MAFELAP 2016

Conference on the Mathematics of Finite Elements and Applications

14–17 June 2016

Mini-Symposium: Boundary-Domain Integral Equations

Organisers:

Sergey Mikhailov and David Natroshvili

Abstracts in alphabetical order

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BOUNDARY-DOMAIN INTEGRAL FORMULATION OF BOUNDARY VALUE PROBLEMS ON SURFACES

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A boundary value problem for the Laplace-Beltrami operator on a smooth two-dimensional surface embedded in $\mathbb{R}^3$ is considered. As in the case of an inhomogeneous heat transfer, a suitable parametrix (Levi function) is found and an integral formulation of the problem is derived. This formulation involves geometrical properties of the surface. Furthermore, besides the usual boundary integrals the integration along the surface is present. The developed approach is also employed to obtain new boundary-domain integral equations for the mean curvature vector.

A numerical method of finding the approximate solution is derived similarly to the corresponding case in $\mathbb{R}^3$. Several key differences and similarities to the popular finite element methods are discussed. Some aspects of implementation are commented on and several numerical examples are presented.

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NUMERICAL SOLUTION OF THE PLANAR DIRICHLET PROBLEM FOR AN ELLIPTIC EQUATION WITH VARIABLE COEFFICIENTS BY AN INTEGRAL EQUATIONS APPROACH

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We present a numerical approximation to the Dirichlet problem for elliptic equations of second-order in divergence form with spacewise dependent coefficients, in planar bounded smooth domains. In the proposed method, the problem is reduced, with the use of the Levi function (parametrix), to a system of domain-boundary integral equations. Making a change of variables involving shrinkage of the boundary curve of the domain, and employing numerical integration, an efficient Nyström scheme is derived for the construction of an approximation of the solution to the obtained system of integral equations. Numerical examples are included showing the feasibility of the proposed approach.

BOUNDARY VALUE PROBLEMS FOR A NONLINEAR BRINKMAN SYSTEM WITH VARIABLE COEFFCIENTS IN SOBOLEV AND BESOV SPACES ON LIPSCHITZ DOMAINS

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We present recent existence and uniqueness results in Sobolev and Besov spaces for boundary value problems of Dirichlet, Neumann or Robin type for a nonlinear Brinkman system with variable coefficients in Lipschitz domains in $\mathbb{R}^3$. Such a PDE system plays a main role in fluid mechanics and porous media. First, we analyze the corresponding boundary value problem for the linear Brinkman system with variable coefficients, and show that this problem is equivalent to a system of Boundary-Domain Integral Equations (BDIEs). Mapping properties of Brinkman Newtonian and layer potential operators are presented in appropriate Sobolev and Besov spaces. We show the existence and uniqueness of the solution of the BDIE system, and accordingly the well-posedness of the boundary value problem for the variable coefficient linear Brinkman system. Then this well-posedness result and a fixed point theorem provide the existence of a solution in L^p-based Sobolev spaces for a boundary value problem corresponding to the variable-coefficient nonlinear Brinkman system. Boundary value problems involving variable coefficient Brinkman and Darcy-Forchheimer-Brinkman systems are also discussed.

A FUNCTIONAL ANALYTIC APPROACH TO THE ANALYSIS OF HOMOGENIZATION PROBLEMS

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This talk is devoted to the homogenization of boundary value problems in a periodically perforated domain by an approach which is alternative to those of asymptotic analysis and of classical homogenization theory.

In particular, we consider a certain linear or nonlinear boundary value problem on a domain with a periodic structure determined by two positive parameters ϵ and δ and we analyze the behaviour of a family of solutions and of the corresponding energy integral as (ϵ, δ) tends to $(0, 0)$.

SEGREGATED BOUNDARY-DOMAIN INTEGRAL EQUATIONS FOR VARIABLE-COEFFICIENT SCALAR BVPS WITH GENERAL DATA

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Segregated direct boundary-domain integral equations (BDIEs) based on a parametrix and associated with the Dirichlet and Neumann boundary value problems for the linear stationary diffusion partial differential equation with a variable coefficient are formulated. The PDE right hand sides belong to the Sobolev space $H^{-1}(\Omega)$ or $\widetilde{H}^{-1}(\Omega)$, when neither classical nor canonical co-normal derivatives are well defined, which complicates the form of the conormal derivative of the third Green identity on the boundary and corresponding boundary-domain integral equations.

Equivalence of the BDIEs to the original BVP, BDIE solvability, solution uniqueness/ non-uniqueness, as well as Fredholm property and invertibility of the BDIE operators are analysed in Sobolev (Bessel potential) spaces. It is shown that the BDIE operators for the Neumann BVP are not invertible, and appropriate finite-dimensional perturbations are constructed leading to invertibility of the perturbed operators. The contribution is based on and develops some results of [1-3].

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NONLINEAR BOUNDARY-DOMAIN INTEGRAL EQUATIONS FOR SCALAR QUASILINEAR ELLIPTIC PDES

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Non-localized and localized nonlinear boundary-domain integral equation (NBDIE) formulations of the Dirichlet, Neumann and Robin boundary value problems are considered for some quasilinear partial differential equations of elliptic type. Using the properties of the non-localized and localized layer and volume potentials it is shown that the NBDIE systems are equivalent to the original classical and/or weak setting of the boundary value problems. For some special type of nonlinearities the corresponding NBDIE systems are analyzed in detail and an iterative method convergence is proved employing the Banach and Leray-Schauder fixed-point theorems. Some applications to the nonlinear problems of mathematical physics are presented.

BOUNDARY DOMAIN INTEGRAL EQUATIONS FOR THE MIXED COMPRESSIBLE STOKES SYSTEM WITH VARIABLE VISCOSITY IN BOUNDED DOMAINS

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The mixed boundary value problem related to the Stokes system is reduced to two different BDIES which are equivalent to the original boundary value problem, see [1, 2, 3]. These Boundary Integral Equation Systems can be expressed in terms of surface and volume parametrix-based potential type operators whose properties are also analysed in appropriate Sobolev spaces. The invertibility and Fredholm properties related to the matrix operator that defines the BDIES are also presented.

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THE FEM FOR COATED PIEZOELECTRIC FIBER IN PIEZOMAGNETIC MATRIX

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From earlier investigations it is well known that some composite materials can provide superior properties compared to their virgin monolithic constituent materials (Ryu et al., 2002). Remarkably larger magnetoelectric (ME) effect is observed in composites as compared to those in either composite constituents (Nan, 1994). The ME effect is intensively studied to utilize it for energy conversion between the magnetic and electric fields and the ME memory elements, smart sensors and transducers (Wang et al., 2005). Pan and Wang (2009) showed larger ME effect in layered composites than in monoliths. Applied magnetic field induces strain in the magnetostrictive constituent of the bilayer mutifferoic composite; this is passed on to the piezoelectric constituent, where it induces an electric polarization. They observed a strong ME effect in artificially fabricated multiferroic composites. The thickness ratio of piezomagnetic and piezoelectric layers has influence on the ME effect (Laletin et al., 2008).

Coating of a base material plays an important role in various engineering applications. Enhancement of ME coupling for fibrous magnetoelectroelastic composites seems to be a great challange for research. The subject of piezoelectric/piezomagnetic fibrous composites with multicoated circular/elliptic fibres is seldomly analyzed in literature. These composites are analyzed only under anti-plane shear deformation (Kuo and Pan 2011). In the present paper influence of coating layer on the ME coefficient in fibrous composites with piezoelectric and piezomagnetic phases is investigated. The effective material parameters are computed on the base of homogenization techniques performed on the RVE. The solution of general boundary value problems for coupled multi-field problems requires advanced numerical methods due to the high mathematical complexity. Such a multi-field problem is described by a system of partial differential equations because of the interactions among the magnetic, electric and mechanical fields involved in the constitutive equations. In this paper the finite element method (FEM) is implemented for investigation of coating layer on effective material properties, particularly the so-called ME effect which is important to optimal design of multiferroics composites.

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