic operator $\mathcal{L} = \mathcal{L}(t)$ in a spatial domain $\Omega \subset R^n$ with Lipschitz boundary, subject to $u(x,0) = \varphi(x)$ for $x \in \bar{\Omega}$ and $u(x,t) = 0$ for $(x,t) \in \partial\Omega \times [0,T]$. We assume that f satisfies $0 \le \gamma^2 \le \partial_z f(x,t,z) \le \bar{\gamma}^2$ for $(x,t,z) \in \bar{\Omega} \times [0,T] \times R$. We are particularly interested in the case $\mathcal{L} := -\varepsilon^2 \triangle$ in the regular ($\varepsilon = 1$) and singularly peturbed ($\varepsilon \ll 1$) regimes.

For this equation, we give computable a posteriori error estimates in the maximum norm. Semidiscrete and fully discrete versions of the backward Euler, Crank-Nicolson and discontinuous Galerkin dG(r) methods are addressed. For their full discretizations, we employ elliptic reconstructions that are, respectively, piecewise-constant, piecewise-linear and piecewise-quadratic for $r = 1$ in time. We also use certain bounds for the Green's function of the parabolic operator.

To give a flavour of our results, in the case of semi-discretizations (in time only) with the discrete solutions $U^j \in H_0^1(\Omega) \cap C(\bar{\Omega})$ associated with $t = t_j$, one gets

$$\begin{aligned}\left\|U^m - u(\cdot,t_m)\right\|_{\infty,\Omega} \le C_1(\kappa_1\,\ell_m + \kappa_2) \max_{j=1,\ldots,m-1} \left\|\chi^j\right\|_{\infty,\Omega} + C_2\,\kappa_0 \left\|\chi^m\right\|_{\infty,\Omega} \\ + \kappa_0 \sum_{j=1}^{m} \int_{t_{j-1}}^{t_j} e^{-\gamma^2(t_m - s)} \left\|\theta(\cdot,s)\right\|_{\infty,\Omega} ds\,.\end{aligned}$$

Here κ_p, $p = 0,1,2$, depend on $\mathcal{M}$ (they appear in the bounds for the parabolic Green's function), $\ell_m = \ell_m(\gamma) := \int_{\tau_m}^{t_m} s^{-1} e^{-\frac{1}{2}\gamma^2 s}\, ds \le \ln(t_m/\tau_m)$. The remaining quantities can be summarized as follows:

	p	χ^{j+1}	θ	C_1	C_2
backward Euler	1	$U^{j+1} - U^j$	$\widetilde{\psi} - \psi^j$ on $(t_{j-1}, t_j]$	1	2
Crank-Nicolson	2	$\tau_{j+1}(\psi^{j+1} - \psi^j)$	$\widetilde{\psi} - I_{1,t}\widetilde{\psi}$	$\frac{1}{8}$	$\frac{1}{2}$
dG(1)-Radau	3	$3\tau_{j+1}(2\psi^j - 3\psi^{j+1/3} + \psi^{j+1})$	$\widetilde{\psi} - I_{2,t}\widetilde{\psi}$	$\frac{2}{81}$	$\frac{1}{6}$

For the evaluation of χ^{j+1} and θ we use

$$\psi^{j+\alpha} := \mathcal{L}(t_{j+\alpha})\, U^{j+\alpha} + f(\cdot, t_{j+\alpha}, U^{j+\alpha}), \qquad \widetilde{\psi} := \mathcal{L}(t)\, \widetilde{U} + f(\cdot, t, \widetilde{U}),$$

where $\alpha \in (0,1]$ is any value for which the approximate solution $U^{j+\alpha}$ at time $t_{j+\alpha} := t_j + \alpha\tau_{j+1}$ is available from the definition of the semidiscrete method. Also, $\widetilde{U}$ is a piecewise-polynomial interpolant of the computed solution of degree $p-1$, while $I_{p-1,t}\widetilde{\psi}$ is a piecewise-polynomial interpolant of $\widetilde{\psi}$ of the same degree using the same interpolation points.

[1] N. Kopteva and T. Linß, Maximum norm a posteriori error estimation for parabolic problems using elliptic reconstructions, SIAM J. Numer. Anal., 51, 2013, pp. 1494–1524.

ADAPTIVE REGULARISATION

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The design of numerical schemes for nonlinear PDEs is delicate. In many important cases, for example when tackling conservation laws, there are infinitely many weak solutions and it is paramount that the underlying scheme respects certain physically motivated selection criteria. In the design of numerical methods for linear problems, high order perturbations tend to be neglected. The main difference in treating nonlinear problems over their linear counterparts is that high order perturbations cannot just be dropped, especially in the case when infinitely many weak solutions may exist.

We propose a methodology of introducing regularisation in an a posteriori fashion. This will allow us to construct numerical approximations of a particularly challenging set of solution concepts, namely entropy and viscosity solutions. These are appropriate “weak” solutions of conservation laws and Hamilton-Jacobi equations. In this talk we illustrate the ideas and application to some simple problems.

CURVE SHORTENING FLOW COUPLED TO LATERAL DIFFUSION

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A semi-discrete finite element scheme for a system consisting of a geometric evolution equation for a curve and a parabolic equation on that evolving curve is presented. More precisely, curve shortening flow with a forcing term that depends on a conserved field is coupled with a diffusion equation for that field. Such a system can be considered as a prototype for more complicated problems as they may arise in applications. Our scheme is based on ideas of Dziuk for the curve shortening flow and Dziuk/Elliott for the parabolic equation on the moving curve. However, additional estimates particularly with respect to the time derivative of the length element are required. Numerical simulation results support the theoretical findings.

FINITE ELEMENT APPROXIMATION OF SEMILINEAR PARABOLIC REACTION DIFFUSION SYSTEMS WITH IMEX TIMESTEPPING

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Coupled systems of semilinear parabolic equations arise in a number of applications in fields such as biology, chemistry and material science. Often the applications are such that the equations are posed on complex or evolving geometries. In this talk we address the design and analysis of finite element approximations of such systems with implicit-explicit time discretisation. The theoretical results will be supported by examples of application driven numerical simulations.