MAFELAP
2016

Conference on the Mathematics
of Finite Elements and Applications

14–17 June 2016

Mini-Symposium: Developments in locally conservative conforming methods for elliptic partial differential equations

Organisers:

Todd Arbogast and Ivan Yotov

Abstracts in alphabetical order

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A MULTISCALE HYBRID-MIXED METHOD FOR THE STOKES AND BRINKMAN EQUATIONS

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In this work a multiscale hybrid-mixed method (MHM) ([1]), applied to the Stokes and the Brinkman equations on heterogeneous media, is introduced and analyzed. Given a coarse partition of the domain and using a hybrid formulation ([2]), the MHM method consists of independent Stokes (or Brinkman) local problems bring together by a face-based weak formulation on the skeleton of the partition. The multiple scales of the media are incorporated in the basis functions which are driven by the local problems with prescribed Neumann boundary conditions. Once available (exactly or approximatively), the multiscale basis functions are used to compute the degrees of freedom from a faced-based global variational problem defined on the skeleton of the partition. The numerical solution shares the important properties of the continuum as the local equilibrium with respect to external forces and local mass conservation. Several numerical tests assess the accuracy and the conservative properties of MHM method on academic and highly heterogeneous cases.

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NEW MIXED FINITE ELEMENTS ON QUADRILATERALS OF MINIMAL DIMENSION

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We develop two families of mixed finite elements on quadrilateral meshes for approximating $(\mathbf{u}, p)$ solving a second order elliptic equation in mixed form. Standard Raviart-Thomas (RT) and Brezzi-Douglas-Marini (BDM) elements are defined on rectangles and extended to quadrilaterals using the Piola transform, which are well-known to lose optimal approximation of $\nabla\cdot\mathbf{u}$. Arnold-Boffi-Falk (ABF) spaces rectify the problem by increasing the dimension of RT, so that approximation is maintained after Piola mapping. Our two families of finite elements are uniformly inf-sup stable, achieve optimal rates of convergence, and have minimal dimension. The elements for $\mathbf{u}$ are constructed from vector polynomials defined directly on the quadrilaterals, rather than being transformed from a reference rectangle by the Piola mapping, and then supplemented by two (one for the lowest order) basis functions that are Piola mapped. One family has full $H(\mathrm{div})$-approximation ($\mathbf{u}$, p, and $\nabla\cdot\mathbf{u}$ are approximated to the same order like RT) and the other has reduced $H(\mathrm{div})$-approximation (p and $\nabla\cdot\mathbf{u}$ are approximated to one less power like BDM). The two families are identical except for inclusion of a minimal set of vector and scalar polynomials needed for higher order approximation of $\nabla\cdot\mathbf{u}$ and p, and thereby we clarify and unify the treatment of finite element approximation between these two classes. The key result is a Helmholtz-like decomposition of vector polynomials, which explains precisely how a divergence is approximated locally. We develop an implementable local basis and present numerical results confirming the theory.

LOCAL BOUNDED COCHAIN PROJECTIONS ON CUBICAL MESHES

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We review the construction of the two main families of finite element differential forms on cubical meshes and construct local bounded cochain projections for the spaces. We provide an outline of the construction of the projections due to Falk and Winther. A crucial step of the construction is an inclusion property for the analogue on cubical meshes of the space of Whitney forms into spaces of finite element differential forms. Preliminary report on a joint work with Ragnar Winther.

ROBUST DISCRETIZATION OF FLOW IN FRACTURED POROUS MEDIA

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Fractures are ubiquitous in natural rocks, and in many cases have a leading order impact on the structure of fluid flow. Since fractures frequently have high aspect ratios, it is appealing to consider them as lower-dimensional features.

We present a modelling approach based on mixed finite element methods and the mortar method which fully couples the physics in domains with different dimensions. In particular, we apply the approach to Darcy flow in fractured media and show how abrupt fracture tips as well as fracture intersections are naturally handled. The proposed discretization is applicable to both two and three spatial dimensions and is capable of handling conductive as well as blocking fractures. Furthermore, the method respects mass conservation and handles non-matching grids. We establish both theoretically and through numerical examples that our method is convergent in all relevant physical limits.

CVD-MPFA DARCY FLUX APPROXIMATION ON UNSTRUCTURED GRIDS

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Control-volume distributed multi-point flux approximations (CVD-MPFA) are presented for unstructured grids in two and three dimensions. These schemes are designed to be consistent and flux-continuous while maintaining the reservoir simulation standard of only employing a single degree of freedom per control-volume, per flow variable, and are consequently proving popular in reservoir simulation. Both cell-centred and cell-vertex approximations are considered. Cell-vertex approximation proves to be advantageous, and requires appropriate grids for Darcy flux approximation, grid generation issues are discussed. The methods are applied to problems involving strong full-tensor permeability fields, faults and layers. The talk will touch on a number of topics (depending on time) including convective and elliptic flux approximation together with fracture models and the resulting flow in fractures.

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GENERALIZED BARYCENTRIC COORDINATES FOR DEGENERATE GEOMETRY IN FEM

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Generalized barycentric coordinates are an essential tool in the growing area of polytopal element methods. Recent applications of the coordinates include the construction of stability matrices for virtual element methods and gradient correction for nonlinear elasticity problems. The success of these and other applications rely on the fact that the coordinates can provide a good-quality interpolation scheme even on elements with (seemingly) poor geometry. We will present recent numerical experiments that exhibit how various types of generalized barycentric coordinates behave under a variety of degenerate geometry scenarios.

STABILIZED HYBRID MIXED FINITE ELEMENT METHODS FOR INCOMPRESSIBLE FLOWS

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We propose a stabilized hybrid mixed method to solve the Stokes problem with velocity and pressure stabilization terms on the edges of the elements. The method is close related to a hybrid DG method proposed by Egger and Waluga but uses Lagrange multipliers associated with the traces of both velocity and pressure fields. As a consequence of this choice, the local problems are stable and all velocity and pressure degrees of freedom can be eliminated at the element level by static condensation. Continuous or discontinuous approximations for the multipliers are adopted. With continuous multiplier approximation, the proposed SHM method presents some similarities with classical Galerkin mixed methods that use C^0 continuous interpolation for both velocity and pressure fields. When discontinuous approximations are adopted for the multipliers, eliminating the Lagrange multiplier we recover a slightly modified version of a symmetric DG method.

AN ARBITRARY ORDER ACCURATE MIMETIC METHOD FOR ELLIPTIC PROBLEMS IN MIXED FORM

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We present an arbitrary-order accurate Mimetic Finite Difference (MFD) method for the approximation of diffusion problems in mixed form on unstructured polygonal and polyhedral meshes. As usual in the mimetic numerical technology, the method satisfies local consistency and stability conditions, which determines the accuracy and the well-posedness of the resulting approximation. The method also requires the definition of a high-order discrete divergence operator that is the discrete analog of the divergence operator and is acting on the degrees of freedom. The new family of mimetic methods is proved theoretically to be convergent and optimal error estimates for flux and scalar variable are derived from the convergence analysis. A numerical experiment confirms the high-order accuracy of the method in solving diffusion problems with variable diffusion tensor. It is worth mentioning that the approximation of the scalar variable presents a superconvergence effect.

SERENDIPITY NODAL VEM SPACES

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We introduce a new variant of Nodal Virtual Element spaces that mimics the "Serendipity Finite Element Methods" (whose most popular example is the 8-node quadrilateral) and allows to reduce (often in a significant way) the number of internal degrees of freedom. When applied to the faces of a three-dimensional decomposition, this allows a reduction in the number of *face* degrees of freedom: an improvement that cannot be achieved by a simple static condensation. On triangular and tetrahedral decompositions the new elements (contrary to the original VEMs) reduce exactly to the classical Lagrange FEM. On quadrilaterals and hexahedra the new elements are quite similar (and have the same amount of degrees of freedom) to the Serendipity Finite Elements, but are *much more robust* with respect to element distortions. On more general polytopes the Serendipity VEMs are the natural (and simple) generalization of the simplicial case.

ENRICHED GALERKIN APPROXIMATION FOR FLOW AND TRANSPORT PROBLEMS

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We present and analyze an enriched Galerkin finite element method (EG) to solve coupled flow and transport system with jump coefficients referred s miscible displacement problems. The EG is formulated by enriching the conforming continuous Galerkin finite element method (CG) with piecewise constant functions. This method is shown to be locally and globally conservative, while keeping fewer degrees of freedom in comparison with discontinuous Galerkin finite element methods (DG). In addition, we present and analyze a fast and effective EG solver simpler than DG and whose cost is roughly that of CG and can handle an arbitrary order of approximations for the flow problem.

Moreover, to avoid any spurious oscillations for the higher order transport system, we employ an entropy residual stabilization technique. Dynamic mesh adaptivity using hanging node is applied to save computational cost for large-scale physical problems. Number of numerical tests in two and three dimensions are presented to confirm our theoretical results as well as to demonstrate the advantages of the EG.

A MULTIPOINT STRESS MIXED FINITE ELEMENT METHOD FOR LINEAR ELASTICITY

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We discuss a new multipoint stress mixed finite element method for elasticity, based on the lowest order mixed finite element spaces with weak symmetry. A trapezoidal-type quadrature rule allows for local stress and rotation elimination and reduces the method to a cell-centered scheme for displacements. Stability and error analysis is performed on simplicial and quadrilateral grids. Numerical experiments are presented to illustrate the convergence of the method and its ability to handle heterogeneous problems.

This is joint work with Ilona Ambartsumyan and Eldar Khattatov from University of Pittsburgh, and Jan Nordbotten from University of Bergen.