MAFELAP 2016

Conference on the Mathematics
of Finite Elements and Applications

14–17 June 2016

Mini-Symposium: A posteriori error estimation and adaptivity

Organisers:

Dirk Praetorius and Martin Vohralik

Abstracts in alphabetical order

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AN HP-ADAPTIVE C^0-INTERIOR PENALTY METHOD FOR THE OBSTACLE PROBLEM OF CLAMPED KIRCHHOFF PLATES

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In this talk we consider an hp-adaptive C^0-interior penalty method for the bilaplace obstacle problem. The a posteriori error estimate consists of two stages. In the first part the error contributions associated with the obstacle condition are split off, and in the second part, a residual based a posteriori error estimate for the simpler biharmonic equation is generalized to higher order h- and p-versions. Essential for the a posteriori error estimate is the computation of a discrete Lagrange multiplier, representing the residual of the variational inequality, either by solving a mixed formulation directly, or by post-processing it after solving a discrete variational inequality. The choice of the finite element sets and whether the discrete inf-sup condition holds uniformly or at all are not of importance for the a posteriori error estimate. Numerical experiments demonstrate the behavior of the a posteriori error estimate and the superior convergence rate of the hp-adaptive scheme compared with uniform and h-adaptive schemes.

AN UPDATE ON THE MAXIMUM STRATEGY

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The adaptive finite element method, with an automatic refinement driven by error estimators, allows to resolve singularities at minimal computational costs. One strategy of refinement is to split those triangles, where the error indicators are almost maximal (maximum strategy). We show optimality of the corresponding adaptive finite element loop. The original result is restricted to the two-dimensional case with linear elements. In this talk we present extensions of these arguments, which include the use of higher order elements.

ADAPTIVE VERTEX-CENTERED FINITE VOLUME METHODS WITH CONVERGENCE RATES

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A classical finite volume method (FVM) describes numerically a conservation law of an underlying model problem. It naturally preserves local conservation of the numerical fluxes. Therefore, FVMs are well-established in the engineering community (fluid mechanics).
We consider an adaptive vertex-centered finite volume method with first-order conforming ansatz functions. The adaptive mesh-refinement is driven by the local contributions of the weighted-residual error estimator. We prove that the adaptive algorithm leads to linear convergence with generically optimal algebraic rates for the error estimator and the sum of energy error plus data oscillations. While similar results have been derived for finite element methods and boundary element methods, the present work appears to be the first for adaptive finite volume methods, where the lack of the classical Galerkin orthogonality leads to new challenges.

For more details we refer to the Preprint [C. Erath and D. Praetorius, Adaptive vertex-centered finite volume methods with convergence rates, 2016, pp. 1-29, arXiv:1508.06155].

AN ADAPTIVE P_1 FINITE ELEMENT METHOD FOR TWO-DIMENSIONAL MAXWELL'S EQUATIONS

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We extend the Hodge decomposition approach for the cavity problem of two-dimensional time harmonic Maxwell's equations to include the impedance boundary condition, with anisotropic electric permittivity and sign changing magnetic permeability. We derive error estimates for a P_1 finite element method based on the Hodge decomposition approach and develop a residual type *a posteriori* error estimator. We show that adaptive mesh refinement leads empirically to smaller errors than uniform mesh refinement for numerical experiments that involve metamaterials and electromagnetic cloaking. The well-posedness of the cavity problem when both electric permittivity and magnetic permeability can change sign is also discussed and verified for the numerical approximation of a flat lens experiment.

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ADAPTIVE FEM FOR ELLIPTIC PROBLEMS WITH GÅRDING INEQUALITY

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Given $f \in L^2(\Omega)$, we consider adaptive FEM for problems of the type

$$a(u, v) + \langle \mathcal{K}u, v \rangle_{L^2(\Omega)} = \langle f, v \rangle_{L^2(\Omega)} \quad \text{for all } v \in H^1_0(\Omega), \tag{1}$$

where $a(\cdot, \cdot)$ is an elliptic and symmetric bilinear form on $H^1_0(\Omega)$ and $\mathcal{K} : L^2(\Omega) \to L^2(\Omega)$ is a continuous linear operator. We suppose that (1) is well-posed and hence admits a unique solution $u \in H^1_0(\Omega)$. This setting is met, e.g., for the Helmholtz equation or second-order linear elliptic problems with reaction and/or convection. For a standard conforming FEM discretization of (1) by piecewise polynomials, usual duality arguments show that the underlying triangulation has to be sufficiently fine to ensure the existence and uniqueness of the Galerkin solution.

Extending the abstract approach of [1], we prove that adaptive mesh-refinement is capable of overcoming this preasymptotic behavior and eventually leads to convergence with optimal algebraic rates. Unlike previous works [2, 3, 4], one does not have to deal with the *a priori* assumption that the initial mesh is sufficiently fine. The overall conclusion of our results thus is that adaptivity has stabilizing effects and can, in particular, overcome preasymptotic and possibly pessimistic restrictions on the meshes.

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THE ROLE OF OSCILLATION IN A POSTERIORI ERROR ANALYSIS

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In a posteriori analysis, it is a common believe that the so-called oscillation is the prize to pay for the 'computability' of the estimator, in particular, for estimating local H^{-1}-norms by scaled L^2-norms.

It is the merit of Cohen, DeVore, and Nochetto [CoDeNo:2012] to uncover that there is a catch: They presented an example, where the error is vanishing faster than the estimator. This implies that even asymptotically, the oscillation cannot be bounded by the error. Interestingly, in this example, the local H^{-1}-norms can be computed exactly and thus computability is not be the reason for the asymptotic overestimation.

In this talk, we shall present a posteriori bounds, where the oscillation appears only because of the computability requirement. In contrast to previous a posteriori analyses, we derive oscillation terms that are dominated by the error irrespective of mesh fineness and regularity of the exact solution. As a consequence, the estimator and the oscillation converge at least as fast as the error.

[CoDeNo:2012] A. Cohen, R. DeVore, and R. H. Nochetto, *Convergence Rates of AFEM with H^{-1} Data*, Found Comput Math **12** (2012):671-718

RELAXING THE CFL CONDITION FOR THE WAVE EQUATION ON ADAPTIVE MESHES

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The Courant-Friedrichs-Lewy (CFL) condition limits the choice of the time-step size for the popular explicit leapfrog method for the wave equation to be bounded by the minimal mesh-size in the spatial finite element mesh. This makes the scheme expensive for locally refined meshes. On the other hand, locally refined meshes are necessary to reveal the optimal convergence rate on domains with re-entrant corners. This talk introduces a reduced ansatz space based on a uniform mesh that allows to balance the CFL condition and adaptive spatial approximation in an optimal way, even in the presence of spatial singularities.

COMPUTABLE A POSTERIORI ERROR ESTIMATORS FOR FINITE ELEMENT APPROXIMATIONS OF AN OPTIMAL CONTROL PROBLEM

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We consider an optimal control problem with control constraints, where the state is governed by a convection–reaction–diffusion equation. We will discuss how computable a posteriori error estimators are obtained for the case when piecewise affine stabilized finite element methods are used to approximate the solutions to the state and adjoint equations and piecewise constants are used to approximate the control. The estimators provide guaranteed upper bounds on the norms of the errors and, up to a constant and oscillation terms, local lower bounds on the norms of the errors. Numerical examples, in two and three dimensions, will be presented to illustrate the theory.

A POSTERIORI ERROR ESTIMATES FOR THE BIOT PROBLEM BASED ON EQUILIBRATED H(DIV)-CONFORMING FLUX RECONSTRUCTIONS

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Over the last few years, adaptive algorithms based on a posteriori error estimates have been put forward, comprising the adaptive stopping of the iterative solvers and the dynamic adaptation of the mesh and the time step (see, e.g. [3]). These two applications are the major motivations for EDF to include a posteriori error estimates in the hydro-mechanical part of their finite element code Code_Aster. We present here an approach allowing us to obtain a posteriori error estimations for a poro-elastic problem, where we handle the hydraulic part as proposed in [3] and develop equivalent techniques for the elasticity. The estimators are obtained by introducing equilibrated reconstructions of the velocity and the mechanical stress tensor, obtained as mixed finite element solutions of local Neumann problems posed over patches of elements (cf., e.g., [4]). With this approach it is possible to distinguish the different error sources: spatial and temporal discretization, and algebraic resolution. In the spirit of [2], the velocity reconstruction is sought in the Raviart-Thomas finite element space, while the difficulty of reconstructing a symmetric $H(\mathrm{div})$-conforming stress tensor is overcome by choosing the mixed finite element space proposed by Arnold and Winther in [1].

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A POSTERIORI ERROR ESTIMATES OF HP-FINITE ELEMENTS FOR MIXED AND MIXED-HYBRID METHODS

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Mixed methods based on the introduction of gradient or stress fields as additional unknowns in $H(\mathrm{div})$-spaces are well-established. They are available for variational equations [1] as well as variational inequalities [2]. In these methods, the discretization of the $H(\mathrm{div})$-space necessitates continuity in the normal direction of the edges of the underlying mesh. Usually, Raviart-Thomas finite elements are used in order to guarantee this continuity condition. Alternatively, one can also apply mixed-hybrid methods where additional Lagrange multipliers on the edges are introduced to enforce the desired continuity.

In this talk, we discuss reliable error estimates and adaptivity of *hp*-adaptive finite elements for mixed and mixed-hybrid methods. In particular, we consider the Poisson problem and the obstacle problem leading to a variational equation and a variational inequality, respectively. The mixed-hybrid approach enables the use of tensor product shape functions based on Lagrange polynomials for all fields and, thus, an effective implementation of assembling routines (numerical integration, static condensation, parallelization) for quadrilateral or hexahedral mesh elements with varying polynomial degree distribution and (multilevel) hanging nodes. The basic idea of the a posteriori error control is to reconstruct the solution of the primal variable in the H^1-space so that error controls for H^1-conforming finite elements can be applied [3]. The reconstruction can be done globally, but also locally in many cases. Several numerical examples confirm the applicability of the proposed techniques within *hp*-adaptive refinements.

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A POSTERIORI ERROR ESTIMATES FOR HIGHER-ORDER TIME DISCRETIZATIONS

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We present equilibrated flux guaranteed a posteriori error estimates with respect to the $L^2(H^1) \cap H^1(H^{-1})$ and $L^2(H^1)$ parabolic energy norms for fully discrete schemes for the heat equation based on high-order conforming FEM in space and high-order discontinuous Galerkin methods in time. Extending the ideas in [2] to high-order methods, the equilibration is obtained by solving, for each timestep, local mixed FEM problems posed on the patches of the current mesh. We further show that the error estimates are locally efficient with respect to the space-time local $L^2(H^1) \cap H^1(H^{-1})$-error and temporal jumps, and, building on [1, 3], we establish full robustness with respect to both the temporal and spatial polynomial degrees, thus making the estimates well-suited for high-order schemes. In the practically relevant situation where the time-step size $\tau \gtrsim h^2$ the mesh-size, the spatial estimators are in addition locally efficient with respect to the space-time local $L^2(H^1)$-error and temporal jumps.

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CONVERGENCE AND OPTIMALITY OF HP-AFEM

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We present an adaptive hp-finite element algorithm. It consists of iterating two routines: *hp*-**NEARBEST** finds a near-best *hp*-approximation of the current discrete solution and data to a desired accuracy, and **REDUCE** improves the discrete solution to a finer but comparable accuracy. The former hinges on a recent algorithm by P. Binev for adaptive *hp*-approximation, and acts as a coarsening step. We prove convergence and instance optimality. For controlling the computational cost, we present results on saturation that are uniform in p.

A GUARANTEED EQUILIBRATED ERROR ESTIMATOR FOR THE $\mathbf{A} - \varphi$ AND $\mathbf{T} - \Omega$ MAGNETODYNAMIC HARMONIC FORMULATIONS OF THE MAXWELL SYSTEM

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Key Words: Maxwell equations, potential formulation, a posteriori estimators, finite element method.

This communication is devoted to the developement and analysis of an equilibrated *a posteriori* error estimator for the harmonic eddy current problems. Therefore the system of interest is given by the quasi-static approximation of Maxwell's equations in the magnetoharmonic regime, completed by the constitutive laws: $\mathbf{B} = \mu\,\mathbf{H}$ in the whole domain D and $\mathbf{J}_e = \sigma\ \mathbf{E}$ in the conductor domain D_c. Here $\mathbf{B}$, $\mathbf{H}$, $\mathbf{J}_e$ and $\mathbf{E}$ represent respectively the magnetic flux density, the magnetic field, the eddy current density and the electric field, while μ stands for the magnetic permeability and σ for the electrical conductivity.
In order to obtain numerical solutions, we solve the two classical potential formulations. The first one is a recast of the original system through a magnetic vector potential $\mathbf{A}$, defined in D, as well as an electrical scalar potential φ, defined only in D_c. The finite element method applied to the $\mathbf{A} - \varphi$ formulation provides the numerical solutions: $\mathbf{B}_h = \text{curl}\ \mathbf{A}_h$ in D and $\mathbf{E}_h = -i\omega\mathbf{A}_h - \nabla\varphi_h$ in D_c. Similarly, a recast of the original system through an electric vector potential $\mathbf{T}$, defined in D_c, as well as a magnetic scalar potential Ω, defined in D, gives the so-called $\mathbf{T} - \Omega$ formulation. The finite element method provides the numerical solutions: $\mathbf{H}_h = \mathbf{H}_s + \mathbf{T}_h - \nabla\Omega_h$ in D and $\mathbf{J}_h = \text{curl}\ \ \mathbf{T}_h$ in D_c, where $\mathbf{J}_s = \text{curl}\ \ \mathbf{H}_s$ denotes the source term.
The aim is to estimate the energy norm of the error ϵ

$$\begin{aligned}\epsilon \ =\ & (||\,\mu^{-1/2}(\mathbf{B}-\mathbf{B}_h)\,||^2_{L^2(D)} + ||\,\mu^{1/2}(\mathbf{H}-\mathbf{H}_h)\,||^2_{L^2(D)} \\ & + ||\,(\omega\,\sigma)^{-1/2}(\mathbf{J}-\mathbf{J}_h)\,||^2_{L^2(D_c)} + ||\,\omega^{-1/2}\,\sigma^{1/2}\,(\mathbf{E}-\mathbf{E}_h)\,||^2_{L^2(D_c)})^{1/2}.\end{aligned}$$

To do that, we derive an error estimator based on the non-verification property of the constitutive laws for the numerical fields [Creusé, S. Nicaise and R. Tittarelli, A guaranteed equilibrated error estimator for the $\mathbf{A} - \varphi$ and $\mathbf{T} - \Omega$ magnetodynamic harmonic formulations of the Maxwell system, IMA Journal of Numerical Analysis, submitted for publication]. Let us denote by $\mathcal{T}_h$ a tetrahedral regular mesh. The estimator η is defined as

$$\eta^2 \ =\ \sum_{T\in\mathcal{T}_h} \eta^2_{m,T} \ +\ \sum_{T\in\mathcal{T}_h, T\subset D_c} \eta^2_{e,T}\,,\ \text{where}$$

$$\eta_{m,T} = ||\mu^{1/2}(\mathbf{H}_h - \mu^{-1}\mathbf{B}_h)||_T \text{ and } \eta_{e,T} = ||\ (\omega\sigma)^{-1/2}\,(\mathbf{J}_h - \sigma\mathbf{E}_h)\ ||_T.$$

First of all, the global equivalence between the error ϵ and the estimator η up to higher order terms (h.o.t.) without unknown constants is proved, that is:

$$\eta^2 = \epsilon^2 + \text{h.o.t.}.$$

Secondly, the local efficiency property is proved *i.e.* $\eta_T = (\eta_{m,T}^2 + \eta_{e,T}^2)^{1/2} \leq \sqrt{2}\,\epsilon_{|T}$ with $T \in \mathcal{T}_h$. This latter inequality gives the key ingredient for driving an adaptive remeshing process. Finally, these theoretical results are validated through an analytical benchmark test.

A POSTERIORI ERROR ESTIMATION, ERROR-DOMINATED OSCILLATION AND OBSTACLES

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Recently, Christian Kreuzer and the author developed an approach to a posteriori error estimation that clarifies the role of oscillation; see also the talk of Christian Kreuzer in this mini-symposium. For Poisson's problem, this approach provides an H^{-1}-oscillation that is bounded in terms of the error.

This talk revisits previous approaches in the a posteriori error analysis with obstacles, assessing their compatibility with this new H^{-1}-oscillation.

POLYNOMIAL-DEGREE-ROBUST ESTIMATES IN THREE SPACE DIMENSIONS

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Braess *et al.* [1] proved for the first time that equilibrated flux a posteriori error estimates for conforming finite elements do not suffer from increased overestimation for higher polynomial degrees, i.e., that they are robust with respect to the polynomial degree. This result has been extended in [2] to a unified framework covering all conforming, nonconforming, discontinuous Galerkin, and mixed finite element discretizations of the Poisson problem, still in two space dimensions. On each patch of elements sharing the given interior vertex, one solves here a homogeneous local Neumann problem by the mixed finite element method to obtain an equilibrated flux reconstruction in $\mathbf{H}(\mathrm{div}, \Omega)$, as well as a homogeneous local Dirichlet problem by the conforming finite element method to obtain a potential reconstruction in $H_0^1(\Omega)$. We extend here this methodology to three space dimensions. Details are given in [3].

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OPTIMAL GOAL-ORIENTED ADAPTIVITY

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Goal-oriented adaptive methods aim to adaptively approximate output quantities of interest of solutions to PDEs, with the least amount of computational effort. In typical adaptive (FEM or BEM) computations, a "double" rate of convergence (the sum of the primal energy-norm rate and the dual energy-norm rate) is observed with respect to the number of degrees of freedom in the approximation space.

In this contribution we will present an analysis of the convergence of goal-oriented adaptivity in abstract settings (the work of which can be found in [1]), which extends all existing prior results for goal-oriented adaptive FEM and goal-oriented adaptive BEM. The setting allows for any linear problem that complies with the Lax–Milgram Lemma, includes axiomatic adaptive components as in [2], and uses an extension of the marking strategy in the seminal work [3] or the one from [4].

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