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A RECOVERY BASED LINEAR FINITE ELEMENT METHOD FOR 4TH ORDER PROBLEMS

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We analyze a gradient recovery based linear finite element method to solve string equations and the corresponding eigenvalue problems. Our method uses only C^0 element, which avoids complicated construction of C^1 elements and nonconforming elements. Optimal error bounds under various Sobolev norms are established. Moreover, after a post -processing the recovered gradient is superconvergent to the exact one. Finally, some numerical experiments are presented to validate our theoretical findings.

AN INTERFACE-FITTED MESH GENERATOR AND VIRTUAL ELEMENT METHODS FOR ELLIPTIC INTERFACE PROBLEMS

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In this work, we develop a simple interface-fitted mesh algorithm which can produce an interface-fitted mesh in two and three dimension quickly. Elements in such interface-fitted mesh are not restricted to simplex but can be polygon or polyhedron. We thus apply virtual element methods to solve the elliptic interface problem in two and three dimensions. We present some numerical results to illustrate the effectiveness of our method.

THEORETICAL ANALYSIS FOR CAPILLARY RISE BETWEEN A FLEXIBLE FILM AND A SOLID WALL

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We study the dynamics of meniscus rise of a liquid contained in a narrow gap between a flexible film and a solid wall. In this talk we will show that the meniscus rises indefinitely expelling liquid from the gap region, and that the height of the rising front $h(t)$ increases with time as $h(t) \propto t^{2/7}$, while the gap distance $e(t)$ decreases as $e(t) \propto t^{-3/7}$. These results are consistent with the experiments of Cambau et al.

A MULTILEVEL CORRECTION METHOD FOR OPTIMAL CONTROLS OF ELLIPTIC EQUATION

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In this talk we present a multilevel correction method to solve optimal control problems constrained by elliptic equations with the finite element method on both uniform and adaptive meshes. In this scheme, solving an optimization problem on the finest finite element space is transformed into a series of solutions of linear boundary value problems by the multigrid method on multilevel meshes and a series of solutions of optimization problems on the coarsest finite element space. Our proposed scheme, instead of solving a large scale optimization problem in the finest finite element space, solves only a series of linear boundary value problems and the optimization problems in a very low dimensional finite element space, and thus can improve the overall efficiency of the solution of optimal control problems governed by PDEs

TRANSMISSION EIGENVALUES AND INVISIBILITY

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We consider a non-self-adjoint fourth order eigenvalue problem using a discontinuous Galerkin (DG) method. For high order problems, DG methods are competitive since they use simple basis functions and have less degrees of freedom. We propose an interior penalty discontinuous Galerkin method using C0 Lagrange elements (C0IP) for the transmission eigenvalue problem and prove the optimal convergence. We also consider invisibility cloaking in acoustic wave scattering. The proposed cloaking device takes a three-layer structure with a cloaked region, a lossy layer and a cloaking shell. This is mainly based on studying a novel type of interior transmission eigenvalue problems and their connection to invisibility cloaking.

TOPOLOGY OPTIMIZATION IN NAVIER–STOKES FLOW WITH A DIFFUSE-INTERFACE APPROACH

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We investigate the problem of finding optimal topologies of fluid domains. In a given hold all domain Ω we search for a topology of a fluid domain, filling at most a given proportion of the container, such that an objective is minimized that might depend on the velocity field and the pressure field inside the domain an the topology itself. Here the velocity and pressure owe to the Navier–Stokes system. This especially contains the problem of minimizing the drag of an obstacle in free flow.

Our approach consists of using a phase field description and a porosity approach. Thus we describe the distribution of the material inside the domain Ω by a phase field variable $\varphi \in H^1(\Omega) \cap L^\infty(\Omega)$ that encodes the obstacle by $\varphi(x) = -1$ and the fluid domain by $\varphi(x) = 1$, while values between -1 and +1 encode a small diffuse region between the fluid domain and the obstacle. By the porosity approach we assume that the obstacle itself is part of the fluid domain, but contains a very dense material with low porosity, that results in an additional Darcy term in the equation. Introducing an interpolation function that interpolated between the dense material and the void we can then extend the fluid equation to the complete domain.

The structure of the final problem is an optimal control problem of a Navier–Stokes equation where the control is given as the phase field and appears as coefficient in the Navier–Stokes equation.

Due to the inherent regularity of the optimization variable, which is $H^1(\Omega)\cap L^\infty(\Omega)$ we can not apply classic descent methods like steepest descent to solve the optimality conditions. Therefore, we apply the variable metric projection type method proposed in [L. Blank and C. Rupprecht, An extension of the projected gradient method to a Banach space setting with application in structural topology optimization, arXiv:1503.03783].

In earlier work also a gradient flow approach was used, see [H. Garcke, C. Hecht, M. Hinze, C. Kahle, Numerical approximation of phase field based shape and topology optimization for fluids, SISC 2015, 37(4), 1846–1871] [H. Garcke, C. Hecht, M Hinze, C. Kahle, K.F. Lam, Shape optimization for surface functionals in Navier–Stokes flow using a phase field approach, IFB 2016, 18(2)]

OPTIMAL PRECONDITIONING OF A CUT FINITE ELEMENT METHOD FOR UNFITTED INTERFACE PROBLEMS

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In recent years *unfitted finite element methods* have drawn an increasing amount of attention. Handling complex geometries without complex and time consuming mesh generation is very appealing. We consider the model interface problem of the type:

$$-\mathrm{div}(\alpha_i \nabla u) = f \text{ in } \Omega_i,\ i = 1,2, \quad [\![\alpha \nabla u]\!]_\Gamma \cdot n_\Gamma = [\![u]\!]_\Gamma = 0 \text{ on } \Gamma, \quad u = 0 \text{ on } \partial\Omega.$$

Here, $\Omega_1 \cup \Omega_2 = \Omega \subset \mathbb{R}^d$, $d = 2,3$, is a nonoverlapping partitioning of the domain, $\Gamma = \bar{\Omega}_1 \cap \bar{\Omega}_2$ is the interface, $[\![\cdot]\!]_\Gamma$ denotes the usual jump operator across Γ and $\alpha_i,\ i = 1,2$ are positive constants. The methodology of unfitted finite element methods for this type of problem, i.e. methods which are able to cope with interfaces Γ which are not aligned to the grid, is often combined with a weak enforcement of interface conditions using Nitsche's method. In the original paper [1] the Nitsche-XFEM technique for interface problem has been introduced and analysed. Let V_h be the space of contiuous piecewise linear finite elements with respect to the computational mesh. Then the Nitsche-XFEM method reads: Find $u_h = V_h|_{\Omega_1} \oplus V_h|_{\Omega_2}$ such that

$$\int_{\Omega_1 \cup \Omega_2} \alpha \nabla u_h \nabla v_h dx - \int_\Gamma \{\!\{\alpha \nabla u_h \cdot n\}\!\} [\![v_h]\!] ds - \int_\Gamma \{\!\{\alpha \nabla v_h \cdot n\}\!\} [\![u_h]\!] ds + \frac{\bar{\alpha}\lambda}{h} \int_\Gamma [\![u_h]\!] [\![v_h]\!] ds = \int_{\Omega_1 \cup \Omega_2} f v_h dx$$

for all $v_h \in V_h^\Gamma$. Here we used the average $\{\!\{w\}\!\} := \kappa_1 w_1 + \kappa_2 w_2$ with an element-wise constant $\kappa_i = \frac{|T \cap \Omega_i|}{|T|}$ as in [1]. In general, the resulting linear systems have very large condition numbers, which depend not only on the mesh size h, but also on how the interface intersects the mesh.

Simple diagonal preconditioning circumvents these problems and achieves condition number bounds of the form ch^{-2} with a constant c that is independent of the location of the interface. The main ingredient in proving this result is the stable subspace splitting between standard degrees of freedom (corresponding to V_h) and extended degrees of freedom which is proven in [2]. Utilizing this property we are able to propose a preconditioner which is optimal in the sense that preconditioning actions have only $\mathcal{O}(N)$ costs (where N is the number of degrees of freedom) and the resulting condition number is independent of the mesh size h and the interface position.

We present the optimal preconditioner, numerical results and outline the main aspects of the analysis.

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RECURSIVE INTEGRAL METHOD FOR A NON-LINEAR NON-SELFADJOINT TRANSMISSION EIGENVALUE PROBLEM

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We propose a robust numerical method to compute complex and real eigenvalues of a nonlinear non-selfadjoint transmission eigenvalue problem. Based on a fourth order formulation, we obtain a quadratic eigenvalue problem.The non-comforming Morley element is used for discretization, leading to a quadratic matrix eigenvalue problem. Then we propose to use a recursive integral method to compute the eigenvalues in prescribed regions on the complex plane. The effectiveness of the proposed method can be validated by numerical examples.

FULLY COMPUTABLE ERROR ESTIMATES FOR EIGENVALUE PROBLEMS

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In the talk, we will present a fully computable error estimate for the eigenvalue problem which is solved by the general conforming finite element methods on the general meshes. Based on the computable error estimate, we also give a guaranteed upper bound of the error estimate for the eigenfunction approximation. Furthermore, we also propose a simple process to compute the guaranteed lower bound of the first eigenvalue based on the upper bound of the eigenfunction error estimate. Some numerical examples are presented to validate the theoretical results

AN ADAPTIVE FINITE ELEMENT METHOD FOR ELECTRICAL IMPEDANCE TOMOGRAPHY

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In this work we discuss an adaptive finite element method for efficiently solving electrical impedance tomography – a severely ill-posed nonlinear inverse problem to recover the conductivity from boundary voltage measurements. The reconstruction technique is based on Tikhonov regularization with a Sobolev smoothness penalty and approximation of the forward model using continuous piecewise linear finite elements. We propose an adaptive finite element algorithm with an a posteriori error estimator involving the concerned state and adjoint variables and the recovered conductivity. The convergence of the algorithm is established, in the sense that the sequence of discrete solutions contains a convergent subsequence to a solution of the optimality system for the continuous formulation. Numerical results are presented to verify the convergence and efficiency of the algorithm.

ANISOTROPIC MESHES AND STABILIZED PARAMETERS FOR THE STABILIZED FINITE ELEMENT METHODS

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In this talk, we demonstrate a numerical strategy to generate anisotropic meshes and select the appropriate stabilization parameter simultaneously for two dimensional convection-dominated convection-diffusion equations by the stabilized continuous linear finite elements. Since the discretization error in a suitable norm can be bounded by the sum of interpolation error and its variants in different norms, we replace them by some terms which contain the Hessian matrix of the true solution, convective fields, and the geometric properties such as directed edges and the area of triangles. Based on this observation, the shape, size and equidistribution requirements are used to derive the corresponding metric tensor and the stabilization parameter. The process of the derivation reveals that the optimal stabilization parameter is coupled with the metric tensor for each element. Numerical results are also provided to validate the stability and efficiency of the proposed numerical strategy.

MIXED ELEMENT METHOD FOR EIGENVALUE PROBLEM OF THE BIHARMONIC EQUATION

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In this talk, a new kind of mixed element method for the eigenvalue problem of the biharmonic equation will be presented. Under the framework of a new mixed formulation of the biharmonic equation, finite element methods are designed so that, firstly, low-degree finite element spaces can be sufficient for the discretization schemes, secondly, an efficient multilevel method can be designed and implemented associated with the schemes, and thirdly, guaranteed upper and lower bounds of the eigenvalues can be computed with the schemes. Numerical experiments are also given for confirmation. This is a joint work with Xia Ji and Yingxia Xi.