Recap of the terminology

Steady:	$\underline{q} = \underline{q}(\underline{r})$, i.e. no time dependence
Two-dimensional:	$\underline{q} = u(x,y)\,\underline{i} + v(x,y)\,\underline{j}$.
Incompressibility:	$\nabla \cdot \underline{q} = 0$.
Stream function ψ:	$\underline{q} = (\nabla\psi) \times \underline{k}$.
Stagnation points:	$\underline{q}(\underline{r}) = \underline{0}$.
Vorticity:	$\underline{\omega} = \nabla \times \underline{q}$.
Irrotational flow:	$\underline{\omega} = \underline{0}$.

With irrotational flow there exists a velocity potential ϕ such that

$$\underline{q} = \nabla\phi = (\nabla\psi) \times \underline{k}.$$

with

$$\nabla^2\psi = \nabla^2\phi = 0.$$

The velocity in Cartesian and polars coordinates

In Cartesian's $\psi = \psi(x,y)$ and

$$\underline{q} = \frac{\partial\psi}{\partial y}\underline{i} - \frac{\partial\psi}{\partial x}\underline{j}$$

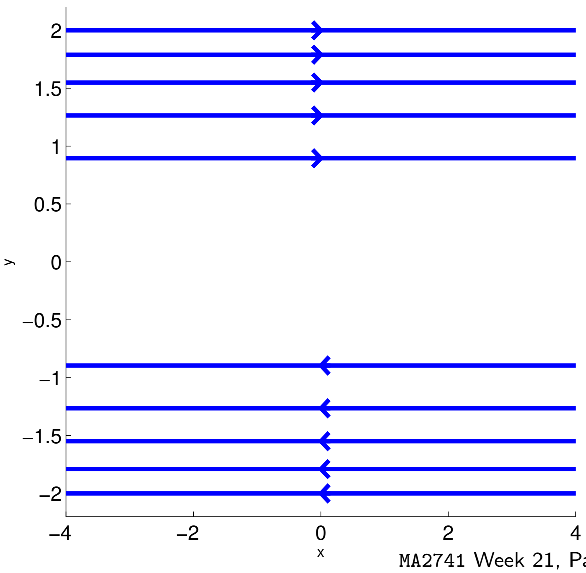
and in polars $\psi = \psi(r,\theta)$ and

$$\underline{q} = \frac{1}{r}\frac{\partial\psi}{\partial\theta}\underline{e}_r - \frac{\partial\psi}{\partial r}\underline{e}_\theta.$$

Curves of the form ψ =const are the streamlines of the flow.

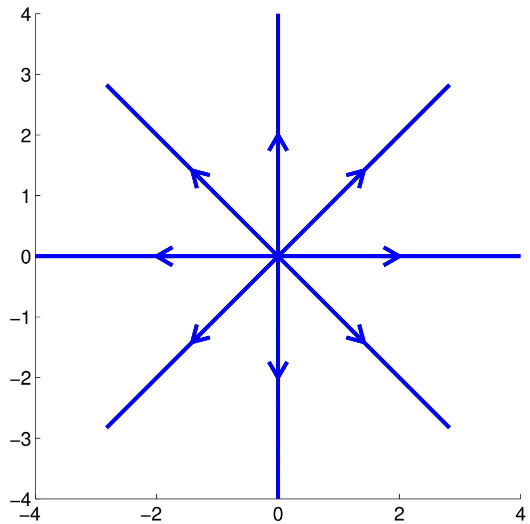
Streamlines for a simple shear flow

$$\psi = \frac{1}{2}\beta y^2 \quad \text{and} \quad \underline{q} = \beta y\,\underline{i}.$$



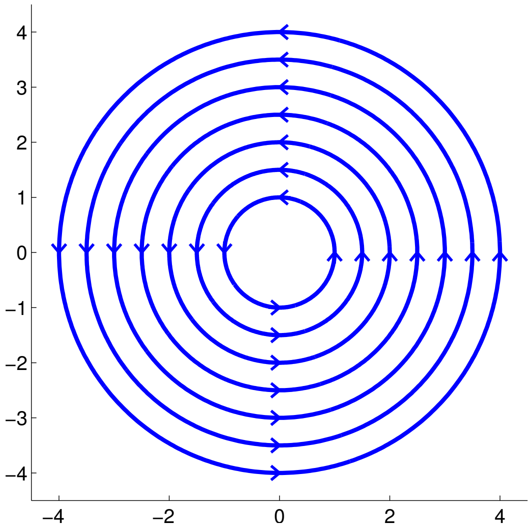
Streamlines for a line source at $r = 0$

$$\psi = A\theta \quad \text{and} \quad \underline{q} = \frac{A}{r}\,\underline{e}_r.$$



Streamlines for a line vortex at $r = 0$

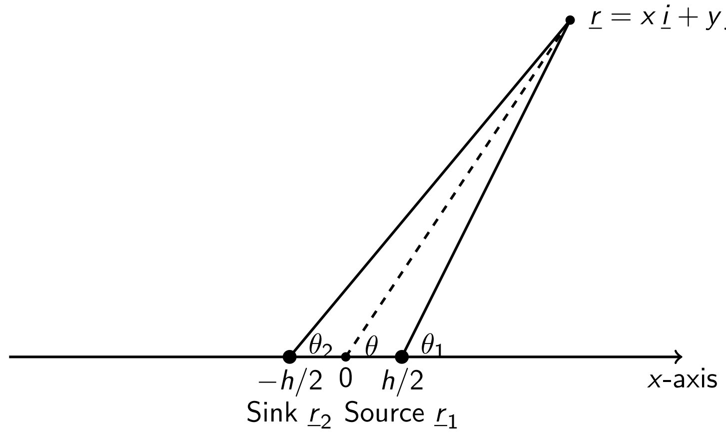
$$\psi(r) = -\frac{\Gamma}{2\pi}\ln\left(\frac{r}{a}\right) \quad \text{and} \quad \underline{q} = \left(\frac{\Gamma}{2\pi}\right)\frac{1}{r}\,\underline{e}_\theta.$$



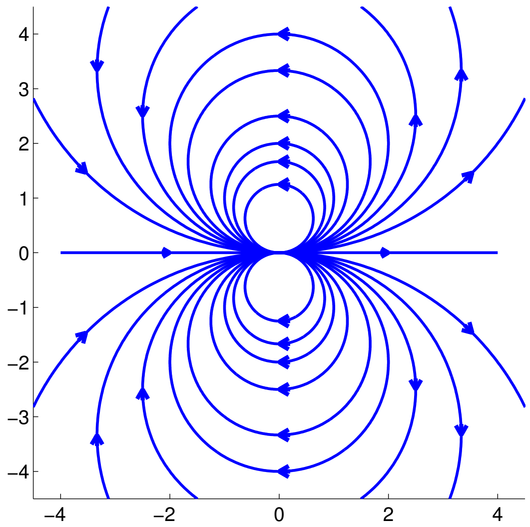
A source and sink close together

If $Ah = \mu$ is constant and $h \to 0$ then we can show that

$$\psi(\underline{r}) = Ah\frac{(\theta_1 - \theta_2)}{h} \to \mu\frac{\sin\,\theta}{r}.$$

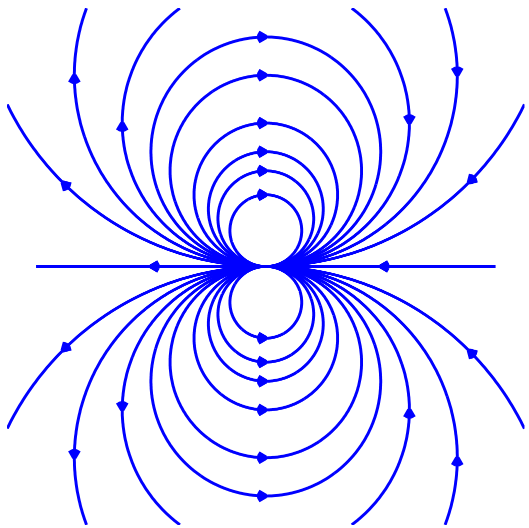


Streamlines for a dipole in the $\underline{i}$ direction



Streamlines for a dipole in the $-\underline{i}$ direction

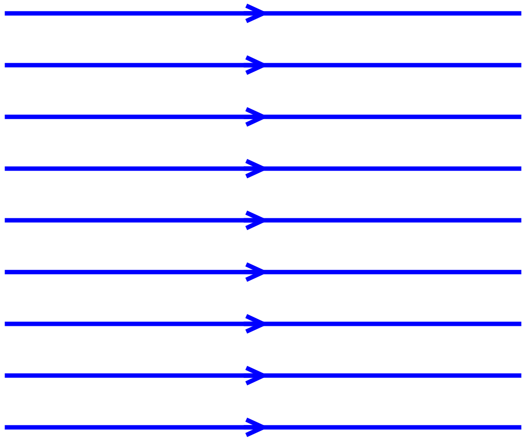
$$\psi = -\mu\frac{\sin\,\theta}{r}$$



This is the same as before with the arrows reversed.

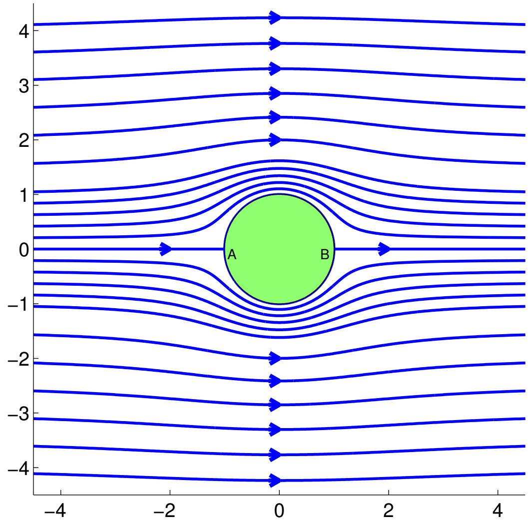
Streamlines for uniform flow in the $\underline{i}$ direction

$$\psi = Ur \sin \theta$$



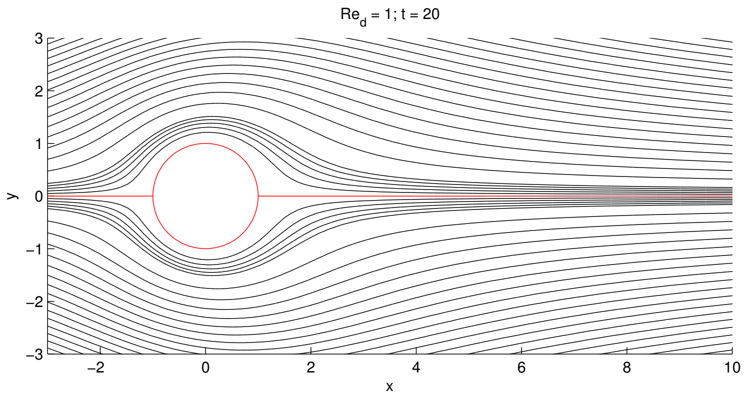
Streamlines for flow round a cylinder

$$\psi = U\left(r - \frac{a^2}{r}\right) \sin \theta$$



Flow around a cylinder – Reynold's number, Re=1

When we have some viscosity we get a similar pattern.



Flow around a cylinder – Re=10

When the viscous effects increase flow differs from the no viscosity case close to the cylinder.