Key identities for later in the module

When we describe incompressible flow: $\nabla \cdot (\nabla \times \underline{F}) = 0$
When we describe irrotational flow: $\nabla \times \nabla \phi = \underline{0}$

Let $\underline{q}$ denote velocity of a fluid flow.

$$\nabla \left(\frac{1}{2} |\underline{q}|^2 \right) - \underline{q} \times (\nabla \times \underline{q}) = (\underline{q} \cdot \nabla) \underline{q}.$$

A bit messy to verify and will appear a few times to justify the inclusion of a few things. The right hand side will appear in the material time derivative expression. It is needed in a further study of fluids.

An incompressible flow is when $\nabla \cdot \underline{q} = 0$ and this will lead to the representation of the velocity using a stream function in the 2D case.

An irrotational flow is when $\nabla \times \underline{q} = \underline{0}$ and this will lead to existence of a velocity potential.

∇ in cartesian coordinates

$$\nabla = \underline{i} \frac{\partial}{\partial x} + \underline{j} \frac{\partial}{\partial y} + \underline{k} \frac{\partial}{\partial z}.$$

∇ in polar coordinates

Base vectors $\underline{e}_r$, $\underline{e}_\theta$ and $\underline{k}$,

$$\begin{aligned} \underline{e}_r &= \cos\theta \underline{i} + \sin\theta \underline{j}, \\ \underline{e}_\theta &= -\sin\theta \underline{i} + \cos\theta \underline{j} \end{aligned}$$

The base vectors vary with θ with

$$\frac{\partial}{\partial \theta} \underline{e}_r = \underline{e}_\theta \quad \text{and} \quad \frac{\partial}{\partial \theta} \underline{e}_\theta = -\underline{e}_r.$$

$$\nabla = \underline{e}_r \frac{\partial}{\partial r} + \frac{1}{r} \underline{e}_\theta \frac{\partial}{\partial \theta} + \underline{k} \frac{\partial}{\partial z}.$$

The following are available on formula sheets

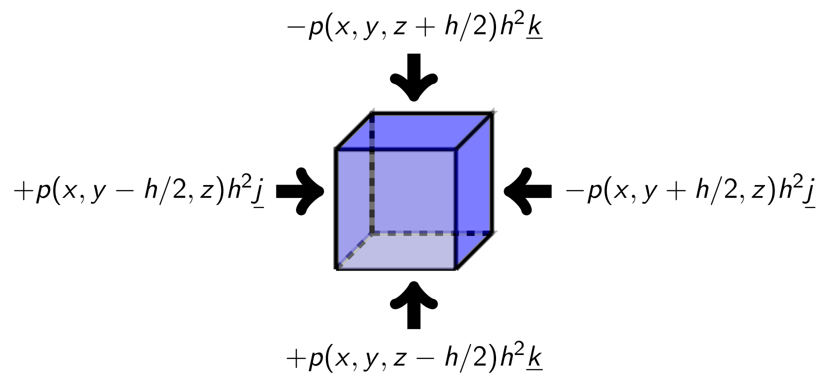
$$\nabla \phi = \frac{\partial \phi}{\partial r} \underline{e}_r + \frac{1}{r} \frac{\partial \phi}{\partial \theta} \underline{e}_\theta + \frac{\partial \phi}{\partial z} \underline{k}.$$

$$\nabla \cdot \underline{F} = \frac{1}{r} \frac{\partial}{\partial r} (r F_r) + \frac{1}{r} \frac{\partial F_\theta}{\partial \theta} + \frac{\partial F_3}{\partial z}.$$

$$\nabla \times \underline{F} = \frac{1}{r} \begin{vmatrix} \underline{e}_r & r\underline{e}_\theta & \underline{k} \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial z} \\ F_r & rF_\theta & F_3 \end{vmatrix}.$$

Pressure in an inviscid fluid

and any fluid in equilibrium



The force on a surface due the pressure is always in the direction of the normal to the surface.