BRUNEL UNIVERSITY

DEPARTMENT OF MATHEMATICS AND STATISTICS

BACHELOR DEGREE EXAMINATIONS

January 1997

MA155A Introduction to Computing

Time Allowed: 2 hours

All questions carry equal marks.
Full marks can be obtained for satisfactory answers to **FIVE** *questions.*
If more than **FIVE** *questions are attempted then only the best* **FIVE** *solutions will be counted.*
In some questions you are asked to give Fortran statements. When you give these statements we will assume that you are putting any statement labels in columns 1 to 5 with the main part of the statement being in columns 7 to 72 as is required in standard Fortran 77.

1. (i) Convert the following into Fortran assignment statements. In your answer you can use one statement or several statements.

(a)

$$y = 1 + x + x^4 - \frac{29}{x^7}$$

(b)

$$z = \left(\frac{\cos(a) - \cos(b)}{\sin(a) + \sin(b)} \right) \mathrm{e}^{3b}$$

(c)

$$d = \left(\frac{3+w}{2+w} \right)^2 - \left(\frac{2+w}{1+w} \right)^2 .$$

For the above `SIN(X)`, `COS(X)` and `EXP(X)` give respectively the values of the intrinsic Fortran functions sine, cosine and exponential at the real point `X`.

(ii) Convert the following Fortran statements into algebraic form.

(a) `Y=1+X/2+3`
(b) `Z=A*(1-R**N)/(1-R)`
(c) `V=EXP(COS(B)-4*TAN(B**2))`
(d) `ANGLE=ACOS((A*A+B*B-C*C)/(2*A*B))`

For the above `EXP(X)`, `TAN(X)` and `ACOS(X)` are respectively the values of the intrinsic Fortran functions exponential, tangent and inverse cosine at the real point `X`.

(iii) Give Fortran statements to compute the sum

$$s = \sum_{n=1}^{1000} \frac{1}{n^2} = 1 + \frac{1}{2^2} + \cdots + \frac{1}{1000^2} .$$

(iv) Give Fortran statements to compute the product

$$s = \prod_{n=1}^{100} \left(1 + \frac{1}{n^2}\right) = (1+1)\left(1 + \frac{1}{2^2}\right) \cdots \left(1 + \frac{1}{100^2}\right) .$$

2. What will be the output to the screen as a result of the following sets of assignment statements.

(i)

```
      A=5.3
      B=3.2
      C=A
      A=B
      B=C
      WRITE(*,*)A,B
      END
```

(ii)

```
      WRITE(*,*)((I*J,J=1,2),I=1,4)
      END
```

(iii)

```
      I=1
      A=I/3
      B=I/3.0
      C=A-B
C
C ::: output a,b and c with 3 digits after the decimal point
      WRITE(*,'(F10.3)')A,B,C
      END
```

(iv)

```
      X=1.0
      H=0.1
      V=(F(X+H)-F(X-H))/(2*H)
      WRITE(*,*)V
      END
      FUNCTION F(X)
      F=X**2
      WRITE(*,*)X
      END
```

(v)

```
      DO 10 I=0,3
      X=0.5*I
      CALL SUB(X,Y,YD)
      WRITE(*,*)X,Y,YD
   10 CONTINUE
      END
      SUBROUTINE SUB(T,Y,YD)
      IF(T.GT.0.75)THEN
      Y=2-T
      YD=-1
      ELSE
      Y=T*T
      YD=2*T
      ENDIF
      END
```

3. Suppose that the first few statements in a program are

```
PARAMETER(N=100)
REAL X(N)
OPEN(10,FILE='marks.dat')
READ(10,*)(X(I),I=1,N)
CLOSE(10)
```

and these are successful in reading 100 values into the array elements `X(1)`, `X(2)`, $\cdots$, `X(100)`.

(i) Give statements which will determine the mean of `X(1)`, `X(2)`, $\cdots$, `X(100)`.

(ii) If `XBAR` and `STDEV` are real variables which store respectively the mean and the standard deviation respectively then give statements which determine the number of the values `X(1)`, `X(2)`, $\cdots$, `X(100)` which are in the interval

[`XBAR-STDEV,XBAR+STDEV`]

(iii) If `XBAR` is a real variable storing the mean, as in part (ii), then give statements which will determine an integer `J` such that the absolute difference `ABS(X(J)-XBAR)` is the smallest of all the 100 differences

`ABS(X(I)-XBAR)`, `I`$= 1, 2, \cdots, 100$.

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4. Suppose that the first few statements in a program are

```
PARAMETER(N=100)
REAL X(N)
OPEN(10,FILE='marks.dat')
READ(10,*)(X(I),I=1,N)
CLOSE(10)
```

and these are successful in reading 100 values into the array elements `X(1)`, `X(2)`, $\cdots$, `X(100)`.

(i) Give statements which will determine the position of the largest of the 100 entries and the position of the smallest of the 100 entries.

(ii) If `IBIG` and `ISMALL` are integer variables storing the position of the largest and the smallest of the entries `X(1)`, `X(2)`, $\cdots$, `X(100)`, with `IBIG`$\neq$`ISMALL`, then give statements which will swap `X(1)` and `X(IBIG)` and give statements which will swap `X(100)` and `X(ISMALL)`. Then give statements which will determine the 2nd largest and the 2nd smallest values.

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5. (i) If a, b, and c are real numbers such that $a \neq 0$ and $b^2 - 4ac \geq 0$ then the quadratic equation

$$ax^2 + bx + c = 0$$

has real roots at

$$x_1 = \frac{-b + \sqrt{b^2 - 4ac}}{2a} \quad \text{and} \quad x_2 = \frac{-b - \sqrt{b^2 - 4ac}}{2a} .$$

Assuming that real variables `A`, `B` and `C` corresponding respectively to a, b and c have already been assigned values, then give statements which will do the following depending on the contents of `A`, `B` and `C`.

(a) Compute and output the two roots if $|a| \geq 10^{-5}$ and $b^2 - 4ac \geq 0$.

(b) Output the character string `'a is too small'` if $|a| < 10^{-5}$.

(c) Output the character string `'The roots are complex'` if $|a| \geq 10^{-5}$ and $b^2 - 4ac < 0$.

For the above you should note that `SQRT` and `ABS` are the Fortran intrinsic square root and absolute value functions.

(ii) Some Fortran compilers warn you about unreliable results if you use statements of the form

```
D=B**2-4*A*C
IF(D.EQ.0)THEN
X1=-B/(2*A)
WRITE(*,*)'double root at x1=',X1
ENDIF
```

where in the above `D` is a real variable. What is the problem and why do you not get the warning with such statements if `D` is of type integer ?

6. Suppose that a function subprogram named `F` has already been successfully created corresponding to the mathematical function $f(x)$ and suppose that `F` has the skeleton form

```
FUNCTION F(X)
REAL F,X
...
F=...
END
```

(i) If f is continuous, $a < b$ and $f(a)f(b) < 0$ then the function f has a root between a and b. The bisection method is a method for numerically determining the root which involves successively computing

$$c = (a+b)/2 \quad \text{and} \quad f(c)$$

and then replacing $[a, b]$ by $[a, c]$ if $f(a)f(c) \leq 0$ and by replacing $[a, b]$ by $[c, b]$ otherwise. Give statements which implements this procedure 10 times.

(ii) Newton's method for numerically solving $f(x) = 0$ involves choosing a starting point x_0 and successively generating

$$x_{k+1} = x_k - \frac{f(x_k)}{f'(x_k)}, \quad k = 1, 2, \cdots .$$

A modified form of the method is obtained by replacing $f'(x_k)$ by

$$\frac{f(x_k + h) - f(x_k)}{h}$$

giving instead the iteration

$$x_{k+1} = x_k - \frac{hf(x_k)}{f(x_k + h) - f(x_k)} \quad k = 1, 2, \cdots .$$

If the variables `X0` and `H` contain suitable values corresponding to x_0 and h respectively then give statements which will generate x_1, x_2, $\cdots$, x_6.

7. The screen of a personal computer in VGA mode has 640 pixels in the horizontal direction and 480 pixels in the vertical direction. Corresponding to this suppose that a program begins with the declaration

```
INTEGER VGA(640,480),COLOUR(16)
```

and suppose that there have been statements which assigns one of the 16 integers $1, 2, \cdots, 16$ to each of the entries `VGA(I,J)`, $1 \le$ `I` ≤ 640, $1 \le$ `J` ≤ 480 to correspond to the colour of the pixel.

(i) Give statements which will determine the 16 entries of the array `COLOUR` so that `COLOUR(K)` contains the number of pixels which have the colour corresponding to the integer `K`.

(ii) An interior pixel on the screen identified by its `(I,J)` position, $2 \le$ `I` ≤ 639, $2 \le$ `J` ≤ 479 is next to the pixels corresponding to the positions

`(I-1,J)`, `(I+1,J)`, `(I,J-1)` and `(I,J+1)` .

Give statements which will determine the number of pixels whose colour is the same as that of the four neighbouring pixels.