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LOCAL ERROR ESTIMATES AND CONVERGENCE OF THE GALERKIN BOUNDARY ELEMENT METHOD ON POLYGONAL DOMAINS

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We consider the local behavior of the Galerkin error of a quasi-uniform boundary element discretization of Symm's equation on polygonal (or polyhedral) Lipschitz domains. It is well-known that the convergence of the global Galerkin error is limited by the regularity of the solution, i.e., singularities (both in the data and geometry) may reduce the global order of convergence. However, on smooth parts of the boundary away from the singularities the behavior of the Galerkin error is much better. For the finite element method this has, e.g., been observed by [1], and for the boundary element method on smooth screens by [2]. In fact, the convergence of the FEM is locally optimal in the energy norm on polygonal domains. For the boundary element method, the local estimates of [2] imply that the local error in the energy norm is at least better than the global error by a factor of square root of the mesh width.

In this talk, we provide local estimates for the L^2- and $H^{-1/2}$-error on a polygonal domain and show that the local error in the L^2-norm converges with the rate of $\mathcal{O}(h^{1/2+\alpha+\alpha_D})$, where α is the global regularity of the solution, and α_D denotes the additional regularity of the dual problem on polygonal domains. The numerical observations also confirm that this rate is optimal. However, the rate of convergence can be improved if the singularities of the data and the dual problem are separated.

References

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SECOND-KIND SINGLE TRACE BOUNDARY INTEGRAL EQUATIONS

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For second-order linear transmission problems involving a single closed interface separating two homogeneous materials, a well-posed second-kind boundary integral formulation has been known for a long time. It arises from a straightforward combination of interior and exterior Calderón identities. Apparently, this simple approach cannot be extended to "composite" settings involving more than two materials.

The key observation is that the same second-kind boundary integral equations (BIE) can also be obtained through a multi-potential representation formula. We can attach a potential to each boundary of a material sub-domain, add them all up to a multi-potential, and then we notice that, thanks to a null-field property, the sum provides a representation of the field solution, when its traces a plugged into the potentials. Taking traces yields a BIE on the skeleton of the sub-domain partition. The skeleton traces of the unknown field will solve it.

Using the fact that multi-potentials for a single homogeneous material must vanish, the BIE can be converted into second-order form: for the scalar case (acoustics) its operator becomes a compact perturbation of the identity in L^2. Galerkin matrices arising from piecewise polynomial Galerkin boundary element (BEM) discretization will be intrinsically well-conditioned.

The new second-kind boundary element method has been implemented both for acoustic and electromagnetic scattering at composite objects. Numerical tests confirm the excellent mesh-size independent conditioning of the Galerkin BEM matrices and the resulting fast convergence of iterative solvers like GMRES. Furthermore, by simple postprocessing, we obtain discrete solutions of competitive accuracy compared to using BEM with the standard first-kind BIE.

Well-posedness of the new second-kind formulations is an open problem, as is the compactness of the modulation of the identity in the case of Maxwell's equations. Reassuringly, computations have never hinted at a lack of stability.

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BEM FOR SOLID MECHANICS WITH DAMAGE AND ITS APPLICATION TO MODELLING COMPOSITE MATERIALS

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For several years, the industry has brought the use of composite materials into focus, e.g. for the construction of wind turbines, aircrafts, and in the automotive industry. There exists a wide variety of possible applications due to the unbeatable advantages over conventional materials such as steel or aluminum; these are mainly the lower weight and an often significantly higher mechanical strength. In contrast to homogeneous materials, the modeling of composites is significantly more complex because of the fine structural features. We use a non linear strain- and stress-based continuum damage model, which was first introduced by Simon and Ju [2], and is well accepted throughout the engineering community [2]. The stress tensor σ is defined by $\sigma(x) = (1 - d(\epsilon, x))\mathbb{C}(x) : \epsilon(x)$, where ϵ is the strain tensor, d the internal damage variable and $\mathbb{C}$ the stiffness tensor. Due to the model we make use of a multi domain Galerkin boundary element method for elasticity [3] coupled with a specific matrix valued radial basis function part to treat the non linear term. To reduce memory requirements of the fully populated matrices, we apply a low rank approximation for the matrices generated by the BEM and RBF parts. The resulting linear system is then solved by the use of specially developed preconditioner technique.

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AN EIGENVALUE ANALYSIS BASED ON CONTOUR INTEGRALS FOR PERIODIC BOUNDARY VALUE PROBLEMS WITH THE BOUNDARY ELEMENT METHOD

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An eigenvalue analysis based on contour integrals for periodic boundary value problems for Helmholtz' equation in 3D with the boundary element method (BEM) is proposed. The Sakurai-Sugiura method (SSM) is one of numerical methods for non-linear eigenvalue problems, which obtains eigenvalues inside a given contour in the complex plane by calculating an integral along the contour. In this paper, we extend integral operators in the BEM to complex phase factor in order to calculate the contour integrals used in the SSM. With the proposed method, we analyse behaviour of reasonance anomalies in some periodic boundary value problems for Helmholtz' equation in 3D.

COMPUTATIONAL ASPECTS OF FAST ADAPTIVE BOUNDARY ELEMENT METHODS

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We will address computational aspects of fast methods in adaptive boundary element methods for 3d computations for the Laplace equation. In the computational examples we will use the (h - h/2)-error estimation strategy [M. Karkulik, G. Of, and D. Praetorius, Convergence of adaptive 3D BEM for weakly singular integral equations based on isotropic mesh-refinement. Numerical Methods for Partial Differential Equations, 29(6):2081-2106, 2013]. An important aspect is the automatic choice of parameters of the Fast Multipole method with respect to error estimation and in adaptive boundary element methods.

OPTIMAL ADDITIVE SCHWARZ PRECONDITIONING FOR THE HP-BEM: THE HYPERSINGULAR INTEGRAL OPERATOR IN 3D

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We consider a discretization of the hypersingular integral operator for the Poisson problem in $\mathbb{R}^3$ using the *hp*-version of the Galerkin boundary element method on a mixed mesh consisting of triangles and quadrilaterals. We propose and analyze a preconditioner based on the overlapping additive Schwarz framework. The underlying decomposition consists of a global block of piecewise linears/bilinears and blocks of higher order polynomials supported on the vertex, edge and element patches. The resulting preconditioned system has a condition number that is uniformly bounded with respect to the mesh size h and the polynomial degree p. We also briefly discuss some options to improve the computational complexity of this preconditioner by replacing the piecewise linears/bilinears with a decomposition of multilevel type and by reducing the higher order block associated with the patches to a finite set of reference configurations.

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MATRIX VALUED ACA FOR HIGH ORDER BEM

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A new variant of the Adaptive Cross Approximation (ACA) for approximation of dense block matrices is presented. This algorithm can be applied to matrices arising from the Boundary Element Methods (BEM) for elliptic or Maxwell systems of partial differential equations. The usual interpolation property of the ACA is generalised for the matrix valued case [1]. Some numerical examples demonstrate the efficiency of the new method. The main example will be the electromagnetic scattering problem, i.e. the exterior boundary value problem for the Maxwell system. Here, we will show that the matrix valued ACA method works well for high order BEM [2] and the corresponding high rate of convergence is preserved. Another example shows the efficiency of the new method in comparison with the standard technique while approximating the smoothed version of the matrix valued fundamental solution of the time harmonic Maxwell system.

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ASYMPTOTIC EXPANSION TECHNIQUES FOR SINGULARLY PERTURBED BOUNDARY INTEGRAL EQUATIONS

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We will consider singularly perturbed elliptic transmission problems in the framework of boundary integral equations and boundary element Galerkin discretisations [1]. For this we demonstrate the use of asymptotic expansion techniques both for establishing regularity results for the solution and for deriving a priori error estimates for boundary element discretisation. The dependence of the corresponding bounds on the singular perturbation parameter is studied in detail. This dependence clearly manifests itself in numerical experiments.

References

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CONVECTION-ADAPTED BEM-BASED FINITE ELEMENT METHOD ON TETRAHEDRAL AND POLYHEDRAL MESHES

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A new discretization method for homogeneous convection-diffusion-reaction boundary value problems in 3D is presented that is a non-standard finite element method with PDE-harmonic shape functions on polyhedral elements, see [1]. The element stiffness matrices are constructed by means of local boundary element techniques. The method, which is referred to as a BEM-based FEM, can therefore be considered a local Trefftz method with element-wise (locally) PDE-harmonic shape functions.

The current research combines the results of [2] with the hierarchical construction of shape functions presented in [3]. The Dirichlet boundary data for these shape functions is chosen according to a convection-adapted procedure which solves projections of the PDE onto the edges and faces of tetrahedral and polyhedral elements, respectively. This improves the stability of the discretization method for convection-dominated problems both when compared to a standard FEM and to previous BEM-based FEM approaches, as we demonstrated in several numerical experiments. Our experiments also show an improved resolution of the exponential layer at the outflow boundary for our proposed method when compared to the SUPG method.

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